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Polarization Characteristics of Bulk Ultrasonic Waves in Acousto-Optic Paratellurite Crystal

N. V. Polikarpova, P. V. Malneva
Oscillation Physics Division, Faculty of Physics, Lomonosov Moscow State University, Moscow 119991, Russia.
malneva@physics.msu.ru

Summary
The paper examines the orientation of the acoustic polarization vector of bulk plane ultrasonic waves propagating along various directions relatively to the crystalline axes of tetragonal acousto-optic crystals. In particular, the strongly anisotropic tetragonal crystal tellurium dioxide (TeO_2) is considered in details in the paper. Theoretical analysis and numerical calculations confirm that in the tetragonal materials the transformation of a quasi-longitudinal wave into a quasi-shear acoustic wave takes place if the elastic coefficient c_{66} exceeds c_{11}. The carried out theoretical analysis also determines the maximum angle at which this transformation of the ultrasonic waves takes place in the tetragonal materials. Furthermore the angular range of the directions in the paratellurite crystal in which quasi-longitudinal waves are transformed into quasi-shear waves is determined.

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1. Introduction

In recent years there has been a considerable growth of interest in natural and artificial media demonstrating unique physical properties [1, 2]. This includes a considerable number of crystalline materials used in modern acousto-optics. Acousto-optic crystals are strongly anisotropic and are characterized by unusual optic and acoustic parameters [3, 4]. For example, the angles between the phase and group velocities of optic and especially acoustic waves in the crystals can reach tens of degrees. Non-coincidence of the velocities of the waves is most pronounced in such acousto-optic materials as paratellurite (TeO_2), calomel (Hg_2Cl_2), tellurium (Te), etc. [1, 5, 6, 7, 8, 9, 10, 11].

It has been observed that the elastic anisotropy in crystals is much higher than the optic anisotropy. As known, the elastic anisotropy results in a strong dependence of the phase velocity of acoustic waves on direction of their propagation. For example, the waves propagate in the tetragonal calomel crystal along the [110] axis with the low phase velocity value $V = 347$ m/s, while the phase velocity along the direction [100] is equal to V = 1305 m/s [9]. In the crystals of mercury bromide (Hg_2Br_2) and mercury iodide (Hg_2I_2), the velocities of the waves along the [110] axis are even lower and correspondingly equal to $V = 281$ m/s and $V = 245$ m/s. On the other hand, the phase velocities in these materials increase to $V = 1240$ m/s and $V = 1200$ m/s if the waves propagate along the [100] axis. Therefore, the ratio of the maximum and the minimum velocities for one and the same acoustic mode in calomel is equal to $r = 3.76$, while in mercury bromide and mercury iodide the coefficient of anisotropy equals to $r = 4.41$ and $r = 4.90$, correspondingly [9]. Moreover, in the paratellurite crystal, the ratio of the velocities is even larger than in the mercury compounds: $r = 4.95$. The strong dependence of the acoustic velocity on the direction of propagation in a crystal results in a deviation of energy flow with respect to the phase velocity vector. It is convenient to evaluate this angular deviation by the so-called energy walk-off angle ψ. As found, the maximal value of the walk-off angle in TeO_2 is equal to $\psi = 74°$ if the slow shear acoustic wave propagates in the (001) plane at the angle $\varphi = 37°$ relative to the [100] or [010] axes [12, 13, 14, 15, 16, 17, 18].

As mentioned, the optical anisotropy in the considered acousto-optic materials is not so strong as compared to the acoustic anisotropy. It is known that the phase velocity of optical waves depends on the value of the refractive index. In the crystals of calomel and mercury bromide, the refractive indexes of the extraordinary polarized waves vary with the direction of propagation only by a factor of 1.5. Due to this feature, the optical walk-off angles in the acousto-optic crystals do not exceed $\psi = 23°$. The maximum recorded optical walk-off angle was found in the double-axis antimony sulfoiodide crystal (SbSI) [19]. This angle in the material is equal to $\psi = 25°$ but is still three times narrower than the acoustic walk-off angle $\psi = 74°$ in paratellurite. That is why, in this paper, we examine only the acoustic anisotropy of the crystalline materials while the optic anisotropy will be studied elsewhere.

