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On the Consistent Use of Space and Time Conventions in Array Processing

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Summary

Processing of microphone arrays of various configurations involves mathematical models of the surrounding sound fields. These models are based on different space and time conventions found throughout the literature. In traditional open array processing, interchanging different space and time conventions can lead to confusion between the arrival and propagation directions. This is relatively easy to observe and adjustments can be made as necessary. However, in the case of spherical arrays, which are a more recent subject of research, the consequences of interchanging between the conventions are not so straightforward. In this paper, an analysis of the misuse of space and time conventions in array processing is presented, with special emphasis on the new results for spherical array configurations. It is shown that, in addition to the reversal of propagation direction, a misuse of notation has the potential of introducing a significant error in the array directivity pattern.

1. Introduction

In the literature on vibrations and acoustics, it is usually required to describe the space and time dependencies of the underlying physical phenomena. These dependencies can be described using several different conventions. For example, in engineering-related literature the harmonic time dependence is usually expressed using $e^{i\omega t}$ [1], while in physics and mathematics the term $e^{-i\omega t}$ is preferred [2], where $i = \sqrt{-1}$, ω is the angular frequency, and t denotes time. These differences in conventions lead to errors and inconsistencies throughout the literature. One of the first discussions of this problem, based on examples from electromagnetics, can be found in [3]. There is also a discussion of the different time dependencies in the fundamental textbook by Morse and Feshbach [4]; in particular, the consequences of changing the time dependence on the resulting expression for the acoustic impedance are described.

The use of different conventions is also found in the array processing literature. In this context, the distinctions between the conventions and the consequences of interchanging between them are generally known. The most obvious consequence is the reversal of the wave propagation direction, which is relatively easy to observe and to deal with [13]. Recently, spherical arrays have received increased attention. These arrays offer several advantages over other geometries, including inherent 3D symmetry and the fact that they facilitate processing in the spherical harmonics (SH) domain.

However, processing in the SH domain leads to the following two shortcomings related to the space and time conventions:

1. In contrast to other array geometries, the consequences of using different space and time conventions in the SH domain are less straightforward. In fact, several inconsistencies can be found in the recent literature, particularly related to spherical arrays. The examples include incomplete [5, 9, 10] and even conflicting [11, 12] descriptions of the conventions.
2. In addition to the reversal of propagation direction, the misuse of the space and time conventions in the SH framework can have a detrimental effect on the performance of various processing methods, including beamforming and source localization.

The current paper aims to clarify the ambiguities related to different space and time conventions encountered in the array processing literature, while placing an emphasis on spherical arrays. The paper starts by reviewing the different space and time conventions. Then, the consequences of using different conventions are analysed in the context of general array processing. Next, the paper presents a new analysis of the ambiguities and errors that arise in the spherical array processing context and which do not exist in the general array processing literature.

2. Wave equation and solutions

Consider a homogeneous medium with no boundaries. Denote the acoustic pressure in the medium by $p(\mathbf{x}, t)$, where $\mathbf{x} = (x, y, z)$ is a point in the standard Cartesian coordinate system. Assuming that the linear approximation holds, the pressure satisfies the following wave equation [2]:

$$\nabla^2 p(\mathbf{x}, t) = \frac{1}{c^2} \frac{\partial^2 p(\mathbf{x}, t)}{\partial t^2}, \tag{1}$$

where ∇^2 denotes the Laplacian sum of the second derivatives [2] with respect to x, y and z. Scalar c denotes the speed of sound in the medium. The four fundamental plane-wave solutions to (1) are given by [8]

$$\begin{aligned} \Psi_1(\mathbf{x}, t) &= e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}, \\ \Psi_2(\mathbf{x}, t) &= e^{i\omega t + i\mathbf{k}\cdot\mathbf{x}}, \\ \Psi_3(\mathbf{x}, t) &= e^{-i\omega t - i\mathbf{k}\cdot\mathbf{x}}, \\ \Psi_4(\mathbf{x}, t) &= e^{i\omega t - i\mathbf{k}\cdot\mathbf{x}}, \end{aligned} \tag{2}$$

