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An Improved Method Combining Beamforming and Acoustical Holography for the Reconstruction of the Sound Pressure on Structure Surface

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Summary

With the increasing velocity of the aerospace vehicle, researchers are increasingly concerned with the study on thermal-acoustic fatigue of the vehicle structure. The distribution and absolute values of the sound loads on the structure surface are essential to build the mechanism model of and the fatigue for the safety evaluation of structural failure. The thermal-acoustic fatigue experiments are, however, conducted in high temperature environment, where the microphone array cannot be placed to acquire the near-field data and the array configuration is limited. On the other hand, by using the measurement in the far-field, only the distribution of the sound loads on the structure surface can be examined. In this paper, an improved method combining beamforming and the acoustical holography to reconstruct the sound pressure value in the sound field is studied. With the proposed method, traditional beamforming is firstly replaced by the Distance-Weighted beamforming to achieve the sound field reconstruction from the far-field to the near-field. The output data from the Distance-Weighted beamforming serves as the data of the virtual hologram surface, which are then used to achieve the reconstruction from the near field to the surface of the sound source by acoustical holography. The method can be used to reconstruct the sound field in the high temperature environment by using the distribution and the value of the acoustic loads on the surface. Extensive sound field reconstruction experiments in free field and common indoor space have been conducted in different sound source conditions, such as different sound pressure amplitude, frequency and frequency band width. The feasibility of this method is verified. The method proposed has advantages in practical application such as feasibility in severe environment, low numbers of microphones, practical measurement in the far field surface, accurate reconstruction of the sound pressure distribution and the amplitude on the surface of sound source.

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1. Introduction

In recent years, the velocity of a near-space vehicle has been improved continuously. Meanwhile it faces an extremely harsh environment during the flight such as aerodynamic heating, noise etc. As the high sound intensity noise and the temperature made by the propulsion system or boundary layer can reach up to 180 dB and 2000 K respectively, an acoustic fatigue crack can easily develop and extend in the weak parts of the aerospace vehicle, which leads to failure of the vehicle, causing fatal accidents such as the fracture of the load-carrying structure [1, 2, 3]. For the reasons above, the influence of the thermal and acoustic compound environment to the vehicle and the feasibility

of the structure must be taken into account in the vehicle structure design. In order to find out the mechanism of the thermal-acoustic fatigue failure, it is necessary to reconstruct the acoustic load field on the surface of a component in an thermal-acoustic fatigue failure environment by acquiring the distribution and amplitude of the acoustic, and then to build the acoustic fatigue mechanism model by combining the acoustic field with fatigue degrees.

In 1990s, some institutions such as NASA [4], US Air Force, IABG (Germany), etc conducted material and structure fatigue failure experiments in harsh thermal and acoustic compound environment. However, the acoustic load was only served as one of the applied loads when testing structure performance. Studies on acoustic fatigue mechanism were not carried out. After the year 2000, NGLT, NASA [5] put forward the next generation launch technology and conducted an amount of thermal amplitude and thermal acoustic experiments in a succession of

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high temperature structural components such as wings and airfoils made of C/C, Si/C, etc. In June 2006, US Air Force [6] used a microphone array with 395 channels to measure the acoustic field of the engine. With the beamforming technology, the noise distribution of the jet flame from the booster engine of the vehicle is accurately obtained. However, the real amplitude of the acoustic loads was not measured and the fatigue mechanism in the region of acoustic load concentration could not be further analysed. In 2008, researchers from NASA Langley [7] conducted a number of thermal-acoustic coupling experiments on ruddervator components made of C/C or Si/C in a reverberation chamber to assess its amplitude index. However, they only used one microphone to measure the average sound pressure in the reverberation chamber as the estimated acoustic load, and the acoustic field was not reconstructed. In 2012, NASA Langley [8] acquired a relatively ideal noise distribution result under high temperature conditions with a random microphone array one metre away from the turbulence thermal noise in the far field, which was produced when the missile jetted. However, the real noise amplitude was not reconstructed to research the acoustic fatigue mechanism.

In 2011, to deal with the high temperature environment in thermal-acoustic experiments, Christian O. Paschereit from TU Berlin and Ingo Röhle from DLR [9] invented an optical microphone based on the theory of optical interference. The microphone could resist high temperature and could be used to measure the sound signal in a combustor. But it didn't have a stable frequency response because of the material limitation of the microphone diaphragm. Thus it can be seen that in spite of the high cost, the technology of the heat-resisting device such as the optical microphone used in the thermal-acoustic fatigue experiments is not matured enough. One of the present feasible methods to reconstruct the sound field in thermal acoustic environment is measuring the sound signal by normal microphone array and improving the existing acoustic field reconstruction, and then reconstructing the information of acoustic load distribution and amplitude accurately by using a thinned microphone array at the far field. This method proved capable of offering the data support to evaluate fatigue failure mechanism.

At present, the main acoustic field reconstruction methods based on the microphone array measurement are beamforming and Near-field acoustic holography.

The increasing maturity of beamforming technology occurred in 1960s [10], when beamforming was widely used in communications fields. And at the end of 1970s, it began to be used in sound source location and acoustic field reconstruction. In 1979, a method of accessing the Direction of Arrival (DOA) of plane sound wave based on eigenvector was proposed by Reddi [11], which indicates that beamforming began to be used in plane sound wave calculation study. Yet, this method was only applied to the measurement in far field. In 1982, Owsley *et al.* [12] studied the spherical sound wave beamforming, which could be applied to the measurement and calculation in the near

field. In 1987, Brooks *et al.* [13] from NASA reconstructed the distribution of the fluid noise source made by helicopter model in a wind tunnel with frequency-domain beamforming. It was also the first time to conduct the aeroacoustics measurement by beamforming. In 1997, in order to optimize the method, M. Viberg *et al.* [14] proposed the Weighted Subspace Fitting to improve the spatial resolution of beamforming. In the same year, Piet *et al.* [15] from ONERA France introduced an optimized microphone array, which used 39 microphones to set up a cross shaped microphone array. And with this configuration, the noise source of an Airbus in a wind tunnel was located and analyzed by beamforming. And the definition of beamforming was improved by this method. In 2004, B&K Denmark [16] efficiently used a random microphone array to reconstruct and identify the noise source of large scale equipment based on beamforming algorithm. In general, the precision of sound source location and identification by beamforming has increasingly improved in recent years. However, for the difference of application background and purpose, the beamforming is mainly applied to the location and identification of sound sources. The reconstruction of the real sound pressure of the sound source still requires further research.

Near-field acoustic holography (NAH) is the optimal method for reconstructing acoustic load distribution and sound pressure. With this method, not only the sound pressure amplitude but also the vibration velocity information at each point in the acoustic field can be reconstructed. The space transformation of sound field (STSF) was firstly proposed by E. G. Williams [17]. As no approximation in calculation is involved in its deduction process, this method possesses a high calculation precision and is still one of the most feasible methods in solving the plane acoustic field problem to this day. In 1992, R. B. Mingsian [18] introduced the Boundary Element Method (BEM) to NAH, which can eliminate the request for the shape of the reconstruction surface and enable any shape of the sound source to be reconstructed by NAH. The difficulty in overcoming the NAH problem by BEM was solved using the NAH based on superposition of waves proposed by Sarkissian [19] in 1994 and the NAH based on Helmholtz equation -least method (HELs) proposed by Wang [20] in 1997. In 2003, E. G. Williams [21] improved the traditional STSF again and proposed the Patch-NAH method. With this method, the acoustic field of the large-scale radiating body could be reconstructed locally, which reduces the measurement difficulty of STSF to a certain degree. Generally speaking, the applied range of Near-field acoustic holography is becoming wider and wider. However, there are still two limitations of this method. One is the requirement for adequate microphones and the other is that the measurement microphones must be placed within a half-wave distance from the sound source, which cannot meet the requirement of the thermal-acoustic fatigue experiments.

In thermal-acoustic fatigue experiments, which are conducted in high sound intensity and temperature environ-

ments, the mainly nonlinear effects are the large amplitude sound wave (also called finite amplitude sound wave) and the large temperature gradient. When the amplitude of sound wave is large enough, the sine wave may be distorted to a sawtooth wave, which is called harmonic wave generating [22, 23], and the amplitude of the sound wave in the sound field does not linearly increase with the sound source power, which is called acoustic saturation [24, 25, 26]. The environment of high temperature and large temperature gradient brings sound ray deviation, which is another nonlinear effect different from the acoustic saturation and harmonic wave generation. The large temperature gradient may cause uneven density of air. The sound ray deviates in the uneven medium based on the refraction rules [27]. In this paper, the proposed method is discussed based on linear acoustics. We only did some static stimulant experiments in ordinary sound intensity and temperature environments for verifying the linear theory of the proposed method. If the linear theory of the proposed method proves correct and useful, we will carry out the experiments next in the a sound intensity and temperature environment and study the nonlinear effects of the proposed method.

In this paper, an acoustic field amplitude reconstruction method combining Distance-Weighted beamforming and the STSF NAH is proposed. Firstly, traditional beamforming is replaced by Distance-Weighted beamforming, which is to achieve the sound field reconstruction from the far-field to the near-field. Then the output data from the Distance-Weighted beamforming is used to complete the reconstruction from the near field to the surface of the sound source by STSF NAH. The acoustic load distribution and amplitude can be acquired with this method, which can solve the difficulty of acoustic field reconstruction in high temperature environment. The flow chart is shown in Figure 1.

The proposed method has two main advantages. The first one is the microphone array for the measurement is placed in the far field, with the combination of Distance-Weighted beamforming and the Near Field Acoustical Holography. This can help deal with the problem that the acoustic load distribution and amplitude information on the material or structure surface cannot be achieved due to the high temperature in the near field with common microphones. The other one is that a relatively precise acoustic field reconstruction can be acquired by the thinned microphone array, which is capable of resolving the problem of the shape configuration limitation of the microphone array.