It should be noted however that in the examined acousto-optic materials not only the ratio of the phase and group velocities is anomalous. Directions of acoustic polarization may also be different from those observed in a ma-

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jority of acoustic crystals. Moreover, taking into account only the magnitudes of the phase velocities in the materials, it is very difficult to predict directions and type of the acoustic polarization in the crystals that demonstrate strong elastic anisotropy. It is known that in isotropic media there exist two basic types of acoustic waves: the longitudinal waves with the polarization vector parallel to the wave vector and the transverse waves (shear waves) with the orthogonal direction of the acoustic polarization. In the crystalline media as much as three acoustic waves may be observed. These waves are a quasi-longitudinal or a longitudinal wave and two shear or quasi-shear waves. The acoustic mode with the polarization close or parallel to its wave vector is the quasi-longitudinal or the longitudinal wave, while the polarization of the shear or the quasi-shear waves is orthogonal or close to orthogonal relative to the acoustic wave vector. All these waves vary from each other by their magnitude of the phase velocity. One of the waves is the fast acoustic mode and the two other waves are the modes propagating with slower velocities. The acoustic polarizations of these waves are mutually orthogonal [12, 13]. It is also known that in the majority of crystalline materials, the fastest elastic mode is usually a longitudinal or a quasi-longitudinal wave. A direction of the acoustic polarization vector in these waves coincides with a direction of the phase velocity vector or slightly deviates from it. If a slow acoustic mode is concerned, the acoustic polarization vector is usually orthogonal or close to orthogonal with respect to the direction of the phase velocity vector.

It was also found that there are acousto-optic crystals demonstrating totally different behavior [11, 12, 13]. The direction of polarization for the fastest acoustic wave in these crystals may be orthogonal to the phase velocity vector. Therefore the wave may be defined as a pure shear acoustic wave. At the same time, the elastic displacement in the slower mode may be directed parallel to the acoustic wave vector. It means that this mode should be defined as a longitudinal wave. On the other hand, the acoustic waves demonstrate an ordinary behavior in a wide range of directions in the crystal. Consequently, a change in the direction of propagation for one and the same acoustic mode may lead to a transformation of the acoustic wave polarization so that a longitudinal mode becomes a shear wave. It should be mentioned however that this phenomenon is unusual and may be observed only in a very limited number of crystals.

The transformation was found in crystalline materials possessing a specific ratio between elastic coefficients [12, 13]. The acousto-optic crystal tellurium dioxide belongs to this rare family of anisotropic crystalline materials demonstrating the extraordinary relation between the coefficients. This acousto-optic material is very well studied. Magnitudes of the acoustic velocities in three basic planes of the paratellurite crystal and also of some other tetragonal acousto-optic media may be found in the literature [6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 20]. Moreover, in reference [7], inhomogeneous and homogeneous quasi-shear elastic waves in a paratellurite crystal were examined not only in the basic planes but also arbitrary directions of acoustic propagation. A three-dimensional acoustic slowness surface may be found in reference [7]. Vectors of acoustic polarization corresponding to the plane bulk quasi-shear mode are also schematically shown in reference [7]. However, a careful analysis of the elastic modes propagating in arbitrary planes of the paratellurite crystal was not performed in the cited papers. Therefore, the goal of this paper is to investigate the polarization characteristics of all bulk plane acoustic waves propagating in the strongly anisotropic paratellurite crystal.

2. Theoretical analysis. Basic parameters of elastic waves in tetragonal crystals

In order to examine unusual cases of acoustic propagation in the crystals, at the beginning we consider basic parameters of elastic waves such as a magnitude of acoustic velocity and a direction of polarization vector. These parameters in the general case of unlimited anisotropic medium may be found from the solution of the Green-Christoffel equation [12, 13],

$$\Gamma_{il} p_l = \rho V^2 p_i, \tag{1}$$

where $\Gamma_{il} = c_{ijkl} n_j n_k$ are the components of the Green-Christoffel tensor, p_i are the components of the polarization vector, ρ is the density of the material, V is the phase velocity of the wave, c_{ijkl} are the elastic coefficients of the crystal, n_j and n_k are the components of the unit vector describing the acoustic propagation [12, 13]. In the general case, a direction of the acoustic wave propagation is given by two angles evaluated in a spherical coordinate system: φ and θ, where φ is the azimuth angle with respect to the X-axis of a material and θ is the polar angle evaluated relatively to the Z-axis. The wave front is then described by its normal vector $\boldsymbol{n}$ with: $n_1 = \cos\varphi \sin\theta$, $n_2 = \sin\varphi \sin\theta$ and $n_3 = \cos\theta$. Solving the Green-Christoffel equation (1) we find eigenvalues $\lambda = \rho V^2$ giving magnitudes of phase velocities of acoustic waves propagating along arbitrary directions [12, 13]. We can also find eigenvectors of acoustic polarization p_i describing particles displacement in a crystal.