where $\mathbf{k}$ is the wave vector with its norm denoted by the wave number $k = \|\mathbf{k}\|$ and related to the angular frequency through $k = \omega/c$. The direction of $\mathbf{k}$ is related to the direction of the wave. In particular, in the case where the signs before the time-dependent factor, $i\omega t$, and the space-dependent factor, $i\mathbf{k}\cdot\mathbf{x}$, in (2) are opposite, vector $\mathbf{k}$ points in the direction of the wave *propagation*. Otherwise, $\mathbf{k}$ points in the direction opposite to the wave propagation direction, which is termed the wave *arrival* direction [8]. This convention arises directly from the constant phase property of the plane wave front.

The above four plane-wave models are used interchangeably throughout the literature on array processing. The implications of the use of different models are discussed in the following section.

3. Processing of open arrays

Array processing methods are often described in the frequency domain and apply to each frequency component independently. The sound field at a given frequency is usually assumed to be composed of a (possibly infinite) sum of plane waves having various amplitudes, phases and directions. Hence, the processing of an array requires a choice of the fundamental solution to be used

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for representing a plane wave. In the current section, the choice of the plane-wave model is discussed and its consequences are analysed for use with open arrays, i.e. arrays with steering vectors consisting only of phase shifts related to the positions of the microphones. In the next section, the use of different plane-wave models will be analysed for arrays that are configured around rigid objects in a spherical geometry.

3.1. Selection of the plane-wave model

The plane-wave model is not always chosen explicitly. Nevertheless, it is determined by the different stages of array processing. The first stage is the transformation to the frequency domain. This step is carried out by applying the Fourier transform to the microphone outputs. Choice of the transform kernel determines the time dependence of the plane-wave model used, i.e. choosing the kernel to be $e^{i\omega t}$, as is preferred in the engineering-related literature, implies that either Ψ_2 or Ψ_4 is being used. Otherwise, choosing the transform kernel to be $e^{-i\omega t}$, which is more common in mathematics and physics, implies that a plane wave is modelled using either Ψ_1 or Ψ_3. The second stage of array processing, which completes the determination of the plane-wave model, is the interpretation of the direction associated with the wave vector $\mathbf{k}$. Note that in both above cases of time dependence, the wave vector $\mathbf{k}$ can point in either the propagation or the arrival direction. Choosing to interpret the direction of $\mathbf{k}$ as the arrival direction will imply that Ψ_2 or Ψ_3 are selected in cases of the positive and negative time dependencies, respectively. Otherwise, when $\mathbf{k}$ is interpreted as the propagation direction, Ψ_1 or Ψ_4 are selected instead. To summarize, the choice of the Fourier transform kernel initially determines the time dependence. This is then complemented by the interpretation of the direction of $\mathbf{k}$, which determines the space dependence of the model.

3.2. Effect on array processing

Here, we demonstrate the consequences of using different plane-wave models and of a failure to adapt the processing algorithms accordingly. The discussion uses spatial filtering or beamforming as an example of a common array processing method. The spatial response of a beamformer is given by

$$B(\mathbf{u}) = \sum_{j=1}^{M} w_j \cdot v_j(\mathbf{u}), \tag{3}$$

where M is the number of microphones in the array and $\{v_j(\mathbf{u})\}_{j=1}^{M}$ are the elements of the array steering vector [13]; these describe the response of all microphones to a unit-amplitude plane wave *travelling in* or *arriving from* the direction indicated by the unit-norm vector $\mathbf{u}$, depending on the chosen convention. Assume, for example, that we model a plane wave using Ψ_2. In this case $v_j(\mathbf{u}) = e^{i\frac{\omega}{c}\mathbf{u}\cdot\mathbf{x}_j}$, where $\{\mathbf{x}_j\}_{j=1}^{M}$ are the positions of the microphones and $\mathbf{u}$ points in the *arrival* direction. The coefficients $\{w_j\}_{j=1}^{M}$ are the array weights, which can be chosen to obtain a required spatial response. As an example, suppose that the response is required to be unity for waves arriving from the direction that $\mathbf{u}_l$ points to, while all the other directions are required to be suppressed. The direction $\mathbf{u}_l$ is usually referred to as the array *look* direction. Such a response can be obtained by using the relatively common *phased array* beamforming method. In this method, the array weights are chosen as [13]

$$w_j = \frac{1}{M} e^{-i\frac{\omega}{c}\mathbf{u}_l\cdot\mathbf{x}_j}. \tag{4}$$