2. Distance-weighted beamforming

Distance-Weighted beamforming was proposed in 2000 by H. Kook *et al.* [28] and widely used [29, 30] in recent years. But the traditional Distance-Weighted methods are different from our method. The traditional distance weight is just the distance r_i between i th microphone and focus point, and the distance is only used to eliminate the attenuation due to spherical spreading in order to keep the

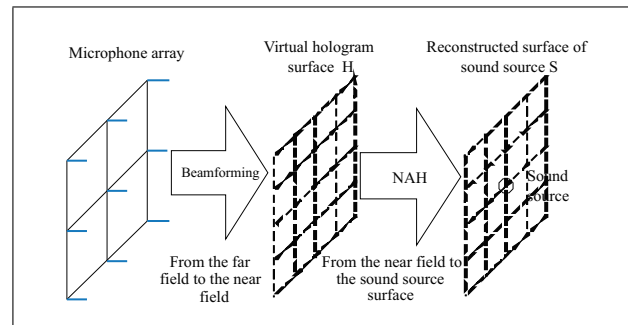


Figure 1. The flow chart of the proposed method.

amplitude of beamforming output constant in any position of array, but not to reconstruct the real sound pressure of reconstructed surface. Some advanced processing methods may be used for the traditional distance weight r_i , such as Maximum Likelihood Estimation in [28], in order to suppress sidelobes but not to reconstruct the real sound pressure of the reconstructed surface as well. In our paper, we have a different purpose to other researchers. We used the distance weight r_f/r_i to reconstruct the real sound pressure of the reconstructed surface, where r_f is the distance from the sound source to the reference microphone and r_i is the distance from the present reconstructed point to the sound source. According to the theory of the acoustic attenuation, there is an equation $P_a/P_b = r_b/r_a$ between any two points a and b in sound field, where P is the sound pressure of the point and r is the distance from the point to the sound source. r_b/r_a , which is the scale factor of sound pressure between point a and b , can be used to solve P_a or P_b . If one of P_a and P_b is known, the other one can be solved by r_b/r_a . Therefore, we can use r_f/r_i to calculate the sound pressure of the i th reconstructed point when the sound pressure acquired by the reference microphone has already known. It can be seen the proposed method is an extension of the traditional distance-weighted beamforming.

2.1. Theoretical background

The core of the beamforming method is beam response, which can be regarded as the bond to establish the one-to-one mapping relationship between the input signals and output ones. The beam response usually consists of two parts, which are array manifold vector and weight vector. Take a one-dimensional linear array, for example, to explain the theory of beamforming, the incidence of the sound wave is shown as Figure 2.

The expression of the array manifold vector $\mathbf{a}$ is

$$\mathbf{a}(\theta) = [e^{-j\psi_1(\theta)}, \dots, e^{-j\psi_m(\theta)}, \dots, e^{-j\psi_M(\theta)}]. \quad (1)$$

In this expression, θ stands for the incidence angle of the sound source whose range is $(-\pi/2, \pi/2)$, M is the quantity of microphones in the array and $\psi_m(\theta)$ is the phase difference of the incoming sound wave between the microphone number m and the reference microphone. $\mathbf{a}(\theta)$ is the set of the phase differences of the receiving sound

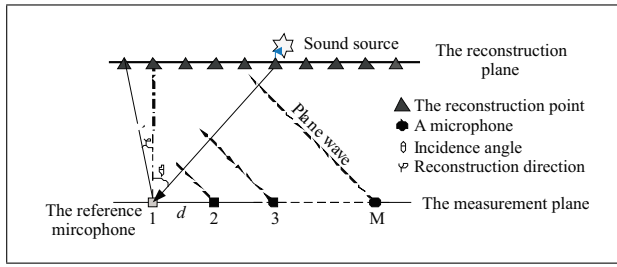


Figure 2. The sketch map of sound wave incidence.

signal between each microphone and the reference microphone in the array, which reflects the property of the array shape.

The weight vector $\mathbf{w}(\varphi)$ is expressed as

$$\mathbf{w}(\varphi) = \frac{e^{-j\psi_1(\theta)}, \dots, e^{-j\psi_m(\theta)}, \dots, e^{-j\psi_M(\theta)}}{M}. \quad (2)$$

In the expression above, φ stands for the reconstruction direction, or the focusing angle, and $\psi_m(\varphi)$ indicates the phase difference between the microphone number m and the inference microphone when the reconstruction direction is φ . The response intensity of the current reconstruction direction is strengthened by the weight vector $\mathbf{w}(\varphi)$ and that of the other directions is weakened as well, forming the spacial directive property.

The beam response expression is

$$q(\varphi, \theta) = \mathbf{w}(\varphi) \cdot \mathbf{a}(\theta). \quad (3)$$

The physical implication of the beam response is the response to a unit power signal of the plane wave in the microphone array. Substituting equation (1) and (2) to equation (3), the beam response is translated into equation (4),

$$q(\varphi, \theta) = \frac{\sum_{m=1}^M e^{j[\psi_m(\varphi) - \psi_m(\theta)]}}{M}. \quad (4)$$

Supposing that θ_0 is the incidence direction of the sound source and φ_i is the reconstruction direction, the relationship between the input and the output in beamforming can be expressed as

$$y_i(t) = q(\varphi_i, \theta_0)x(t), \quad (5)$$

where $x(t)$ is the receiving signal of the microphone array from the direction θ_0 , and $y_i(t)$ is the acoustic signal at the reconstruction direction φ_i whose amplitude only reflects the relative intensity of the signals from different reconstruction directions.

In practical applications of the beamforming method, the objective space is scanned by changing the reconstruction direction φ . And then the incidence direction of the sound source is located according to the relative value of $y(t)$. Thus it can be seen that the traditional beamforming method is mainly aiming at identifying of a sound source. However, it is difficult for the method to acquire the accurate value of acoustic load and amplitude information. Accordingly, it is necessary to improve the beamforming and the theory of the improvement is acoustic attenuation theory.

2.2. The theory of acoustic attenuation

As a kind of mechanical wave, a sound wave propagates by the alternate sparsity and denseness of the acoustic medium. In microcosm, the mutual transformation between the kinetic energy and the potential energy happens constantly during the propagation of the sound wave. At a macroscopic level, for a short propagation distance of the sound, the quantity of the energy transforming to internal energy is so low that the energy loss can be neglected. Thus the process of sound wave propagation can be seen as a conservation of mechanical energy. In this situation, the acoustic attenuation is not caused by energy loss in the free field. In fact, the total energy in the wave surface is invariant while the surface of the wave surface becomes bigger and bigger. Consequently the energy distribution becomes sparser and sparser, leading to the sound pressure attenuation. Theoretically, only the ideal plane wave does not diffuse when propagating. However there are few kinds of ideal plane wave in existence. The sound wave of almost all sound sources will diffuse when propagating, that is to say the surface of the wave surface becomes bigger and bigger.

Based on the conservation of mechanical energy, at any moment the acoustical power at the same wave surface can be described as

$$W_1 = W_2. \quad (6)$$

The calculation of the acoustical power is given as

$$W = 4\pi r^2 I, \quad (7)$$

where r is the radius of the wave surface and its unit is m and I is the sound intensity, whose unit is W/m^2 and it can be written as

$$I = \frac{P^2}{\rho c}. \quad (8)$$

Here, P is the sound pressure, ρ is the density of the air and c is the sound velocity. In a stable environment, ρ and c are both constants.

After putting equations (7) and (8) into equation (6), we acquire

$$4\pi r_1^2 \frac{P_1^2}{\rho c} = 4\pi r_2^2 \frac{P_2^2}{\rho c}. \quad (9)$$

After simplification of this equation, the relationship between the sound pressure P at any point of the spacial acoustic field and the distance to the sound source r can be acquired as

$$P_1 r_1 = P_2 r_2 = \dots = P_i r_i = z, \quad (10)$$

where z is a constant only relating to the acoustical power of the sound source.

From the above equation, the farther the sound source is, the more acoustical attenuation will occur. Based on the calculation formula of the sound pressure level, it can

be estimated that if the propagation distance doubles, the sound pressure level will increase 6.02 dB. In the common beamforming method, because the beam response $q(\varphi, \theta)$ does not exclude the compensation of the acoustic attenuation, the acquired reconstruction of sound pressure nephogram only includes the relative intensity of the sound pressure, without the real sound pressure value of the reconstruction surface. There is no influence for the general sound source location and identification. For the acoustic fatigue mechanism research of the material or structure, however, it is difficult to establish the fatigue mechanism model only using the relative intensity of the acoustic load. Therefore it is necessary to introduce the acoustical attenuation compensation weight, making the calculation result on the reconstruction surface become the real sound pressure value.

2.3. Distance-weighted beamforming method

Equation (10) shows the relationship between the sound pressure and the distance from the sound source at any time. Converted to the explicit expression about the time t , equation (10) can be expressed as

$$P_1(t_0)r_1 = P_2\left(t_0 + \frac{r_2 - r_1}{c}\right)r_2. \quad (11)$$

The physical meaning of equation (11) can be stated as follows. On one of the wave surfaces in space, the sound wave propagates to point 1 at some time t_0 and to point 2 at $(t_0 + (r_2 - r_1)/c)$, where $(r_2 - r_1)/c$ stands for the time delay between the two points caused by their distance. If the sound pressure P_1 at Point 1, the distance r_1 from the sound source to Point 1 and the distance r_2 from the sound source to Point 2 are known, the sound pressure P_2 at Point 2 will be acquired by equation (11).

For a continuum acoustic field during a period of time, it is made up of quite a few numbers of sound surfaces in essence, thus equation (11) can be extended to the application to a continuum acoustic field. Let $x(t)$ and $y(t)$ are the steady-state time domain waveform at Point 1 and Point 2, respectively, then equation (11) can be expressed as

$$x(t)\frac{r_1}{r_2} = y\left(t + \frac{r_2 - r_1}{c}\right). \quad (12)$$

Thus if the steady-state time domain waveform $x(t)$ at Point 1, the distance r_1 and the distance r_2 are known, the steady-state time domain waveform $y(t + (r_2 - r_1)/c)$ at Point 2 can be reconstructed. Because in common indoor measurements, the duration of propagation time delay $(r_2 - r_1)/c$ is only a few milliseconds which can be neglected, the signals at these two points are regarded as synchronous signals. Thus based on equation (12), equation (5) is added to distance weight r_f/r_i as

$$y_{iH}(t) = q(\varphi_i, \theta_0)\left[x(t)\frac{r_f}{r_i}\right] = \left[q(\varphi_i, \theta_0)x(t)\right]\frac{r_f}{r_i}. \quad (13)$$

In equation (13) the input signal $x(t)$ is the steady-state time domain waveform at the reference point, $y_{iH}(t)$ is

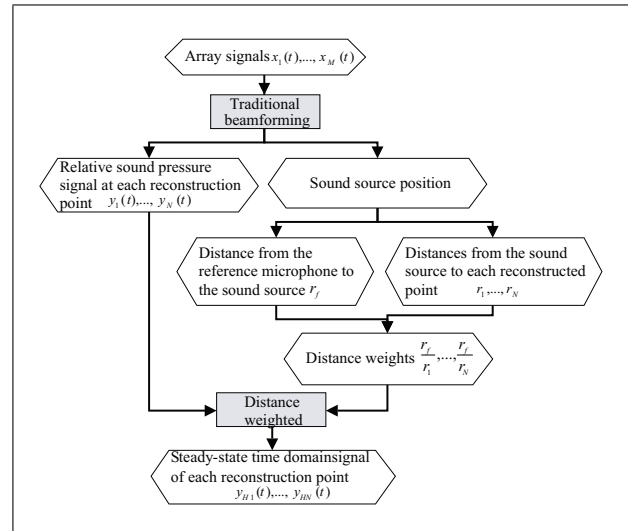


Figure 3. The flow path of the Distance-Weighted beamforming.

the steady-state time domain waveform at the i th point on the reconstruction plane, r_f is the distance from the sound source to the reference microphone and r_i is the distance from the present reconstruction point to the sound source. Based on the equation (4) of the beam response q , only the phase change is calculated while the sound pressure amplitude is unchangeable. Therefore if the $x(t)$, r_i and r_f are known, the steady-state time domain waveform $y_{iH}(t)$ can be achieved by equation (13).