The examined crystal of paratellurite represents the tetragonal symmetry group 422. It is known that the acoustic properties of the tetragonal materials belonging to the crystalline classes 42m, 422, 4mm and 4/mmm are defined by six elastic constants c_{ijkl}. Mapping indexes in Voigt notation results in constants c_{11}, c_{12}, c_{13}, c_{33}, c_{44} and c_{66}. The components of the Green-Christoffel tensor in the tetragonal crystals [12, 13] are then equal to

$$\begin{cases} \Gamma_{11} &= c_{11} n_1^2 + c_{66} n_2^2 + c_{44} n_3^2, \\ \Gamma_{12} &= \left(c_{12} + c_{66}\right) n_1 n_2 = \Gamma_{21}, \\ \Gamma_{13} &= c_{44} n_1 n_3 = \Gamma_{31}, \\ \Gamma_{22} &= c_{66} n_1^2 + c_{11} n_2^2 + c_{44} n_3^2, \\ \Gamma_{23} &= \left(c_{13} + c_{44}\right) n_2 n_3 = \Gamma_{32}, \\ \Gamma_{33} &= c_{44}\left(n_1^2 + n_2^2\right) + c_{33} n_3^2. \end{cases} \tag{2}$$

It is shown [6, 7, 8, 9, 10, 11, 12, 13] that the tetragonal crystals demonstrate the strongest anisotropy in the XOY plane; in this particular case of the acoustic propagation, the Green-Christoffel equation is rather simple so that the problem may be solved analytically. In the XOY plane the phase velocities are determined by the following expressions [12, 13]

$$V_{1,2}^2 = \frac{c_{11} + c_{66} \pm \sqrt{(c_{11} - c_{66})^2 \cos^2 2\varphi + (c_{12} + c_{66})^2 \sin^2 2\varphi}}{2\rho},$$
$$V_3^2 = c_{44}/\rho. \quad (3)$$

Then the two acoustic waves $V_{1,2}(\varphi)$ posess polarization vectors directed orthogonal to the Z-axis. Their different from zero projections on the X- and Y-axes may be represented by the expressions [12, 13]

$$\begin{cases} p = \frac{\Gamma_{12}}{\sqrt{\Gamma_{12}^2 + (\Gamma_{11} - \rho V_{1,2}^2)^2}}, \\ q = \frac{\rho V_{1,2}^2 - \Gamma_{11}}{\sqrt{\Gamma_{12}^2 + (\Gamma_{11} - \rho V_{1,2}^2)^2}}, \end{cases} \quad (4)$$

where $\Gamma_{11} = c_{11} \cos^2 \varphi + c_{66} \sin^2 \varphi$ and $\Gamma_{12} = (c_{12} + c_{66}) \cos \varphi \sin \varphi$. A third acoustic mode with the velocity V_3 is not dependent of the angle φ and is polarized along the Z-axis. We used equation (4) to determine the components p and q of the polarization vector in the XOY plane in the tetragonal crystals. Based on the values of the vector components, we determined the direction of the displacement relative to the X-axis as $\tan \beta = q/p$. Finally, the angle between the displacement, i.e., the acoustic polarization and the acoustic wave vector is calculated by means of the formula $\gamma = \beta - \varphi$, where φ is the angle between the phase velocities and the X-axis.

It is known that one of the three existing acoustic modes is referred to a quasi-longitudinal wave if its polarization angle belongs to the interval $0 < \gamma < 45°$. The angular range $45° < \gamma < 90°$ corresponds to a quasi-shear wave. This means that at $\gamma = 45°$, a transformation of one type of elastic wave to another takes place. We define the direction of the waves transformation by the corresponding angle of propagation φ^*. It means if the angle φ is continuously varying from $\varphi = 0$ to $\varphi = 45°$ then at $\varphi = \varphi^*$ the angle of polarization is equal to $\gamma = 45°$. Therefore the quasi-longitudinal wave is transformed into the quasi-shear wave. Moreover both slow and fast acoustic modes demonstrate similar effects of the polarization change [11, 12, 13, 18, 21].