In this way, the response in the direction $\mathbf{u} = \mathbf{u}_l$ is $B(\mathbf{u}_l) = 1$, while all the other directions are summed out-of-phase and, therefore, attenuated. Now, suppose that the plane-wave model is changed to Ψ_3, which can be the consequence of using a negative time dependence instead of a positive one when computing v_j. In this case, the steering-vector elements are given by $v_j(\mathbf{u}) = e^{-i\frac{\omega}{c}\mathbf{u}\cdot\mathbf{x}_j}$, while $\mathbf{u}$ still represents the arrival direction. Now, using the array weights in (4) without the appropriate adaptation will yield a unit response for waves arriving from the direction opposite to the required one, i.e. $\mathbf{u} = -\mathbf{u}_l$. Hence, changing the time dependence of the plane-wave model without the appropriate adaptation of the array weights has the potential to reverse the array spatial response. Similarly, changing the spatial dependence of the plane-wave model and misinterpreting the direction of the wave vector will reverse the response of the filter.

To summarize, for open arrays, either of the four fundamental solutions can be used to represent a plane wave, while failure to adopt the processing algorithm appropriately can cause confusion between arrival and propagation directions. In contrast to open arrays, interchanging the plane-wave models when using rigid spherical arrays may cause errors and reduced performance in various processing methods. This is discussed in the following section.

4. Processing of rigid spherical arrays

In the last decade, spherical microphone arrays have been developed and analyzed for a wide range of applications [5]. One of the most common spherical microphone array configurations is based on a rigid sphere with microphones positioned on its surface [6, 7, 5], providing improved numerical conditioning as compared to that obtained with the open sphere configuration [5]. In the literature on rigid spherical arrays, similarly to the literature on open array processing, the sound field component at a given frequency is usually assumed to be composed of plane waves. The acoustic pressure on the surface of a rigid sphere due to an incident single unit-amplitude plane wave is commonly expressed as [5]

$$p(\theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} v_{nm}(\theta_0, \phi_0) Y_n^m(\theta, \phi), \tag{5}$$

where (θ, ϕ) denotes the standard spherical coordinate system [8] with its origin located at the center of the sphere. Function $Y_n^m(\cdot)$ denotes the spherical harmonic (SH) of order n and degree m and (θ_0, ϕ_0) denotes the plane wave arrival or propagation direction, depending on the time and space conventions that are employed. The coefficients $v_{nm}(\theta_0, \phi_0)$ are the Spherical Fourier Transform (SFT) coefficients of the acoustic pressure due to the incident plane wave and are given by

$$v_{nm}(\theta_0, \phi_0) = b_n^{(q)}(kr)[Y_n^m(\theta_0, \phi_0)]^*. \tag{6}$$

The expressions for coefficients $b_n^{(q)}$, in accordance with the plane-wave model indicated by q, are summarized in Table I. In Table I, the wave vector $\mathbf{k}$ points in the direction (θ_0, ϕ_0) with magnitude k, $j_n(\cdot)$ is the spherical Bessel function of the first kind, and $h_n^{(1)}(\cdot)$ and $h_n^{(2)}(\cdot)$ are the spherical Hankel functions of the first and second kinds, respectively, with n indicating their order and $(\cdot)'$ denoting the derivative of the function. A derivation of the expression for $b_n^{(1)}$ in Table I can be found in [8]. A derivation for the other expressions can be obtained by changing the signs and following similar steps, while keeping in mind that the scattered part of the field should propagate outwards from the sphere.