The steady-state time domain waveform $x(t)$ at the reference point can be acquired by the reference microphone. And based on equation (13), whether adding the weight to input signal $x(t)$ or conducting the conventional beamforming first, both results are absolutely equivalent. Thus the conventional beamforming can be conducted first in order to acquire the spatial position of the sound source. Now that the spatial positions of the reference microphone and each point on the reconstruction plane are given information, the distances r_f and r_i can be easily solved using the above information. Then putting $x(t)$, r_i and r_f to equation (13), the steady-state time domain waveform $y_{iH}(t)$ can be reconstructed.

The flow chart of the Distance-Weighted beamforming is shown as Figure 3. In the practical engineering measurement, the following requirement of the Distance-Weighted beamforming should be noted.

1. The real sound source usually has directivity, however for the semi-space of the sound source direction or within limits of radiation angle of the sound source, the acoustical field can be regarded as uniform field.
2. Choosing a different microphone as the new reference microphone, the value of r_f will change as well as the amplitude of the corresponding actual measurement signal $x_f(t)$. But according to equation (10), it can be seen that the product of the sound pressure amplitude at any moment and r_f at the corresponding position is a constant. As a result the choice of the reference microphone does not affect the amplitude reconstruction.

3. In theoretical derivation, the calculation of r_f and r_i are both based on the hypothesis of the point acoustic source. Thus the distance in the calculation is the length between the unknown point and the sound source. However, in practical applications the calculated distance will not be neglected if the volume of the sound source is large, generating a large deviation based on this hypothesis. If the size of the sound source approximates the calculation distance or is even larger, the scatter degree of the sound wave will be deemed to be small and the acoustic field can be regarded as the plane wave model. Thus the distance between the surface of the sound source and the plane containing the calculation point can be used instead.
4. From equation (10), it can be seen that if r_i approaches zero, that is to say, near the surface of the sound source plane, the value of P will approach infinity, and the acoustic attenuation formula will become ineffectual, which leads to the failure of Distance-Weighted beamforming. As a result, the precise acoustic field reconstruction at the plane near the surface of the sound source cannot be obtained only by the method of Distance-Weighted beamforming. The combination of Distance-Weighted beamforming and the NAH is needed here.

3. NAH with virtual holographic data

Due to the limitations of the acoustic attenuation formula, the sound source surface could not be reconstructed by the Distance-Weighted beamforming.

In this section, the STSF NAH is selected as the near-field resolution, which is a classical and effective method of NAH. Plane sound sources or the sound sources which can be expanded into the plane sound sources can be accurately reconstructed by the STSF NAH.

3.1. Theoretical background

The STSF NAH is based on the wave equation

$$\nabla^2 p(x, y, z, t) - \frac{1}{c^2} \frac{\partial^2 p(x, y, z, t)}{\partial t^2} = 0, \quad (14)$$

where the coordinate (x, y, z) is the location of real space, t is the time, p is the sound pressure at point (x, y, z) , c is the sound velocity, and ∇ is the Laplace operator. For a stable or quasi-stable sound field, the Helmholtz equation, obtained from Fourier transformation of the wave equation, can solve any point's amplitude and velocity of vibration. The solving process can be simplified by the Helmholtz equation which does not depend on the time. The Helmholtz equation is given as

$$\nabla^2 p(x, y, z) + k^2 p(x, y, z, t) = 0, \quad (15)$$

with fixed $k = 2\pi f/c$, which is wave number, and f is the frequency of sound. The Helmholtz equation can be

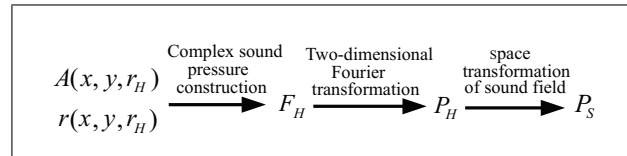


Figure 4. The flow path of STSF NAH.

solved by the two-dimensional Fourier transformation and the Green function. Its solution is given as

$$P_S(k_x, k_y, r_S) = P_H(k_x, k_y, r_H) / e^{jk_z(r_H - r_S)}. \quad (16)$$

Equation (16) represents the transformation of k space from the holographic surface H to the reconstructed surface S . P_H is the data of k space which is obtained from the Fourier transformation of the complex sound pressure on the holographic surface; P_S is the data of k space which is obtained from the Fourier transformation of the complex sound pressure on the reconstructed surface; $e^{jk_z(r_H - r_S)}$ is the G operator which is the parameter of the transformation between the holographic surface and the reconstructed surface; r_H is the distance between the holographic surface and the source surface; r_S is the distance between the reconstructed surface and the source surface. When the reconstructed surface is the source surface, $r_S = 0$. However, due to the vibration amplitude of the sound source, r_S should be a little greater than the vibration amplitude. The coordinate (k_x, k_y, k_z) is the location of k space which represents the part of wave number along the (x, y, z) direction. k_z is expressed as

$$k_z = \begin{cases} \sqrt{k^2 - (k_x^2 + k_y^2)}, & k_x^2 + k_y^2 \leq k^2, \\ j\sqrt{(k_x^2 + k_y^2) - k^2}, & k_x^2 + k_y^2 > k^2. \end{cases} \quad (17)$$

The complex sound pressure matrix F_H of the holographic surface H is given as

$$F_H(x, y, r_H, k) = A(x, y, r_H) e^{jkr(x, y, r_H)}, \quad (18)$$

where the coordinate (x, y) is the location of the surface H ; $A(x, y, r_H)$ is the sound pressure value of point (x, y) . $r(x, y, r_H)$ is the distance between the point (x, y) and the sound source.

Based on equation (18), if the complex sound pressure F_H on the holographic surface H is known, the complex sound pressure on any surface between the holographic surface and the source surface could be acquired by P_H which is obtained from the two-dimensional Fourier transformation of F_H . The flow of STSF NAH method is shown as Figure 4.

In traditional applications, F_H could be measured by a microphone array placed on the holographic surface H . In order to decrease deviation, the distance between the holographic surface H and the reconstructed surface S should be less than half of the sound wavelength. Thermal-acoustic fatigue experiments are, however, conducted in high temperature environment, where the microphone array cannot be placed to acquire the near-field data. Therefore, an improved method to obtain F_H should be studied.

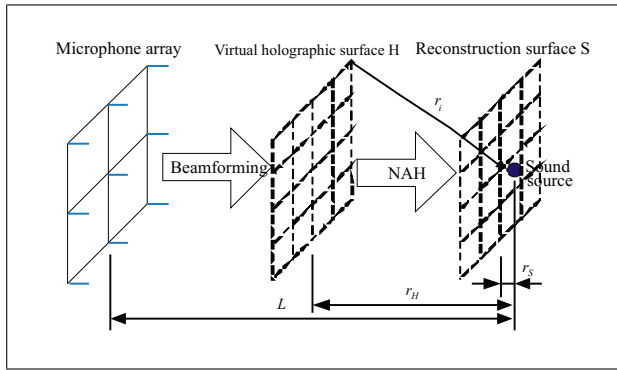


Figure 5. The sketch map of the geometrical relationship of the surfaces and the points in space.

According to equation (18), under the situation that the frequency of sound source has been already known, the unknown parameters contain $A(x, y, r_H)$ and $r(x, y, r_H)$. That is to say if the sound pressure values of the surface H and the position of sound source are known, F_H can be acquired, and then a virtual holographic surface can be constructed as the input data to STSF NAH method.

3.2. The STSF NAH based on the virtual holographic surface

The results obtained from the Distance-Weighted beamforming are the sound pressure values of the surface H and the position of the sound source, which can be used to construct an integrated virtual holographic surface as the input data to STSF NAH method.

Firstly, the stable time domain signals $[y_{H1}(t), \dots, y_{HN}(t)]$ of N points on the holographic surface H can be achieved by Distance-Weighted Beamforming. And then the effective sound pressure values $(A_{H1}, \dots, A_{HN})$ can be acquired by $[y_{H1}(t), \dots, y_{HN}(t)]$, which are treated as the amplitude of complex sound pressure on the surface H . Thirdly, with the position of sound source and the geometrical relationship between the points in space, the distances $(r_{H1}(t), \dots, r_{HN}(t))$ between the points on surface H and the sound source can be acquired. The geometrical relationship of the surfaces and the points in space is shown as Figure 5, where L is the distance between the microphone array surface and the source surface, r_H is the distance between the virtual holographic surface H and the reconstructed surface S , and r_i is the distance between the point i on surface H and the sound source.

Substituting $(A_{H1}, \dots, A_{HN})$ and $(r_{H1}(t), \dots, r_{HN}(t))$ into equation (18), the complex sound pressure matrix F_H of the virtual holographic surface H could be acquired.

$$F_H(k) = [A_{H1}e^{-jkr_{H1}}, \dots, A_{HN}e^{-jkr_{HN}}]. \quad (19)$$

Then the data $P_H(k_x, k_y, r_H)$ of virtual holographic surface H in k space could be achieved by the two-dimensional Fourier transformation of $F_H(k)$.

Substituting $P_H(k_x, k_y, r_H)$ into equation (16), the data $P_S(k_x, k_y, r_S)$ of reconstructed surface S in k space could be acquired. It should be noted that r_S should have been

equaling zero. Due to the vibration amplitude of the sound source, however, r_S should be a little greater than the vibration amplitude. According to the vibration amplitude of the actual sound source, r_S is 5 mm in this paper.

Finally, the complex sound pressure matrix F_S of the reconstructed surface S can be acquired by two-dimensional inverse Fourier transformation of $P_S(k_x, k_y, r_S)$.

$$F_S(k) = [A_{S1}e^{-jkr_{S1}}, \dots, A_{SN}e^{-jkr_{SN}}]. \quad (20)$$

The amplitudes $(A_{S1}, \dots, A_{SN})$ of F_S are the end results.

The flow of STSF NAH method based on the virtual holographic surface is shown in Figure 6. Since the amplitudes $(A_{S1}, \dots, A_{SN})$ contain the information of location coordinates (x, y, R_s) in real space, they can be used to plot the reconstructed nephogram of sound source surface.

From the reconstructed nephogram, both the distribution and the value of the acoustic load on the sound source surface could be acquired, and then the mechanism model of the relationship between the acoustic loads and the level of fatigue can be resolved, which can be used for the safety evaluation of structural failure.

4. Discussion of key parameters

In order to get accurate acoustic loads, two key parameters need to be further discussed.

The first one is the filter parameter. There is a difference between the distribution of complex sound pressure on the virtual holographic surface acquired by beamforming and the real distribution. Since the difference influenced by sound frequency could lead noise into K space, it is necessary to study the relationship between the filter parameter and sound frequency in order to filter the noise of different frequency out exactly and reduce the deviation of calculation.