We revealed a dependence of the angle φ^* on the magnitudes of the elastic coefficients. Analysis of the components of the Green-Christoffel tensor equation (2) enables the derivation of an expression for φ^*. We found that the angle of the wave type transformation at $\gamma = 45°$ in the tetragonal crystals in XOY-plane is

$$\varphi^* = \frac{1}{2} \arctan \sqrt{\frac{c_{66}/c_{11} - 1}{c_{12}/c_{11} + c_{66}/c_{11}}}. \quad (5)$$

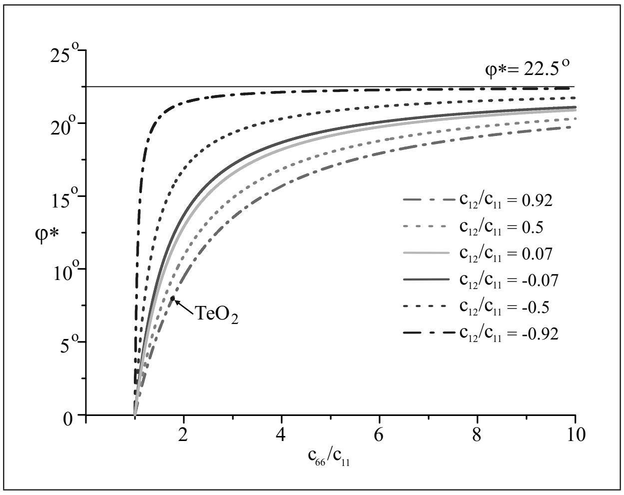

Figure 1. Dependences of the acoustic propagation angle φ^* in the XOY-plane of paratellurite on the ratio of the elastic constants c_{66}/c_{11}.

Applying equation (5) we plot the dependence of the transformation angle φ^* versus the ratio of the elastic coefficients c_{66}/c_{11} for different values of the parameter c_{12}/c_{11}. It should be noted that the ratio c_{12}/c_{11} does not exceed unity. The dependence equation (5) is presented in Figure 1. The elastic coefficient c_{12} in the majority of crystalline materials is positive. However, in the tetragonal potassium dihydrogen phosphate (KDP) crystal, the elastic coefficient is negative [19]. That is why, in the analysis, we considered the ratio c_{12}/c_{11} in the range $-1 < c_{12}/c_{11} < 1$. In order to reveal the dependence of the transformation angle φ^* on the coefficients ratio c_{66}/c_{11}, we calculated the angle φ^* at six values of the elastic constants ratio: $c_{12}/c_{11} = -0.92$, $c_{12}/c_{11} = -0.5$, $c_{12}/c_{11} = -0.07$ as well as $c_{12}/c_{11} = 0.07$, $c_{12}/c_{11} = 0.5$ and $c_{12}/c_{11} = 0.92$. These values were chosen because the ratio 0.92 characterizes the crystal of paratellurite, the ratio -0.07 describes the KDP crystal and the four other values were considered to cover range $-1 < c_{12}/c_{11} < 1$ in the most complete way. As it is seen in equation (5), the transformation of the waves exists only in the case $c_{66}/c_{11} > 1$. Since the ratio c_{12}/c_{11} is limited by 1, the angle φ^* versus the ratio c_{66}/c_{11} is included in a narrow range of magnitudes. This is confirmed by data presented in Figure 1. Therefore it may be stated that the transformation angle φ^* in the XOY-plane of the tetragonal crystals does not exceed the magnitude $\varphi^* = \pi/8$. It should also be noted that the discussed anomaly in the acoustic polarization occurs very seldom.

To the best of our knowledge the examined polarization effect is typical only for paratellurite [10]. As for materials of other crystalline classes, the unusual polarization phenomenon exists only in the cubic crystals TmSe [22] and in the trigonal tellurium crystal Te [11]. To prove this we applied the analysis to materials other than tetragonal acousto-optic paratellurite. It was mentioned earlier that in the crystals of Hg_2Cl_2, Hg_2Br_2 and Hg_2I_2, the velocity of the slow shear acoustic wave propagating along the