Table I. Coefficients $b_n^{(q)}$ for each of the four plane-wave models along with the associated incident part of the field, the interpretation of (θ_0, ϕ_0) with regard to the direction of the incident part, and the appropriate Fourier transform kernel $\kappa(t, \omega)$. For completeness, the Fourier transform of $p(t)$ is defined as: $p(\omega) = \int_{-\infty}^{\infty} p(t)\kappa^*(t, \omega)dt$.

Model	$\kappa(t, \omega)$	incident	interpretation of (θ_0, ϕ_0)	$b_n^{(q)}(kr)$
Ψ_1	$e^{-i\omega t}$	$e^{i\mathbf{k}\cdot\mathbf{x}}$	propagation	$b_n^{(1)}(kr) = 4\pi i^n\left(j_n(kr) - \frac{j_n'(kr)}{h_n^{(1)'}(kr)} h_n^{(1)}(kr)\right)$
Ψ_2	$e^{i\omega t}$	$e^{i\mathbf{k}\cdot\mathbf{x}}$	arrival	$b_n^{(2)}(kr) = 4\pi i^n\left(j_n(kr) - \frac{j_n'(kr)}{h_n^{(2)'}(kr)} h_n^{(2)}(kr)\right)$
Ψ_3	$e^{-i\omega t}$	$e^{-i\mathbf{k}\cdot\mathbf{x}}$	arrival	$b_n^{(3)}(kr) = 4\pi(-i)^n\left(j_n(kr) - \frac{j_n'(kr)}{h_n^{(1)'}(kr)} h_n^{(1)}(kr)\right)$
Ψ_4	$e^{i\omega t}$	$e^{-i\mathbf{k}\cdot\mathbf{x}}$	propagation	$b_n^{(4)}(kr) = 4\pi(-i)^n\left(j_n(kr) - \frac{j_n'(kr)}{h_n^{(2)'}(kr)} h_n^{(2)}(kr)\right)$

The spatial response of a beamformer based on a spherical microphone array can be expressed in a similar manner to (3). The response is typically written in the SH domain as

$$B(\theta_0, \phi_0) = \sum_{n=0}^{N} \sum_{m=-n}^{n} w_{nm}^* v_{nm}(\theta_0, \phi_0). \tag{7}$$

Coefficients w_{nm} are the SFT coefficients of the complex conjugate of the array weighting function and $N = \lceil kr \rceil$ is the effective SH order of the sound field [5]. Similarly to (3), these coefficients can be chosen to obtain a required spatial response. A common choice for w_{nm} is the maximum-directivity beamformer [5], for which the weights are given by

$$w_{nm}^* = Y_n^m(\theta_l, \phi_l)/b_n^{(q)}(kr), \tag{8}$$

where (θ_l, ϕ_l) is the array look direction.

Similarly to the processing of open arrays, the misuse of time and space conventions may lead to ambiguities and errors. In order to show this, we divide the array processing procedure into three steps:

Step I: Calculation of the Fourier transform of the individual microphone outputs, i.e. time domain to frequency domain transformation. The outcome of this step depends on the kernel of the Fourier transform, which may have either positive time dependence, $e^{i\omega t}$, or negative time dependence, $e^{-i\omega t}$.

Step II: Calculation of the SFT at each frequency, i.e. space domain to SH domain transformation [5]. Note from (5) that the acoustic pressure on the sphere surface, $p(\theta, \phi)$, is related to its SFT coefficients, v_{nm}, through $Y_n^m(\theta, \phi)$, which does not depend on the choice of the plane-wave model. Hence, the calculation in this step has no effect on the choice of the time and space conventions.

Step III: Calculation of the array output using (7). The array weights, w_{nm}, are obtained through (8), where $b_n^{(q)}$ should be chosen from Table I in accordance with the Fourier transform kernel used in Step I and the preferred interpretation of (θ_0, ϕ_0). Failure to recognize this can lead to errors. The nature of these errors may differ from the errors encountered in open array processing, as is demonstrated in the following section.