Secondly, the procedure of dividing wideband noise into narrowband noise is also necessary to study. NAH is the method which is only applied to single frequency wave. Therefore, the wideband noise needs to be divided into narrowband noise for using NAH method.

4.1. Filtering in k space

In this paper, the process of reconstruction has two steps. Firstly, use the Distance-Weighted beamforming method to achieve the virtual holographic surface which is the input data of STSF NAH method. Secondly, use STSF NAH method to acquire the distribution and the value of the acoustic load on the sound source surface. It is obvious that the result of NAH is influenced by the result of beamforming. The key parameter is the width of main lobe of beamforming. It influences the result of the beamforming directly, and then has an effect on the result of NAH indirectly. The width of main lobe bw is given as

$$BW = \arcsin \left[\sin \varphi + \frac{c}{Df(1 + M/(M-1))} \right] - \arcsin \left[\sin \varphi - \frac{c}{Df(1 + M/(M-1))} \right], \quad (21)$$

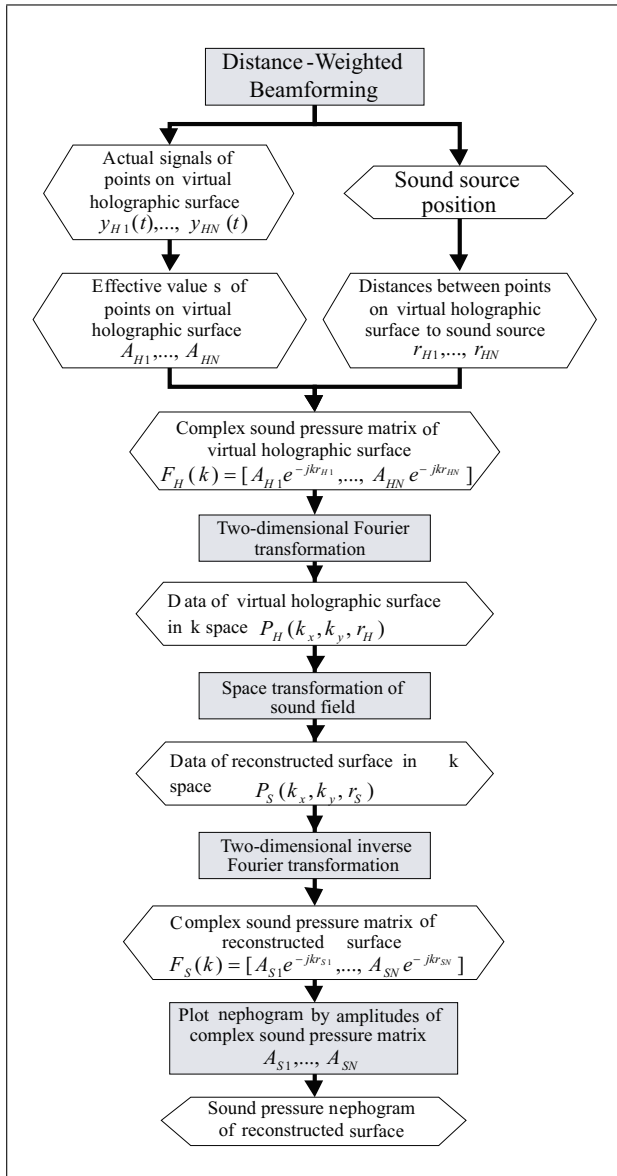


Figure 6. The flow path of STSF NAH method based on the virtual holographic surface.

where φ is focus angle, D is the maximum dimension of microphone array, M is the count of microphones, and f is sound frequency. According to the equation (21), BW is inversely proportional to f . Because of the theory of beamforming, the incident signal of sound source could get the maximum response value in the area of BW . The wider BW is, the larger the area of the maximum response becomes, and then the larger the area of sound source image becomes. Therefore, BW is directly proportional to the area of sound source imaging, and then f is inversely proportional to the area of sound source image.

For the reason above, there is an unavoidable deviation between the distributed area of complex sound pressure on the virtual holographic surface which is obtained from beamforming and the real one. It is more serious that the deviation could be amplified in the process of transformation from P_H to P_S . In high k space, $k_x^2 + k_y^2 \gg k^2$, and

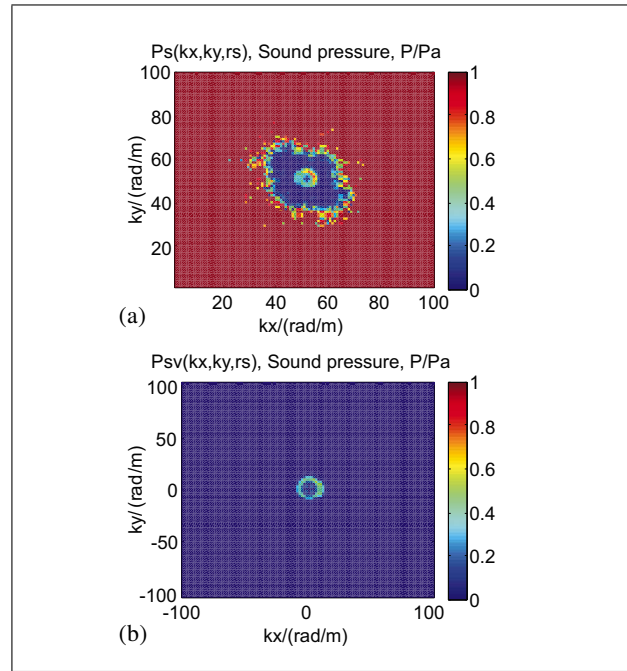


Figure 7. The comparison of P_S on the reconstructed surface S noises. (a) The modulus of P_S of actual experiment, (b) The modulus of P_{SV} data in space of simulation.

based on equation (17), $k_z = j\sqrt{(k_x^2 + k_y^2) - k^2}$. Substituting k_z into the G operator,

$$e^{jk_z(r_H - r_S)} = e^{-(r_H - r_S)\sqrt{(k_x^2 + k_y^2) - k^2}}. \quad (22)$$

It can be seen from equation (22), when $k_x^2 + k_y^2 \gg k^2$, the exponent part trends to $-\infty$, and then the G operator tends to infinitesimal. Due to the G operator is denominator of equation (16), the deviation of P_H could be amplified exponentially in the process of transformation from P_H to P_S . The larger k space is, the larger the deviation is amplified.

Figure 7 illustrates the comparison between the experimental (a) and the simulative (b) P_S on the reconstructed surface S in the case of 1000 Hz sound source. Figure 7a shows the modulus of P_S in k space, which is calculated from experimental sound signal acquired by 3×3 grid-style array in anechoic chamber. Figure 7b shows the modulus of P_{SV} in k space, which is calculated from a simulative sound source. The parameters of the simulative sound source, such as frequency, meshing of reconstructed surface, are the same to the real experiment.

From the comparison, it can be seen that the central circular zone is effective energy. Theoretically, the energy outside the central circular zone should be 0 or a minimum value as shown in Figure 7b. Since the deviation exists and is amplified, however, the amplitude of the noise outside the central circular zone increases exponentially as shown in Figure 7a. The noise makes the effective energy of central circular zone very weak and brings the large deviation into reconstructed result.

To filter the noise out, the following filter function [31] is used:

$$W_{k_r} = \begin{cases} 1 - \frac{e^{(k_r/k_c-1)/\alpha}}{2}, & k_r \leq k_c, \\ \frac{e^{(1-k_r/k_c)/\alpha}}{2}, & k_r > k_c, \end{cases} \quad (23)$$

where $k_r = \sqrt{k_x^2 + k_y^2}$, k_c is cutoff wave number, which expresses a circle with diameter of k_c centered at the origin. α is steepness parameter. The smaller α is, the steeper filter function is. In this paper, $\alpha=0.2$ in order to make filter function steep to filter the noise effectively.

As already discussed, using the result of beamforming as the input data of NAH could lead to deviation of calculation unavoidably. And the deviation could be amplified exponentially in high k space. That is why the k_c should be chosen appropriately in order to filter the noise out. According to the study of beamforming, the deviation changes with sound frequency f , the expression of k_c should be a function of f in this paper.

In this section, the relationship between k_c and f is studied in the view of energy. After the Fourier transformation of F_H , the area S_E of central circular zone in k space expresses the value of effective energy E . The equation (24) shows that S_E is directly proportional to E ,

$$S_E \propto E. \quad (24)$$

If the sound waves have the same amplitude, the vibrational displacements of microphone diaphragms made by sound waves are the same as well. However, the higher the sound frequency is, the more times the vibrations of microphone diaphragm happen. For this reason, it could be considered that the energy E which is generated by the sound wave acting on the microphone diaphragm is directly proportional to f ,

$$E \propto f. \quad (25)$$

Based on equations (24) and (25), S_E is directly proportional to f ,

$$S_E \propto f, \quad (26)$$

where $S_E = 4\pi k_c^2$. Substituting S_E into equation (26),

$$k_c^2 \propto f. \quad (27)$$

Because k_c and f are both positive numbers, equation (27) can be changed to

$$k_c \propto \sqrt{f}. \quad (28)$$

Based on k_c acquired by equation (28) and the filter function shown as equation (23), the noise of P_S existed outside the radius circle with diameter of k_c can be filtered out. If the k_c is just equal to the radius of central circular zone, the noise can be filtered out without any loss of effective energy.

Incidentally, the applicable frequency range of the proposed method and equation (21) in this section are also related. In this paper, NAH is not limited by the microphone size because the virtual microphone on the virtual holographic surface is only a point which does not occupy space. That is to say, the applicable frequency range of the proposed method mainly depends on beamforming. According to equation (21) and some experience of sampling, the applicable frequency range of beamforming is depends on the array size, reconstruction surface size and sampling frequency. A larger array, a larger reconstruction surface or a higher sampling frequency can obtain a wider applicable frequency range.

4.2. Wideband dividing method for wideband noise field solution

Due to the proposed method is largely used in the reconstruction of wideband noise field, a method solving wideband noise needs to be further studied, which is based on the method of single frequency and wideband dividing.

If there are M microphones in the array, M wideband signals will be acquired at the same time. The M wideband signals can construct a signal matrix $\mathbf{X}_{M \times l}[\vec{x}_1, \dots, \vec{x}_M]^T$, where l is the length of each signal. Firstly, with a suitable dividing bandwidth b , each signal $\vec{x}_i$ could be divided into N narrowband signals $[\vec{x}_i^1, \dots, \vec{x}_i^N]$ by the Butterworth Filter. Secondly, the narrowband signals in the same narrowband could construct a new narrowband signal matrix $\mathbf{X}^n = [\vec{x}_1^n, \dots, \vec{x}_M^n]$, where $n = 1, 2, \dots, N$. $\mathbf{X}^n$ expresses the n th narrowband signal matrix which has M microphones' signals. After the process of wideband dividing, each $\mathbf{X}^n$ could be treated as single frequency signal matrix approximately. The smaller the dividing bandwidth b is, the more similar the narrowband signal divided out is to the single frequency signal, and then the smaller the deviation of approximate calculation is.

