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Volterra Filtering Scheme using Generalized Variable Step-size NLMS Algorithm for Nonlinear Acoustic Echo Cancellation

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Summary

This correspondence presents a nonlinear acoustic echo cancellation algorithm, which includes two distinct modules in cascade. The first module is a polynomial Volterra filter, which is an equivalent paradigm for a loudspeaker with nonlinear distortion. The second module in the presented cascaded structure is a linear tapped-delay- line (finite impulse response) filter, which is analogous to the impulse response of the acoustic path. In the proposed adaptive structure, the adaptive nonlinear filter in the first module tackles the nonlinear constituents of the Volterra model, which uses the conventional fixed step-size normalized least mean square (FSS-NLMS) algorithm. However, the adaptive linear filter in the second module deals with the linear constituents of the Volterra model as well as the linear impulse response of the acoustic path, in which the generalized variable step-size (GVSS) NLMS algorithm is incorporated to suppress the adverse effects of nonstationarity / distortion. Computer simulation results demonstrate that the presented GVSS-NLMS algorithm based approach outperforms the FSS-NLMS algorithm based Volterra filtering, as far as convergence and tracking characteristics are concerned. In simulations of the real-time environment and appropriate parameter setting for the third-order polynomial model, it provides approximately 5 dB performance advantage over the conventional nonlinear filtering approach in the tracking mode, in terms of the reduction in mean square error. Moreover, the presented adaptive technique exhibits lower computational complexity than the conventional NLMS based polynomial Volterra filtering used for the acoustic echo cancellation.

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1. Introduction

Nonlinear echo in acoustic systems arises due to the nonlinear characteristics of amplifiers/loudspeakers, which degrades the performance of speech and audio communication systems. These nonlinear echo components can not be cancelled by conventional linear acoustic echo cancellers (AECs). High quality multimedia services require echo-free speech and audio transmission and reception. However, a nonlinear echo of saturation-type is commonly perceived as a major degradation factor [1]. Various nonlinear AECs have been presented in the literature [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12], but form only the tip of the iceberg in the domain of audio and speech signal processing that combat nonlinear distortions. For mild nonlinear distortions, second – order Volterra filters are found to be efficient. However, saturation-type nonlinear distortions require the application of higher-order polynomial Volterra filters, which encounter a high eigenvalue spread of the input signal vector and also the problem of over-parameterization. This leads to an alleviated rate of convergence for such nonlinear AECs [6].

The affine projection and recursive least squares algorithms are commonly used to deal with ill- conditioned autocorrelation matrix of the input signal, which has deleterious effects on the performance characteristics of LMS algorithm based linear AECs [13, 14]. Moreover, saturation [15] and nonlinearities containing memory [16] have severe influence on classical linear AECs. For a loudspeaker nonlinearity with memory, the efficiency of second order Volterra filters (SOVFs) supersedes linear adaptive filters [17, 18], in which the number of coefficients grows with the square of the memory size of SOVF. It is reported in [18] that the response of SOVF is concentrated around the main section of the linear response, which corresponds to the delay introduced due to the linear acoustic path. However, the computational complexity can be reduced by decomposing the higher-order Volterra kernels into the product of lower-order kernels [19, 20].

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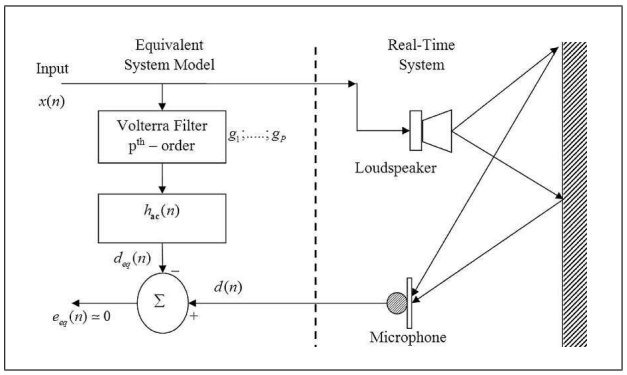

Figure 1. Real-time loudspeaker, acoustic path and microphone assembly with equivalent mathematical system model.

In this paper, we consider a cascaded structure consisting of two modules. The first module identifies the loudspeaker parameters responsible for the nonlinearity, which is based on a polynomial Volterra paradigm. The second module identifies the parameters of the linear acoustic path impulse response as in [21]. The normalized-LMS (NLMS) adaptive algorithm is proposed as an amicable solution in [21] to mitigate the instability problem arising due to the filter misadjustment in the transient phase. However, the nonlinear constituents vary rapidly with the acoustic path due to the effect of their convolution, which also restricts the convergence rate of adaptive algorithms. To enhance the convergence rate in the presence of ill-conditioned autocorrelation matrix of the input signal, we propose the usage of GVFF-NLMS [22] adaptive algorithm for AEC. The presented adaptive cascaded structure processes the second- and third-order Volterra kernels in the first module. Subsequently, the second module processes the first-order Volterra kernels and the linear acoustic path channel coefficients jointly, as in [21]. It reduces the computational complexity as compared to the recursive least squares algorithm based approaches for AEC.