5. Model errors in b_n

In this section we discuss the consequences of selecting $b_n^{(q)}$ in Step III that is inconsistent with the choice of the Fourier transform kernel in Step I and/or the interpretation of (θ_0, ϕ_0) in Step III. Consider a general case where the kernel and the interpretation were chosen such that the correct coefficients that should be used for the calculation of the array weights in (8) are $b_n^{(q_{corr})}$. Suppose that incorrect coefficients $b_n^{(q_{incorr})}$ are employed instead, with $q_{incorr} \neq q_{corr}$. In this case, the incorrect beamformer weights, $\left[w_{nm}^{incorr}\right]^*$, are related to the correct weights, $\left[w_{nm}^{corr}\right]^*$, through:

$$\begin{aligned}\left[w_{nm}^{incorr}\right]^* &= \frac{Y_n^m(\theta_l, \phi_l)}{b_n^{(q_{incorr})}} \\ &= \frac{Y_n^m(\theta_l, \phi_l)}{b_n^{(q_{corr})}} \frac{b_n^{(q_{corr})}}{b_n^{(q_{incorr})}} \\ &= \left[w_{nm}^{corr}\right]^* \frac{b_n^{(q_{corr})}}{b_n^{(q_{incorr})}}.\end{aligned} \tag{9}$$

Hence, the error factor in the array weights is given by:

$$\epsilon_n(kr) = \frac{b_n^{(q_{corr})}}{b_n^{(q_{incorr})}}. \tag{10}$$

Next, note from Table I that there are two different kinds of error that may arise. The first is an error in the kind of Hankel function, as would be the case if $b_n^{(1)}$ is confused with $b_n^{(2)}$. The second error is the sign before i, as would be the case if $b_n^{(1)}$ is confused with $b_n^{(3)}$. The other possible combinations can be decomposed into the above two cases.

5.1. Incorrect kind of the Hankel function

Error in the kind of Hankel function is equivalent to an error in the time dependence of the model, i.e. $e^{\pm i\omega t}$ (see Table I). In order to demonstrate its consequences, suppose that the weights of the maximum-directivity beamformer in (8) are calculated using $b_n^{(2)}$ instead of $b_n^{(1)}$. In this case, the error factor in the array weights is $\epsilon_n(kr) = b_n^{(1)}(kr)/b_n^{(2)}(kr)$. Recall that $j_n(kr)$ is real valued and $h_n^{(1)}(kr) = \left[h_n^{(2)}(kr)\right]^*$. Utilizing these properties and using the Wronskian relation for spherical Bessel functions [8] yields

$$\begin{aligned}\epsilon_n(kr) &= e^{-2i\angle\left(j_n(kr) - \frac{j_n'(kr)}{h_n^{(1)'}(kr)} h_n^{(1)}(kr)\right)} \\ &= -e^{i2\angle h_n^{(1)'}(kr)}.\end{aligned} \tag{11}$$

It can be shown [8] that $\epsilon_n(kr) \xrightarrow{n \gg kr} 1$. However, as can be seen from Figure 1, a substantial error is introduced at higher frequencies and lower orders.

The effect of this error on array processing can be demonstrated by considering the response of a spatial filter given by (7). In order to calculate the response, the coefficients $v_{nm}(\theta_0, \phi_0)$ were simulated according to (6) by using $b_n^{(1)}$. Next, if the

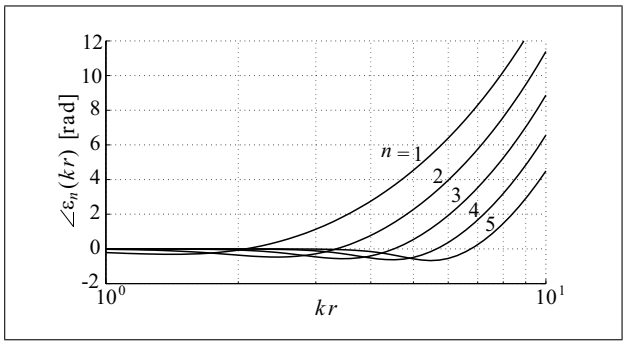


Figure 1. Plot of the phase of the error factor $\angle\epsilon_n(kr)$.