From the calculation of N narrowband signal matrixes $\mathbf{X}^1, \dots, \mathbf{X}^N$ with the proposed method, N complex sound pressure matrixes $\mathbf{F}_S(k_1), \dots, \mathbf{F}_S(k_N)$ could be acquired, where k_n corresponds to the center frequency of the n th narrowband. Each calculated frequency is the center frequency of the calculated narrowband. $\mathbf{F}_S(k_n)$ expresses the amplitude and phase of stable time-domain sine wave field with the wave number k_n . According to the addition rule of sine wave, the addition between the sine wave fields of different frequency is not equivalent to the addition between amplitude and phase of these sine wave fields. Therefore, because of the difference between the frequencies of $\mathbf{X}^1 \dots \mathbf{X}^N$, the matrixes $\mathbf{X}^1 \dots \mathbf{X}^N$ should not be added to each other directly. For respecting the addition rule of sine wave, it is necessary to expand $\mathbf{F}_S(k_n)$ to stable sine wave field with the amplitude and phase of $\mathbf{F}_S(k_n)$, and then add all sine wave fields together.

The sine wave should be constructed as

$$y(t) = A_{\max} \sin(2\pi ft + \varphi), \quad (29)$$

where f is the frequency of sine wave, equaling to the center frequency of each narrowband. $A_{\max}$ and φ are the amplitude and phase of sine wave, which need to be solved.

The complex sound pressure expression of the i th reconstructed point in the complex sound pressure matrix $F_S(k_n)$ is $A_{ni}e^{-jk_n r_i}$. A_{ni} is effective value of time-domain signal. The relationship between the peak value $A_{\max,ni}$ and the effective value A_{ni} of sine signal is shown as

$$A_{\max,ni} = \sqrt{2}A_{ni}. \quad (30)$$

Assume the initial phase of sound source is equal to 0. The phase of the point where is r away from the sound source is given as

$$\varphi = kr. \quad (31)$$

Based on the position of sound source and the geometrical relationship of the points in space as shown in Figure 4, the distance r_{ni} between the i th reconstructed point and the sound source could be acquired, and then the phase φ_{ni} of the i th reconstructed point in the n th narrowband could be achieved:

$$\varphi_{ni} = k_n r_{ni}. \quad (32)$$

Substituting equations (30) and (32) into equation (29), the time-domain wave signal of the i th reconstructed point on source surface in the n th narrowband can be acquired as

$$y_i(t) = \sqrt{2}A_{ni} \sin(2\pi f_n t + k_n r_{ni}). \quad (33)$$

At last, the time-domain wave signal of the i th reconstructed point on source surface in the wideband can be calculated with

$$y_i(t) = \sum_{n=1}^N y_{ni}(t). \quad (34)$$

After the calculation of all reconstructed points on source surface, reconstructed time-domain signals $y_1(t), \dots, y_G(t)$ of all points on source surface can be acquired, where G is the count of all reconstructed points on source surface. The effective values $A_{S1}, \dots, A_{SG}$ of the signals $y_1(t), \dots, y_G(t)$ could be used to plot the reconstructed nephogram. From the reconstructed nephogram, both the distribution and the value of the acoustic load on the sound source surface could be obtained, and then the mechanism model of the relationship between the acoustic loads and the level of fatigue can be developed, which offers the data support to the safety evaluation of structural failure.

The flow of wideband sound field reconstruction based on wideband dividing is shown as Figure 8.

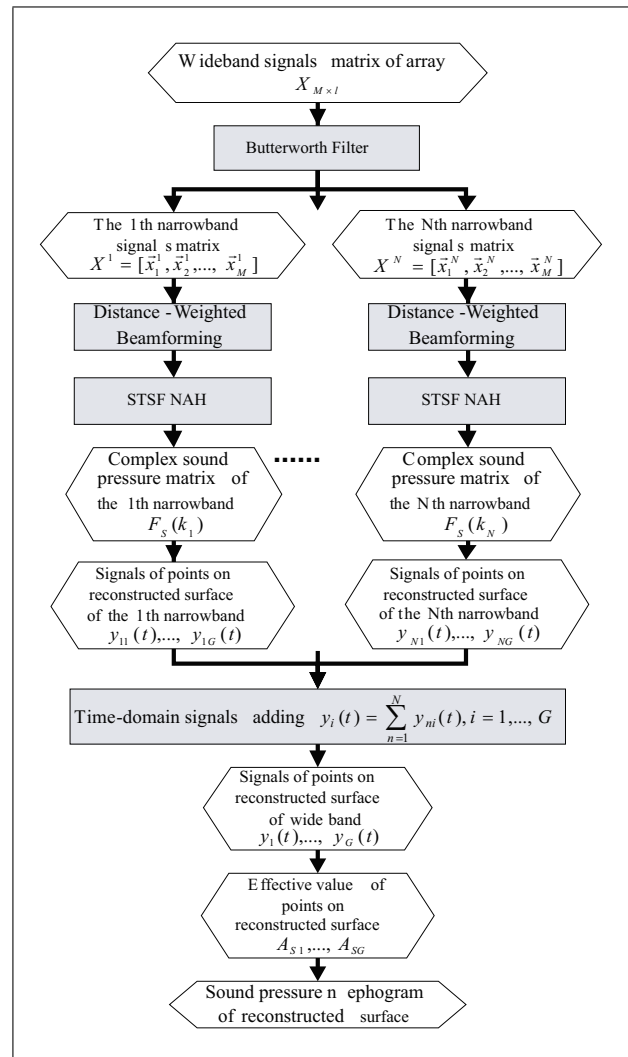


Figure 8. The flow of wideband sound field reconstruction based on wideband dividing.

5. Experimental validation

The experimental validation of the proposed method has been carried out to verify the accuracy of the reconstructed results which contain both the distribution and the value of the acoustic load on the sound source surface. In order to investigate the performance of the proposed method in different conditions of sound source, the experiments have been divided into the single frequency sound source experiment and the wideband sound source experiment.

5.1. Experimental environment

The experiments of single frequency sound source were conducted in anechoic chamber. The size of the anechoic chamber was 6.8 m × 4.2 m × 4.2 m. The sound absorption coefficient of the anechoic chamber is 0.99, so the anechoic chamber is free-field environment.

The experimental setup uses a spherical loudspeaker with the diameter of 0.12 m. The sine and Gaussian wideband sound signal are generated by the computer and played by the loudspeaker.

Table I. The experimental value results of cutoff wave number k_c for different frequencies f .

f [Hz]	k_c [rad/m]
900	25.1
1100	26.6
1300	27.1
1500	29.5
1700	32.7
1900	34.3

The 3×3 grid-style microphone array is built by MPA201 free-field microphones made by the BSWA Technology Company of China. The microphone spacing is 0.28 m. The sound signal acquisition device is built by PXIe bus device made by the NI Company, which is a synchronous multi-channel device. The sampling frequency is 22050 Hz.

The position the of experimental devices in space is as follows: the sound source is placed to the centre of the array with the distance of 1 m to the array; for acquiring the real sound signal, the NO.10 microphone is placed to the centre of the sound source with the distance of 0.005 m to the sound source; the reconstructed surface is set to the centre of the array with the distance of 0.005 m to the sound source, and its size is $1\text{ m} \times 1\text{ m}$ which is meshed to $100 \times 100 = 10000$ reconstructed points with the microphone spacing of 0.01 m; the virtual holographic surface is set to the center of the array with the distance of 0.06 m to the sound source, and its size and meshing are the same to the reconstructed surface

The photo of the single frequency experiment is shown as Figure 9a, and the positions of the experimental devices are shown as Figure 9b.

5.2. The solution of k_c in signal filtering

As already discussed in section 4.1, the input data of the STSF NAH method, acquired by beamforming method, can bring serious noise into high k space in the STSF NAH process, and then bring a large deviation into reconstructed results. That is why the k_c should be chosen appropriately to filter the noise out in k space without any loss of effective energy.

The sound source frequencies are 900 Hz, 1100 Hz, 1300 Hz, 1500 Hz, 1700 Hz, and 1900 Hz. The iteration calculation continues with the step length of 0.1 rad/m until the deviation between the reconstructed sound pressure value and the real sound pressure value of the sound source position reaches the minimum value, and then record the value of k_c . Based on the processes above, we can achieve an optimal value of k_c for each sound source frequency which are shown in Table I.

Based on equation (28), the expression of k_c is shown as

$$k_c = u\sqrt{f}. \quad (35)$$

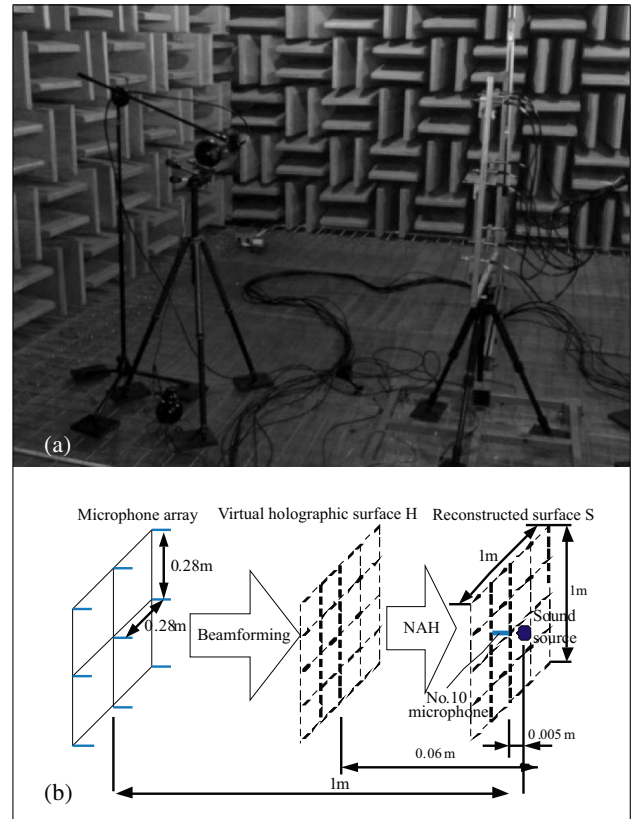


Figure 9. The photo and sketch map of single frequency experiments. (a) The photo of single frequency experiments, (b) The sketch map of the experimental devices positional relationship.

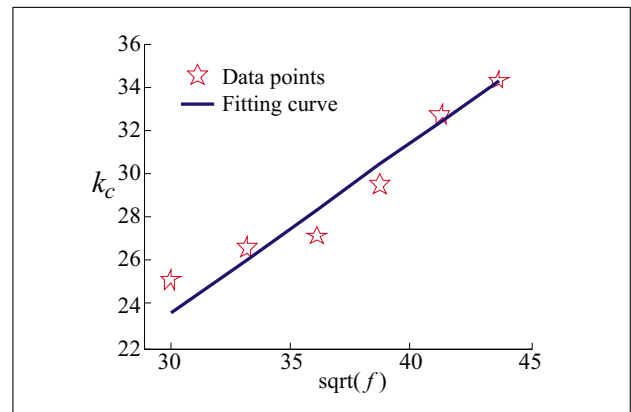


Figure 10. The straight line of linear fitting.