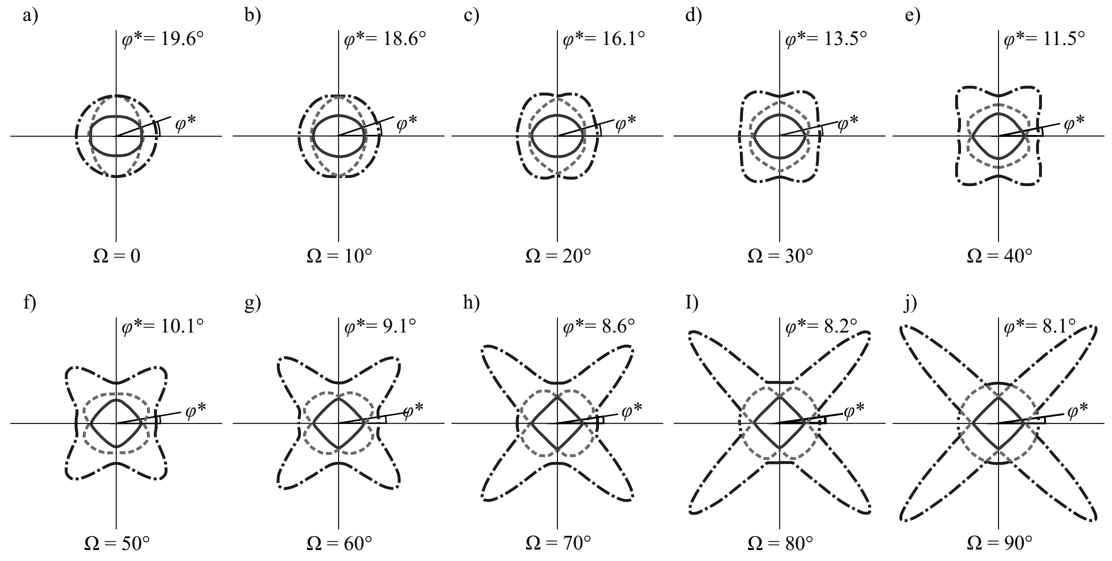

Figure 2. Cross section of the slowness surfaces for the paratellurite crystal in the planes rotated around the X-axis by the angle Ω. The case $\Omega = 0$ illustrates the XOZ-plane, the plane XOY is described by $\Omega = 90°$. The angle φ^* indicates the direction of the wave type transformation.

[110] direction is even lower than that in paratellurite. As for the parameters of elastic anisotropy, they are also large and very close to the record magnitude $r = 4.95$ in the tellurium dioxide. However, there are no principal changes of the type of the waves in the considered mercury compounds. Absence of a strong change in the direction of the acoustic polarization vector is explained by the ordinary, i.e., $c_{66}/c_{11} < 1$ relation between the magnitudes of the elastic coefficients. A review of available data presented in the literature indicates that the value of the elastic constant c_{11} exceeds the magnitude of c_{66} in the majority of tetragonal materials [9]. Therefore, unlike the paratellurite crystal, in the mercury halides the pure longitudinal wave along the [100] axes possesses a higher phase velocity value than the corresponding pure shear wave.

3. Velocity and polarization of elastic waves in a paratellurite crystal

Here we quantitatively determine the phase velocities and the directions of acoustic polarization specifically for the tellurium dioxide crystal. The characteristics of the elastic waves in the XOY-plane of this crystal were estimated using equations (3)–(4) with the density $\rho = 6000\,\mathrm{kg/m^3}$ and the values of the elastic coefficients $c_{11} = 5.612 \cdot 10^{10}\,\mathrm{N/m^2}$, $c_{12} = 5.155 \cdot 10^{10}\,\mathrm{N/m^2}$ and $c_{66} = 6.614 \cdot 10^{10}\,\mathrm{N/m^2}$ [23]. Based on magnitudes of other elastic coefficients $c_{13} = 2.2 \cdot 10^{10}\,\mathrm{N/m^2}$, $c_{33} = 10.6 \cdot 10^{10}\,\mathrm{N/m^2}$ and $c_{44} = 2.66 \cdot 10^{10}\,\mathrm{N/m^2}$ we numerically solved equation (1) and obtained the phase velocities along all directions in the crystal in arbitrary oriented planes. A family of cross sections of the acoustic slowness $(1/V)$ surfaces is shown in Figure 2. Calculations have been carried out for various rotation angles Ω. The angle Ω characterizes rotation of the cross section around the X-axis. It is measured in the YOZ-plane with respect to the Z-axis. The case $\Omega = 90°$ evaluated between the Z- and Y-axes corresponds to the cross section made by XOY-plane (Figure 2j), while the case $\Omega = 0$ describes the cross section by XOZ-plane (Figure 2a). The dash-dotted line in the figure represents the slowest acoustic mode, the solid line describes the fast acoustic mode, and the dotted line shows the mode with the intermediate velocity value. Note that the data summarized in Figure 2a and Figure 2j are in a good agreement with the results presented in the literature [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. As for data in the Figures 2b–2h, they are new and these cross sections have not been analyzed in details.