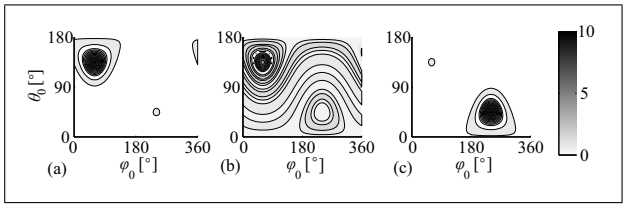


Figure 2. Comparison of spatial responses when using (a) correct $b_n^{(q)}$, (b) incorrect time, and (c) and incorrect spatial dependencies.

array weights are calculated in (8) using the correct coefficients, i.e. $b_n^{(1)}(kr)$, then the spatial response in (7) reduces to $B(\theta_0, \phi_0) = \sum_{n=0}^{N}\sum_{m=-n}^{n} Y_n^m(\theta_l, \phi_l)\left[Y_n^m(\theta_0, \phi_0)\right]^*$. An example of this response is plotted in Figure 2a, where $kr = 7$, $(\theta_l, \phi_l) = (135°, 60°)$, and $N = 7$. It can be seen that, as expected, the response is maximal in the array look direction and is reduced for other directions.

Now, in order to see the effect of an error in the kind of Hankel function used, suppose that instead of using $b_n^{(1)}$, the array weights are erroneously calculated as $w_{nm}^* = Y_n^m(\theta_l, \phi_l)/b_n^{(2)}(kr)$. In this case, substitution of the erroneous w_{nm}^* into (7) will result in the error factor, $\epsilon_n(kr)$, given in (11), i.e. the response will be

$$B(\theta_0, \phi_0) = \sum_{n=0}^{N}\sum_{m=-n}^{n}\left(-\mathrm{e}^{\mathrm{i}2\angle h_n^{(1)'}(kr)}\right) Y_n^m(\theta_l, \phi_l)\left[Y_n^m(\theta_0, \phi_0)\right]^*.$$

An example of this effect is plotted in Figure 2b. It can be seen that the response differs significantly from the correct response in Figure 2a. As discussed above, the effect of this error is expected to be stronger for higher frequencies and lower orders.

5.2. Incorrect sign before i

An error in the sign before i is equivalent to an error in the space dependence of the incident plane wave, i.e. $\mathrm{e}^{\pm\mathrm{i}\mathbf{k}\cdot\mathbf{x}}$ (see Table I). In order to analyze its consequences, suppose that the weights of the maximum-directivity beamformer in (8) are calculated using $b_n^{(3)}$ instead of $b_n^{(1)}$. In this case, the error factor is $\epsilon_n(kr) = b_n^{(1)}(kr)/b_n^{(3)}(kr) = (-1)^n$. In order to see the effect of this error, note that $(-1)^n Y_n^m(\theta_l, \phi_l) = Y_n^m(\pi - \theta_l, \phi_l + \pi)$. Direction $(\pi - \theta_l, \phi_l + \pi)$ denotes the reciprocal direction with respect to (θ_l, ϕ_l) [8]. Hence, the array look direction in (8) is reversed. Thus, similarly to open arrays, an incorrect sign before i inverts the spatial response of the filter, as demonstrated in Figure 2c.

6. Conclusion

The potential ambiguities found in array processing related to inconsistent use of space and time conventions have been discussed. The discussion started with an overview of the ambiguities in conventional open array processing. Then, a new analysis of the ambiguities was presented for the more recent spherical array systems. It was shown that, in addition to the confusion between arrival and propagation directions encountered in open arrays, a misuse of space and time conventions in spherical arrays may lead to a significant error in the response of spatial filters.

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