With the data in Table I, the coefficient u can be solved by the linear fitting of equation (35). The straight line of linear fitting is shown in Figure 10.

It could be solved with $u = 0.79$. Substituting the value of u into equation (35), the expression of k_c becomes

$$k_c = 0.79\sqrt{f}. \quad (36)$$

With the filtering parameter k_c achieved by equation (36), the noise in k space could be filtered out without any loss of effective energy, and the deviation could be decreased as well.

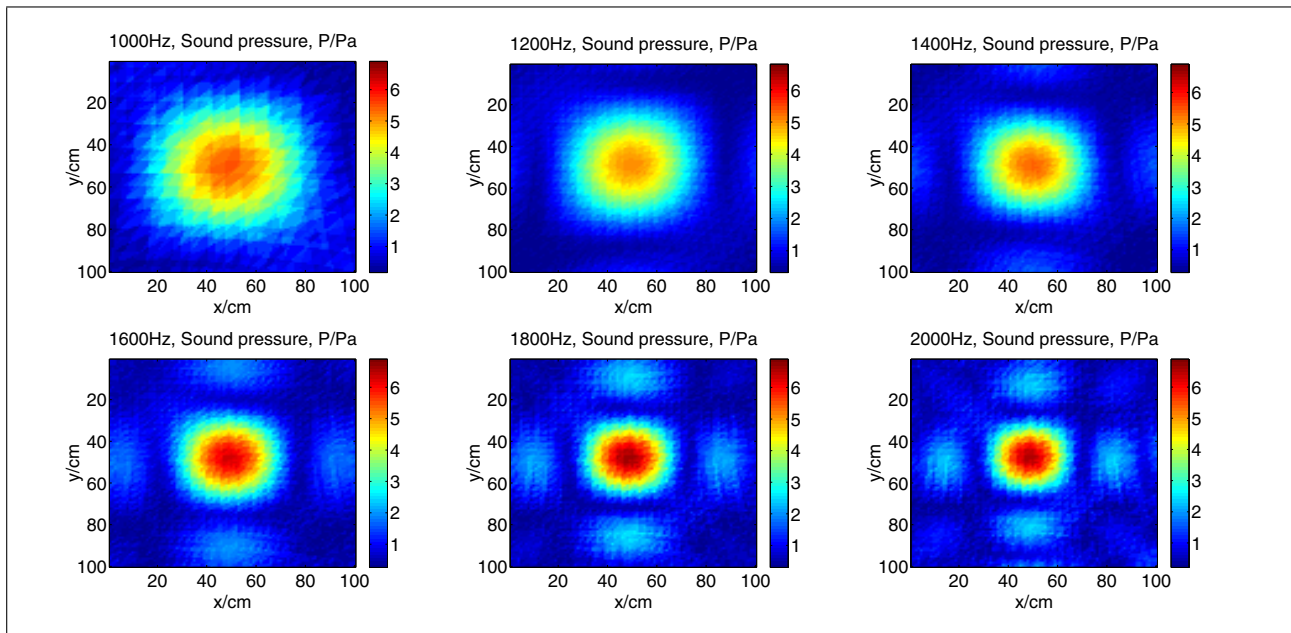


Figure 11. The sound pressure nephograms of the virtual holographic surface H with different frequencies.

5.3. The experimental verification of single frequency sound source

The experimental verification of single frequency sound source is divided into two parts. The first experiment is changing the frequency of the sound source. The second one is changing the sound pressure value of the sound source. Based on the experiments above, we can see the proposed method is effective in dealing with different conditions of sound source

5.3.1. Sound source with different frequencies

This experiment has been performed in order to verify the proposed method is an effective way to the sound field reconstruction with different frequencies. The sound frequencies are 900 Hz, 1100 Hz, 1300 Hz, 1500 Hz, 1700 Hz and 1900 Hz, avoiding the frequencies which were used in the solution of k_c .

The real pressure value of sound source is acquired by the NO.10 microphone. The relative deviation ε between the calculated sound pressure value P and the real value P_R is given as

$$\varepsilon = \frac{|P - P_R|}{P_R} \cdot 100\%. \quad (37)$$

The sound pressure nephograms of the virtual holographic surface H , reconstructed by the Distance-Weighted beamforming method, is shown in Figure 11. The sound pressure nephograms of the reconstructed surface S before filtering is shown as Figure 12, and the one after filtering is shown as Figure 13. The real pressure value P_R of sound source, the reconstructed sound pressure value P' of the sound source position before filtering, the reconstructed sound pressure value P of the sound source position after filtering, and the relative deviation ε between P and P_R are shown in Table II.

Table II. The experimental results for sound frequency f , sound pressure value measured P_R sound pressure value calculated before filtering P' , sound pressure value calculated after filtering P [Pa] and relative deviation ε with different frequencies of the sound source.

f	P_R	P'	P	ε
1000 Hz	11.93	$3 \cdot 10^8$	11.21	6.0%
1200 Hz	11.22	$0.1 \cdot 10^8$	10.67	4.9%
1400 Hz	11.47	$0.2 \cdot 10^8$	11.32	1.3%
1600 Hz	12.07	$2.5 \cdot 10^8$	12.37	2.5%
1800 Hz	11.84	$1.3 \cdot 10^8$	12.53	5.8%
2000 Hz	11.55	$0.8 \cdot 10^8$	11.42	1.1%

Through the analysis of the experimental results, the following conclusions could be obtained.

1. It could be seen from Figure 11, because of the characteristic of beamforming, the area of sound source imaging is inversely proportional to the sound source frequency. The lower the sound source frequency is, the wider the main lobe, and the larger the sound source image. In this experiment, each sound source image is larger than the area of real sound source. Accordingly, this phenomenon brings the deviation between the distributed area of complex sound pressure on the virtual holographic surface and the real one.
2. It can be seen from Figure 12 and the 3th column of Table II, the deviation from beamforming method is amplified exponentially by the G operator in the high k space, and then produces the high noise. If the noise is not filtered out, the reconstructed results of both location and sound pressure value of the sound source are different from the real value significantly.
3. It can be seen from Figure 13 and the 4th, 5th column of Table II, using the proposed method, after filtering

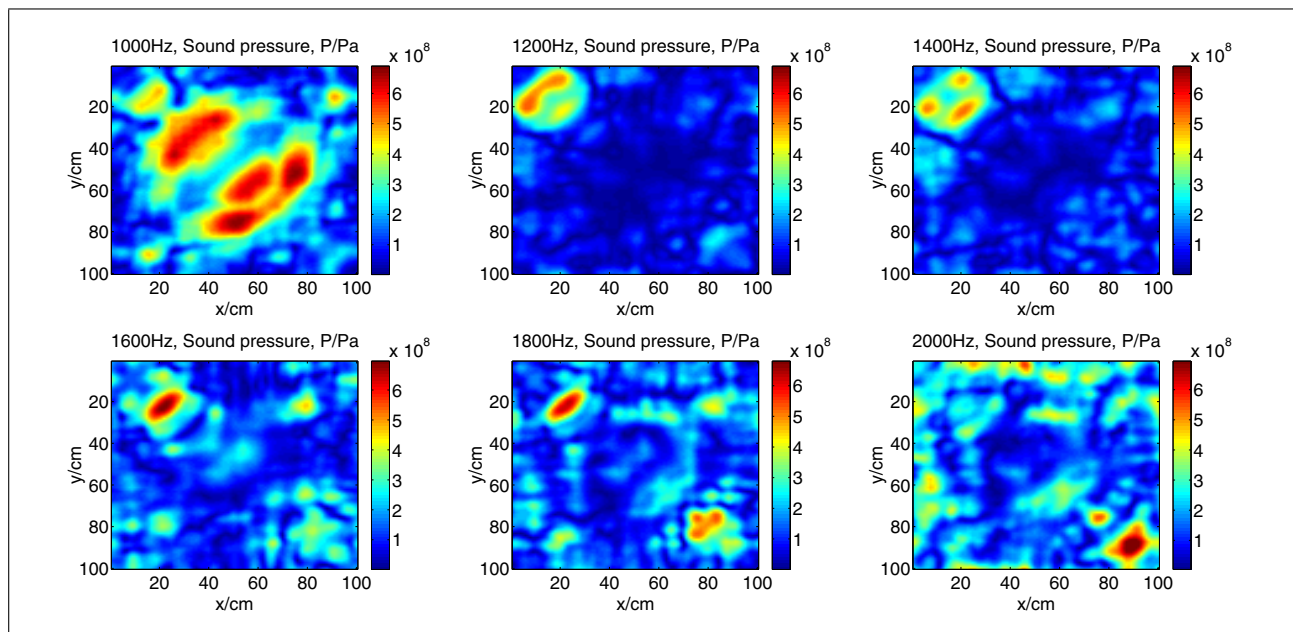


Figure 12. The sound pressure nephograms of the reconstructed surface S before filtering with different frequencies.

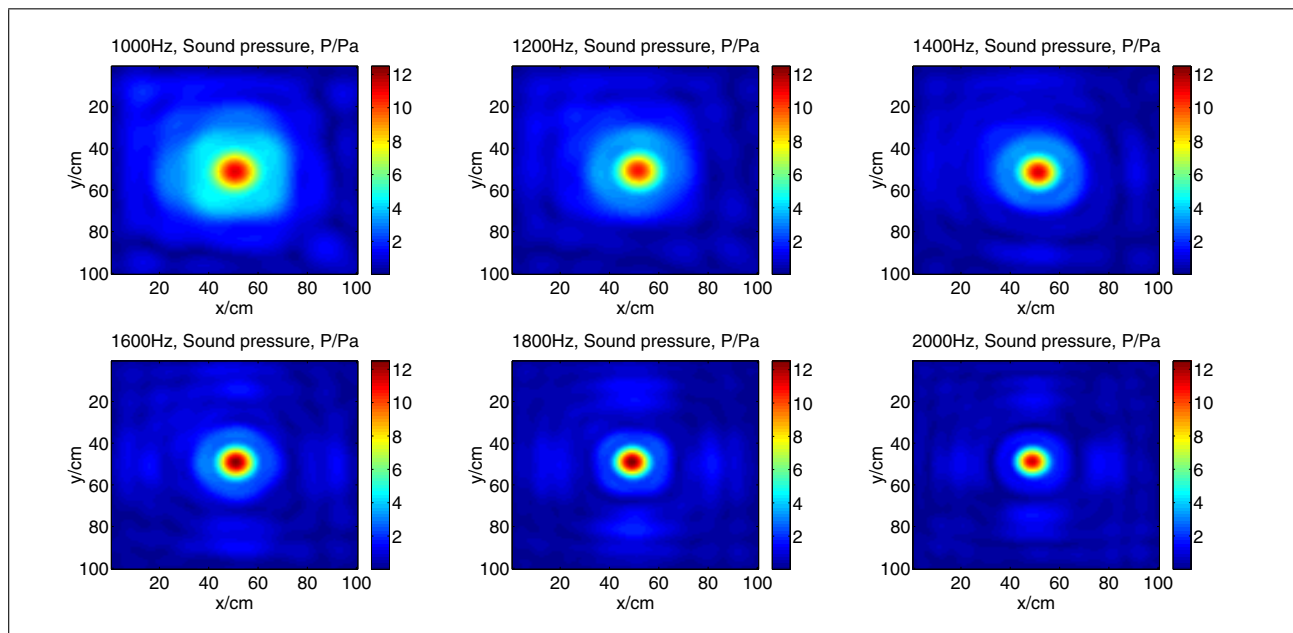


Figure 13. The sound pressure nephograms of the reconstructed surface S after filtering with different frequencies.

the noise out in k space with k_c acquired by equation (36), the sound source locations of different frequencies are accurate, and the area of sound source imaging is closer to the area of real sound source with the diameter of 0.12 m than the result acquired by beamforming method. For the reconstruction of the pressure value of the sound source, the maximum relative deviation is 6%, the minimum relative deviation is 1.1%, and the average relative deviation is 3%.