The carried out calculations prove that the velocity of the slow acoustic mode along the directions [110] and [1$\bar{1}$0], i.e. at the angles $\varphi = \pm 45°$ and $\Omega = 90°$, demonstrates the lowest acoustic velocity $V_1 = 616\,\mathrm{m/s}$. Along the X and Y axes, i.e., at the angles $\varphi = 0°$ and $\varphi = 90°$ ($\Omega = 90°$), the acoustic velocity is equal to $V_1 = 3050\,\mathrm{m/s}$. As seen in Figure 2j, the slowness curves demonstrate a strong dependence of the acoustic velocities on the direction of the acoustic propagation. The magnitudes of the velocities V_1 at the angles $\varphi = 0°$ and $\varphi = \pm 45°$ differ by a factor of 4.95, as mentioned in the beginning of the paper. It should be noted that the slowness curves shown in Figure 2 may be useful in the design of new acousto-optic and acousto-electronic devices [10, 16, 24, 25, 26].

We also determined the directions of the acoustic polarization vectors by means of calculation of the Green-Christoffel equation (1). Data in Figure 3 present the calculated dependences of the polarization angle γ versus the direction of the acoustic wave propagation for different angles Ω. The angle γ in the figures is measured between the polarization vector and a normal $\boldsymbol{n}$ directed orthogonal to

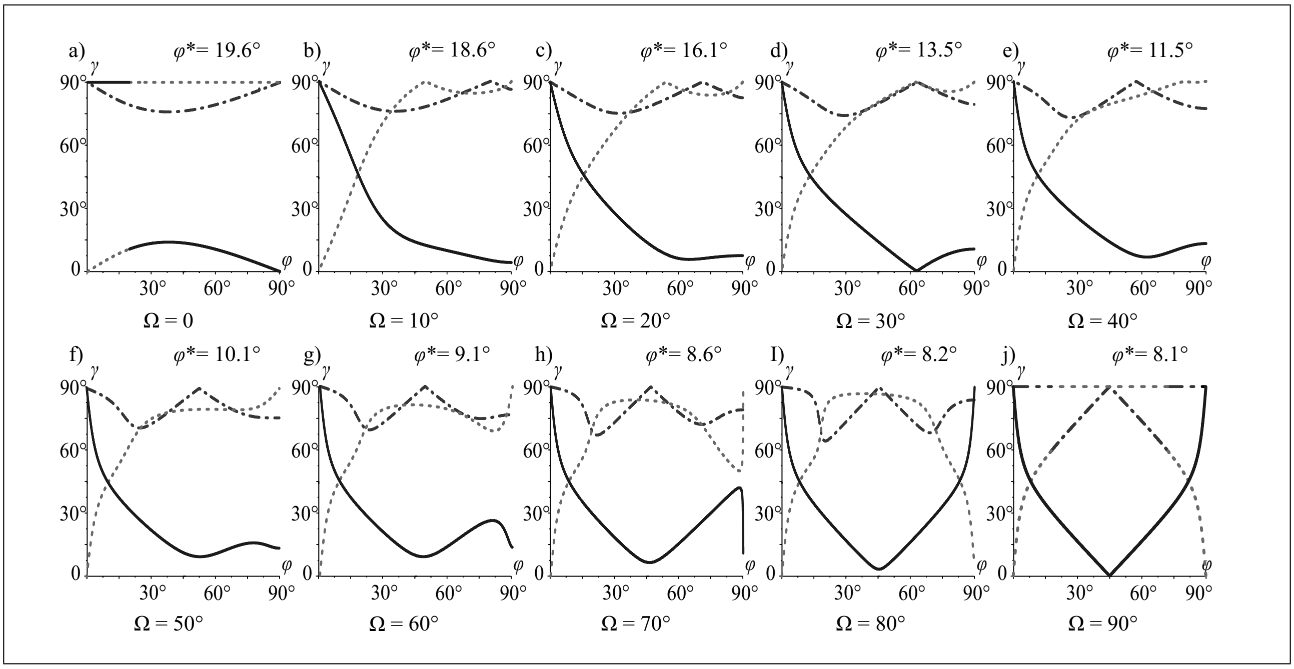

Figure 3. A family of curves illustrating the dependence of the acoustic polarization angle on direction of propagation in the planes rotated around the X-axis by the angle Ω. The solid line corresponds to the fastest acoustic mode, the dot-dashed line shows the slowest mode and the dashed line illustrates the mode with the intermediate velocity. The figure shows the angle φ^* at which the transformation of the wave type takes place.

the acoustic wave front. Similar to that in Figure 2, the angle determines rotation of the propagation plane about the X-axis. The solid curve shows the polarization behavior for the fast acoustic mode, the dash-dotted line presents the slowest mode and the dotted graph describes the mode with the medium phase velocity value.