In summary, the experiment with the different frequencies of the sound source can verify the accuracy of the deviation analysis, the necessity of the filtering in k space, and the effectiveness of the solution of k_c . It has also shown

that the proposed method is an effective way to the sound field reconstruction of different frequencies.

5.3.2. The experiment with different sound pressure values

The experiment has also been performed in order to verify the proposed method is an effective way to the sound field reconstruction with different sound pressure values. The sound source frequency is 1000 Hz of which the deviation is the maximum. The sound pressure values of sound source are 1.07 Pa, 3.82 Pa and 11.93 Pa.

The sound pressure nephograms of the virtual holographic surface H , reconstructed by the Distance-Weigh-

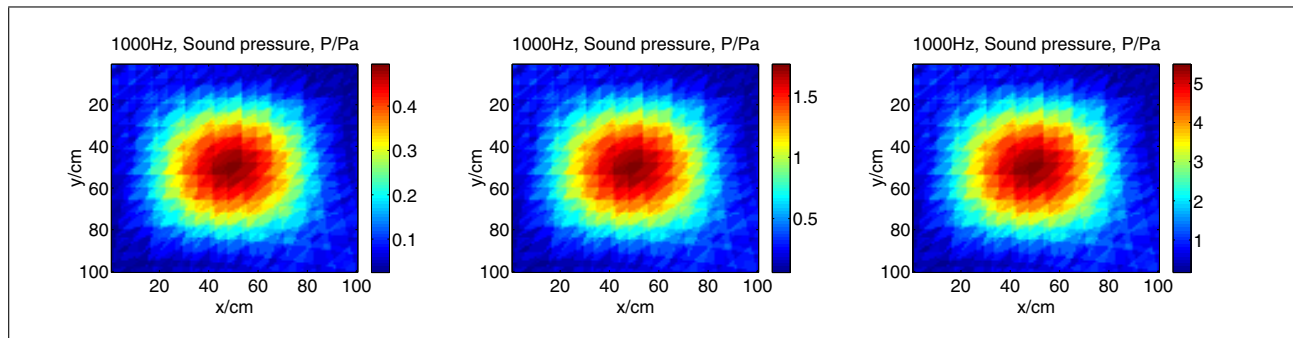


Figure 14. The sound pressure nephograms of the virtual holographic surface H with different sound pressure.

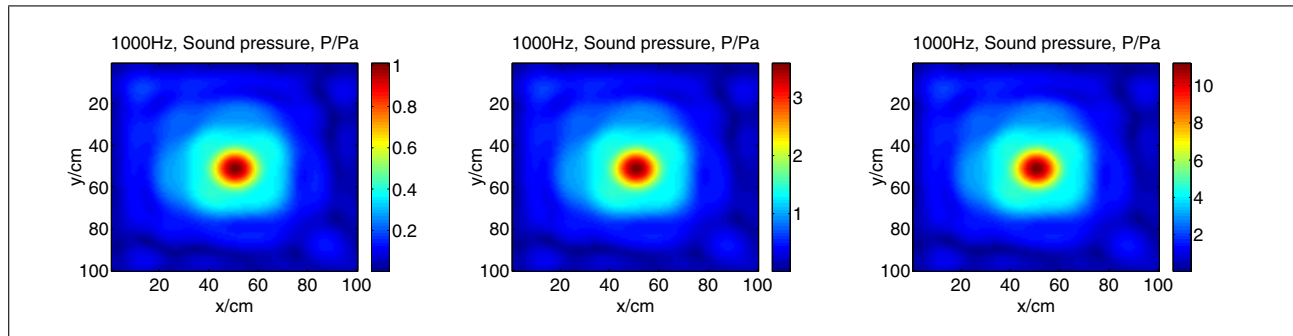


Figure 15. The sound pressure nephograms of the reconstructed surface S after filtering with different sound pressure.

Table III. The experimental results with different sound pressure values. Sound pressure value measured P_R , sound pressure value calculated P and relative deviation ϵ .

P_R	P	ϵ
1.07	1.01	5.6%
3.82	3.59	6.0%
11.93	11.21	6.0%

ted beamforming method, is shown as Figure 14. The sound pressure nephograms of the reconstructed surface S after filtering is shown as Figure 15. The real sound pressure value P_R of the sound source, the reconstructed sound pressure value P at the sound source position after filtering, and the relative deviation ϵ between P and P_R are shown in Table III.

Through the analysis of the experimental results, the following conclusions can be drawn.

1. It can be seen from Figure 14, if the frequency is constant, the operation of changing the sound pressure value of the sound source only changes the sound pressure amplitude, but not the sound pressure distribution acquired by the beamforming method. Therefore, it can be further verified that the area of sound source image acquired by the beamforming method is related to the sound source frequency.
2. It can be seen from Figure 15 and Table III, using the proposed method, the sound source locations of different sound pressure values are accurate, and the area of sound source image is closer to the area of real sound source with the diameter of 0.12 m than the result

achieved by beamforming method. For the reconstruction of the sound pressure values, the relative deviations are stabilized at 6%, which does not change with the change of the sound pressure value of the sound source.

In summary, the experiment with different sound pressure values has verified that the proposed method is an effective way to the sound field reconstruction with different sound pressure values, and the relative deviations are unrelated to the sound pressure values of the sound source.

5.4. Verification of wideband sound source

In order to verify the effectiveness of the proposed method in the wideband sound field, an experiment of wideband sound field reconstruction may be carried out in an ordinary room which is classified as an irregular indoor sound field environment. The size of the room is 8 m × 5 m × 6 m. The 3 × 3 grid-style microphone array used in the single frequency experiments is also used in this experiment.

Because of the uncertainties of measurements in room acoustics [32], there are more uncertain factors in the ordinary room than anechoic chamber. For this reason, it is necessary to check if the reflected sound in the room may interfere with the sound source location. Therefore, with the origin which is the centre point of the sound source surface, the loudspeaker is placed at the spatial coordinate (0.05 m, -0.03 m) of the sound source surface.

5.4.1. Experiment of the proposed method

The purpose of this experiment is to verify that the proposed method is an effective way for the wideband sound field. The settings of experiment parameters, such as the

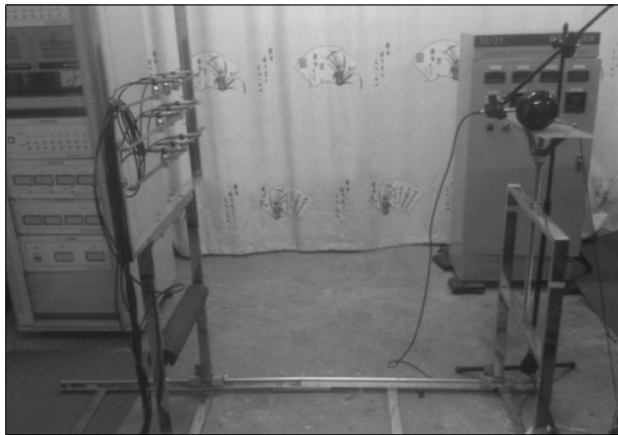


Figure 16. The photo of wideband experiment environment.

Table IV. The experimental results of wideband sound source. Sound frequency f , bandwidth of narrowband b , sound pressure value measured P_R , sound pressure value calculated P , relative deviation ϵ .

f	b	P_R	P	ϵ
1000–2000	10	13.12	11.78	10.2%
2000–3000	10	12.14	10.86	10.5%
3000–4000	10	13.48	11.77	12.7%

Table V. The experimental results of 3000–4000 Hz sound source with different bandwidth. Sound frequency f , bandwidth of narrowband b , sound pressure value measured P_R , sound pressure value calculated P , relative deviation ϵ .

f	b	P_R	P	ϵ
3000–4000	6	13.48	12.54	7.0%
3000–4000	10	13.48	11.77	12.7%
3000–4000	20	13.48	11.52	14.5%

NO.10 microphone, the reconstructed surface and the virtual holographic surface, are the same as in the experiment of single frequency. The photo of the wideband experiment environment is shown as Figure 16.

The sound signals of the experiment, using Gauss white noise, are 1000–2000 Hz, 2000–3000 Hz and 3000–4000 Hz.

The sound pressure nephograms of the virtual holographic surface H , reconstructed by the Distance-Weighted beamforming method, are shown in Figure 17. The sound pressure nephograms of the reconstructed surface S after filtering are shown as Figure 18. The real sound source pressure value P_R of the sound source, the reconstructed sound pressure value P of the sound source position after filtering, and the relative deviation ϵ between P and P_R are shown in Table IV.

Through the analysis of the experimental results, the following conclusions can be obtained.

1. It can be seen from Figure 17, the characteristic that the area of sound source image acquired by the beamforming method is inversely proportional to the sound

source frequency. It is also applicable to the wideband noise. Therefore, on the virtual holographic surface, the characteristic also brings the deviation between the distribution of complex sound pressure and the real one. The method for deviation analysis and noise filtering, which is used in the single frequency reconstruction, is applicable to the wideband noise calculation as well.

2. It can be seen from Figure 17 and Table IV, using the proposed method, the sound source locations of different wideband are accurate, and the area of sound source image is closer to the area of real sound source with the diameter of 0.12m than the result acquired by beamforming method. For the sound pressure value reconstruction of the wideband sound source, the maximum relative deviation is 12.7%, the minimum relative deviation is 10.2%, and the average relative deviation is 11.13%.

Comparing with the single frequency experiment, the deviations of the wideband experiment increase by 8% and the calculated values are always less than the real values. The main reason for this phenomenon is the loss of energy generated from the approximate calculation. When the single frequency method is used as a substitute for the narrowband method approximately, the loss of energy will be generated. The smaller the dividing bandwidth b of the narrowband is, the closer the narrowband to the single frequency sound, and the smaller the loss of energy generated from the approximate calculation.