4. Transformation of wave type in paratellurite crystal

As mentioned in section 3, the acoustic slowness curves $(1/V)$ in XOY plane of paratellurite are shown in Figure 2j while the curves in Figure 3j present the dependence of the polarization angle γ on the angle φ of the acoustic propagation. The results of the calculations confirm that the acoustic polarization vectors in the XOY-plane of paratellurite change their direction of orientation in a very peculiar manner. Data in Figure 3j prove that the transformation of the quasi-shear acoustic wave into the quasi-longitudinal wave takes place at the propagation angle $\varphi^* = 8.1°$. It means that in the case of the fast acoustic mode, the following effect is observed in the crystal along the X-axis at $\varphi = 0°$. Therefore the wave is principally shear because $\gamma = 90°$. In the range of the propagation angles $0 < \varphi < 8.1°$, the mode becomes a quasi-shear wave with $45° < |\gamma| < 90°$. Then in the range of the propagation angles $8.1° < \varphi < 45°$, the quasi-shear wave is transformed into the quasi-longitudinal wave ($0° < |\gamma| < 45°$). Finally, along the direction [110], i.e., at the angle of acoustic wave propagation $\varphi = 45°$, the wave is a pure longitudinal mode with $|\gamma| = 0$. So the calculations show that the transformation angle φ^* corresponding to the polarization angle $\gamma = 45°$ in the XOY-plane ($\Omega = 90°$) of paratellurite is equal to $\varphi^* = 8.1°$.

It means that the fast acoustic mode behaves, in a considerable range of directions, as if it were the shear or the quasi-shear acoustic wave. On the other hand, according to expectations, this fast elastic mode behaves along other directions like the longitudinal or the quasi- longitudinal wave. It means that principally different polarization angles γ, i.e., $\gamma > 45°$ and $\gamma < 45°$ correspond to various propagation directions in the case of one and the same acoustic mode.

The acoustic slowness curves and the acoustic polarizations were additionally examined in the XOZ-plane of paratellurite. Data summarized in Figure 2a and Figure 3a prove that there also exists an effect of the change in the type of acoustic wave for two elastic modes. The third acoustic mode illustrated in the figure by the dot-dashed line is characterized by the polarization directed orthogonal to the acoustic wave vector or very close to it. We found that there were no changes of the type of this wave with direction of its propagation. The calculation showed that the effect of the wave transformation for the two other acoustic modes in the XOZ-plane (dashed and solid lines in Figure 3) occurs along the direction of the angle $\varphi^* = 19.6°$.

These peculiarities have in general been mentioned in the literature [11, 12, 13, 18, 21, 22]. However, the effect was considered in the planes XOY and XOZ of paratellurite, mainly along the X axis. A detailed analysis of the phenomenon along other directions has not been carried out. Therefore, it was necessary to examine general cases of the wave type transformation. Calculation of the prop-

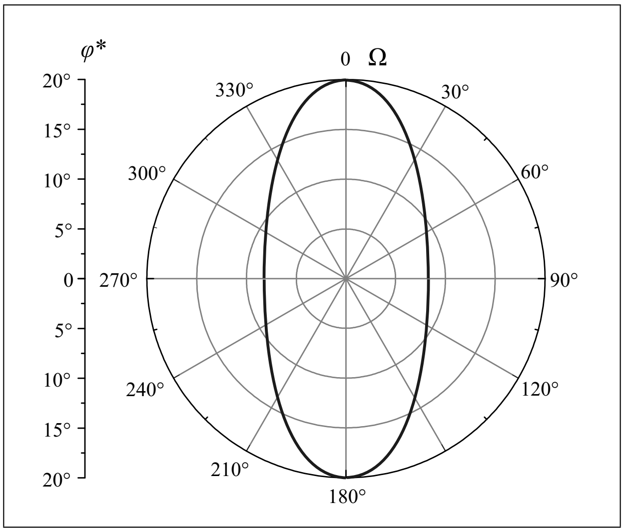

Figure 4. The propagation angle φ^* versus the rotation angle Ω in the planes around the X-axis.

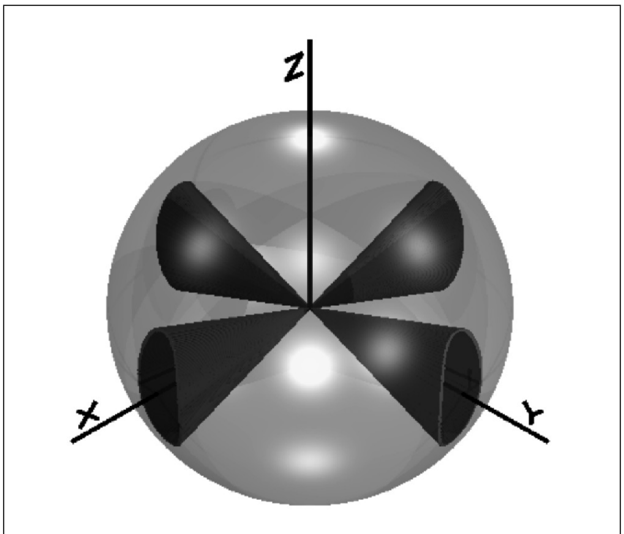

Figure 5. Range of directions at which the transformation of the acoustic wave type takes place in the paratellurite crystal.

agation angles φ^* for different values Ω at which the direction of the acoustic polarization corresponds to $\gamma = 45°$ was also the goal of the present investigation. For this purpose, we examined all directions of the acoustic propagation in the paratellurite crystal. A set of cross sections of acoustic slowness surface was carefully considered. Out of the symmetry planes, the family of curves in Figures 3b–3i shows orientation of the polarization vector. The curves were obtained by the rotation of a plane over X-axis by the angle Ω. It is seen in the graphs that at arbitrary cuts of the slowness surfaces, the unusual transformation of the wave type takes place in the crystal. This conclusion is confirmed by the fact that each graph in Figure 3 includes the angle φ^* corresponding to the transformation of the elastic wave type. For example, Figure 3b ($\Omega = 10°$) demonstrates that the angle of polarization transformation for the fast acoustic wave is equal to $\varphi^* = 18.6°$. At the same time, the rotation angle $\Omega = 20°$ corresponds to the transformation angle $\varphi^* = 16.1°$. The angle $\Omega = 30°$ is described by the value $\varphi^* = 13.5°$. Further increasing of the rotation angle Ω keeps the tendency of φ^* angle diminution. For example, at the magnitude $\Omega = 60°$, the angle of the transformation is equal to $\varphi^* = 9.1°$. Using the results of calculations we plotted the curve in a polar system of coordinates $\varphi^*(\Omega)$ (Figure 4), where the angle Ω indicates the direction, while the angle φ^* represents a radial vector. Therefore the graph $\varphi^*(\Omega)$ plotted in Figure 4 in polar coordinate system illustrates the spatial distribution of the angles φ^* at which the effect of the wave type change may be observed in tellurium dioxide. This dependence proves that there are extended regions in space in which the unusual effect may be observed. In order to clearly represent this phenomenon, we also plotted a corresponding three-dimensional picture shown in Figure 5. The figure reveals the areas in which the shear acoustic waves propagate with a higher phase velocity than the longitudinal acoustic waves. Using the presented data, it was straightforward to find, in paratellurite, all the directions characterized by the effect of the wave transformation. It should be emphasized that the examined phenomenon must be taken into consideration during the design of new acousto-optic devices such as modulators, deflectors and filters [24, 25, 26] based on off-axis cuts of the tetragonal crystals.

5. Conclusion

We examined basic parameters of acoustic waves propagating in the tetragonal crystals. We revealed the influence of the magnitudes of the elastic coefficients in the materials on the characteristics of the acoustic modes. It is proved that if the elastic constant c_{66} is larger than the constant c_{11} in the tetragonal crystals, a transformation of the quasi-longitudinal acoustic waves into the quasi-shear waves and vice versa appears. We found that this effect of the wave transformation may take place in a considerably wide range of propagation directions. We derived an analytical expression for the magnitude of the propagation angle corresponding to the transformation of the acoustic polarization in XOY-plane of the tetragonal materials. We predicted that the maximum possible transformation angle in this plane of a tetragonal crystal is limited by the value $\varphi^* = \pi/8$, while in the paratellurite crystal this angle is equal to $\varphi^* = 8.1°$. All other directions characterized by the change of acoustic wave type were also found in paratellurite. The revealed regular trend helps to predict behavior of elastic waves in the crystalline media or even to stimulate growth and development of new materials with proposed elastic properties.

Finally, it should be noted that the majority of acousto-optic devices based on the paratellurite crystal such as deflectors and tunable acousto-optic filters, use the slow shear acoustic mode propagating along off-axis directions in the crystal. As shown here, these elastic modes can change their type with variation of their direction of propagation. It means that the results of this research are useful for the design of devices based on off-axis propagation of acoustic waves. For example, in order to launch a slow acoustic mode in paratellurite along a direction close to the X-axis, one should use a piezoelectric transducer for the generation of the longitudinal waves; and not the shear waves as for all other tetragonal crystals.

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