To verify the accuracy of the deviation increasing analysis, the wideband noise of 3000–4000 Hz, which has the maximum deviation, is recalculated with the bandwidth b of 6 Hz, 10 Hz and 20 Hz. The nephograms of the reconstructed surface are almost the same as shown in Figure 18, but the values of the sound source pressure reconstruction vary greatly, which are shown in Table V.

It can be seen from Table V, the smaller the dividing bandwidth b of the narrowband is, the smaller the relative deviation ϵ . It can be verified that the main reason of the increasing deviation is the loss of energy generated from the approximate calculation of using the single frequency method instead of the narrowband method. It should be noted that, however, the smaller the bandwidth b is, the greater the amount of calculation. Therefore, in the practical application, the bandwidth b should be chosen appropriately in order to obtain both the smaller deviation and the higher calculated efficiency. In this paper, $b = 10$ Hz, which can meet the requirements.

5.4.2. Experiment of the distance-weighted beamforming

The purpose of this experiment is to verify that there is an acceptable deviation between the sound pressure value of the source point in the virtual holographic surface and the measured one in the same position in an ordinary room. The settings of the experiment parameters are almost the same as the parameters in the experiment of wideband in section 5.4.1. The only difference from the experiment in section 5.4.1 is the position of the NO.10 microphone,

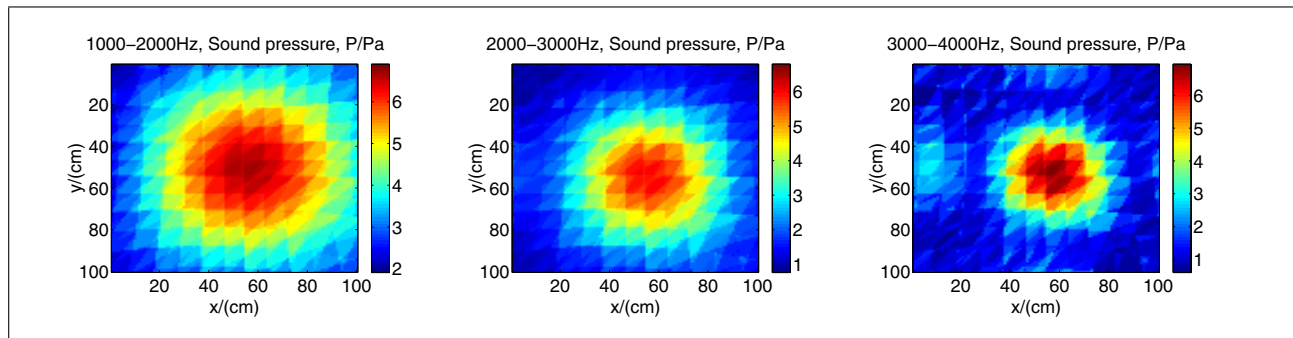


Figure 17. The sound pressure nephograms of the virtual holographic surface H with different wideband.

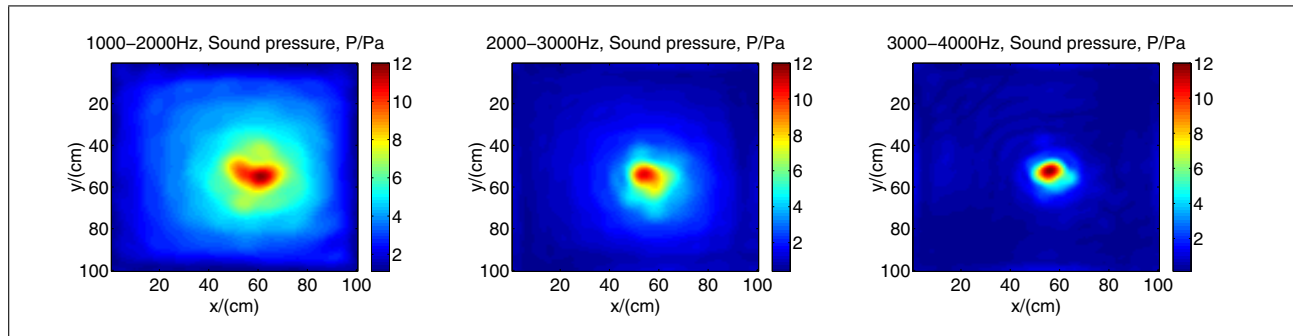


Figure 18. The sound pressure nephograms of the reconstructed surface S with different wideband.

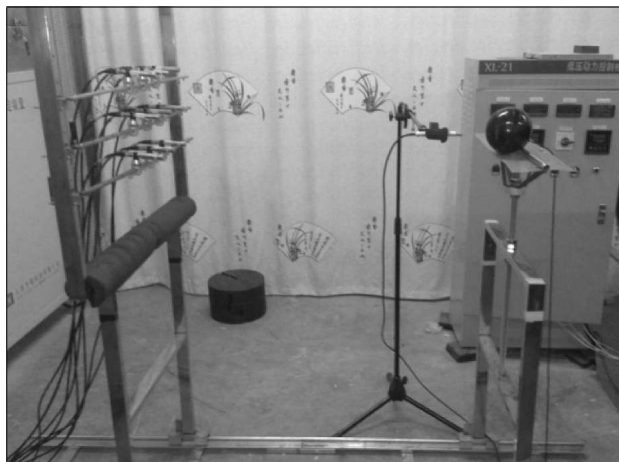


Figure 19. The photo of wideband experiment of Distance-Weighted beamforming.

which is 0.06m away from the centre of the sound source. The virtual holographic surface is 0.06m away from the sound source as well, so the NO.10 microphone could measure the real sound pressure value of the hot point in the virtual holographic surface. The photo of the wideband experiment is shown as the Figure 19:

The sound pressure nephograms of the virtual holographic surface H , reconstructed by the Distance-Weighted beamforming method, is shown in Figure 20. The bandwidth b which divided the wideband signal into narrowband signals, the pressure value measured P_R of the hot point in the virtual holographic surface H , the pressure value calculated P of the hot point in the virtual holo-

Table VI. The experimental results of hot point of wideband sound. Sound frequency f , bandwidth of narrowband b , sound pressure value measured P_R , sound pressure value calculated P , relative deviation ϵ .

f	b	P_R	P	ϵ
1000–2000	10	8.09	7.75	4.2%
2000–3000	10	7.49	7.03	6.1%
3000–4000	10	8.17	7.60	6.9%

graphic surface H , and the relative deviation ϵ between P and P_R are shown in Table VI.

It can be seen from Figure 20 and Table VI, the hot point locations of different wideband sound are accurate. The relative deviations ϵ between P and P_R with different wideband sound are 4.2%–6.9%. That is to say the pressure value calculated cannot be accurately equal to that measured, but the deviation between them is acceptable. The proposed method in this paper is an approximate method for the severe environments, so the error is unavoidable. In terms of approximation of the sound source pressure, the proposed method is feasible.

6. Conclusions

Because the microphone array cannot be placed to acquire the near-field data in high temperature environment, the values of the sound loads on the structure surface could not be acquired accurately. An improved method combining the beamforming and the acoustical holography to re-

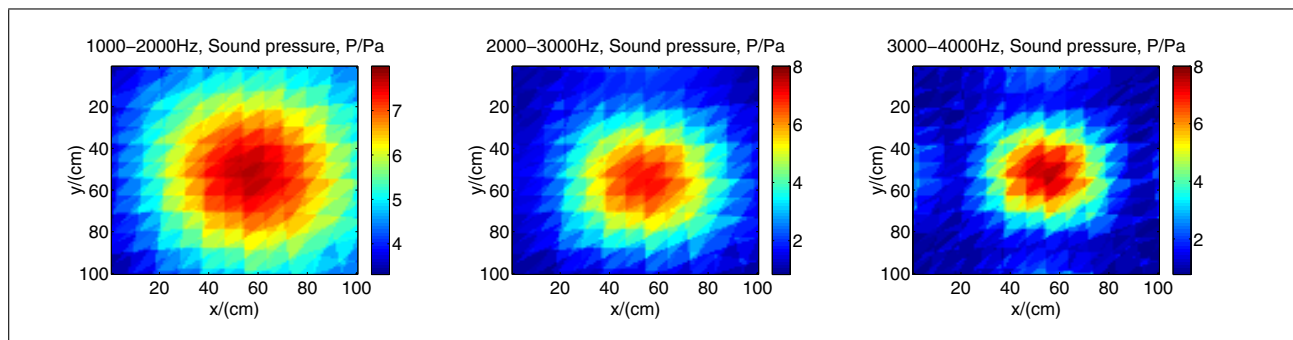


Figure 20. The sound pressure nephograms of the virtual holographic surface H with different wideband sound.

construct the sound pressure value in the sound field is proposed, and the following conclusions can be drawn:

1. According to the theory of the acoustic attenuation, the traditional beamforming is added to the distance weight which is the ratio of the distance from the sound source to the reference microphone and the distance from the present reconstruction point to the sound source. After the calculation of the distance weighting, both the sound source position and the real sound pressure value of the near-field surface could be obtained accurately, and then the virtual holographic surface can be acquired, which can be treated as the input data of the STSF NAH method.
2. Based on the virtual holographic surface obtained from the Distance-Weighted beamforming, the distribution and the value of the sound source surface can be reconstructed by the STSF NAH. With the filter parameter k_c acquired by the method combining the actual measurement and the emulation, the deviation from beamforming can be eliminated by filtering the noise out in the high k space. This method provides a new way to the measurement and analysis of the thermal-acoustic fatigue of material or structure in high temperature environment.
3. For the reconstruction of wideband noise, a method with wideband division is proposed. Firstly, divide the wideband noise into narrowband noises with an appropriate division bandwidth. Secondly, calculate the narrowband noises with single frequency method. Finally, add all results of the narrowband noises together. Based on this method above, the problem of wideband noise field reconstruction can be solved.
4. Based on the 3×3 grid-style sparse microphone array with the distance of 1 m from sound source, a plurality of experimental verification in different conditions of sound source are performed in the free-field and irregular indoor sound field environments. The results of the experiments have shown that, firstly, for the single frequency sound source, the sound source locations of different frequencies and sound pressure values are accurate, and the deviation range of sound pressure value reconstruction is between 1.1% and 6% with the average value of 3%; secondly, for the wideband sound source, the sound source locations of different widebands are

accurate, and the deviation range of sound pressure value reconstruction is between 10.2% and 12.7% with the average value of 11.1%. The study further shows the deviation of the wideband experiment can be decreased to 7% by reducing the dividing bandwidth b of narrowband.

According to the results of the experiments above, the proposed method combining the Distance-Weighted beamforming and the STSF NAH is effective for the sound pressure distribution and amplitude reconstruction of different frequencies, sound pressure values and widebands. The method proposed has advantages in practical application such as feasibility in severe environments, low numbers of microphones, practical measurement in the far field surface, accurate reconstruction of the sound pressure distribution and the amplitude on the surface of sound source.

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