This paper is organized as follows. We first describe the mathematical system modeling of the loudspeaker and linear acoustic path in Section 2 (as shown in Figure 1). We next provide details of the proposed adaptive filtering paradigm for the nonlinear system identification in Section 3. Subsequently, the application of GVSS-NLMS is introduced for the adaptive acoustic echo cancellation using Volterra filtering. Further, the simulation results are presented in Section 4 to demonstrate the efficacy of the presented scheme, by comparing it with the conventional NLMS algorithm based AECs using the polynomial filtering approach. Finally, concluding remarks and future scope are presented in Section 5.

2. Nonlinear system model for loudspeaker and acoustic path

Let us consider a cascaded approach to model loudspeaker and acoustic path (as shown in Figure 1). The loudspeaker is assumed to be a polynomial filter because these conventionally have memory embedded in their nonlinear part, which usually results in harmonic distortion (based on the Taylor series expansion [23]). At the nth instant of time, the input to the loudspeaker is $x(n)$. The corresponding output of loudspeaker is given as

$$\begin{aligned} x_{vf}(n) \;=\; & \sum_{\alpha_1=0}^{+\infty} g_1(\alpha_1;n)\, x(n-\alpha_1) \\ & + \sum_{\alpha_1=0}^{+\infty}\sum_{\alpha_2=\alpha_1}^{+\infty} g_2(\alpha_1,\alpha_2;n)\, x(n-\alpha_1)\, x(n-\alpha_2) \\ & + \cdots + \sum_{\alpha_1=0}^{+\infty}\sum_{\alpha_2=\alpha_1}^{+\infty} \cdots \\ & \sum_{\alpha_p=\alpha_{p-1}}^{+\infty} g_p(\alpha_1,\alpha_2,\ldots,\alpha_p;n)\cdot x(n-\alpha_1)\ldots x(n-\alpha_p), \end{aligned} \tag{1}$$

which is based on the causal pth order nonlinear Volterra filter response. The kernel g_1 is responsible for the linear response, and the kernels $g_2, g_3, \ldots, g_p$ are responsible for the higher-order nonlinear response. Based on audio and speech signal analysis under realistic practical conditions, it is demonstrated in [21] that the third-order Volterra model is a close approximation of the loudspeaker nonlinear response, such that

$$x_{eq}(n) \;=\; x_{vf}(n)\,\big|_{p=3} = x_1(n) + x_2(n) + x_3(n), \tag{2}$$

with

$$x_1(n) \;=\; \sum_{\alpha_1=0}^{+\infty} g_1(\alpha_1;n)\, x(n-\alpha_1) = g_1(n) * x(n), \tag{3}$$

$$x_2(n) \;=\; \sum_{\alpha_1=0}^{+\infty}\sum_{\alpha_2=\alpha_1}^{+\infty} g_2(\alpha_1,\alpha_2;n)\, x(n-\alpha_1)\, x(n-\alpha_2), \tag{4}$$

$$\begin{aligned} x_3(n) \;=\; & \sum_{\alpha_1=0}^{+\infty}\sum_{\alpha_2=\alpha_1}^{+\infty}\sum_{\alpha_3=\alpha_2}^{+\infty} g_3(\alpha_1,\alpha_2,\alpha_3;n) \\ & \cdot\, x(n-\alpha_1)\, x(n-\alpha_2)\, x(n-\alpha_3), \end{aligned} \tag{5}$$

where the symbol * indicates the linear convolution operator. If the impulse response of the linear acoustic path is $h_{ac}(n)$, then it can be shown that

$$x_{23}(n) \;=\; x_2(n) + x_3(n). \tag{6}$$

The output of presented equivalent system model is

$$\begin{aligned} d_{eq}(n) \;&=\; h_{ac}(n) * x_1(n) + h_{ac}(n) * x_{23}(n) \\ &=\; h_{ac}(n) * g_1(n) * x(n) + h_{ac}(n) * x_{23}(n). \end{aligned} \tag{7}$$

The linear convolution of the acoustic path impulse response with the nonlinear input $x_{23}(n)$ makes the situation more cumbersome to handle. If the actual output of microphone is $d(n)$, then for the mathematical equivalence of two models in Figure 1, the stringent requirement is $d(n) - d_{eq}(n) = e_{eq}(n) \simeq 0$.

3. Adaptive filtering paradigm for AEC using GVSS-NLMS

To design the adaptive filtering paradigm for AEC, Equation (7) is rearranged to provide

$$d_{eq}(n) = h_{eq}(n) * x(n) + h_{ac}(n) * x_{23}(n), \quad (8)$$

with $h_{eq}(n) = h_{ac}(n) * g_1(n)$.

The discrete-time Fourier transform on both sides of the above equation results in

$$D_{eq}\{e^{j\omega}\} = H_{eq}\{e^{j\omega}\}\left[X\{e^{j\omega}\} + \frac{H_{ac}\{e^{j\omega}\}}{H_{eq}\{e^{j\omega}\}} X_{23}\{e^{j\omega}\}\right]. \quad (9)$$

Let us consider that

$$\hat{H}\{e^{j\omega}\} = H_{ac}\{e^{j\omega}\}/H_{eq}\{e^{j\omega}\}, \quad (10)$$

$$\hat{H}\{e^{j\omega}\} \xleftrightarrow{F} \hat{h}(n) \quad \text{(Fourier transform pair)}. \quad (11)$$

Taking the inverse discrete-time Fourier transform of Equation (9) gives

$$\begin{aligned} d_{eq}(n) &= h_{eq}(n) * \left[x(n) + \hat{h}(n) * x_{23}(n)\right] \\ &= h_{eq}(n) * x_{eq}(n), \end{aligned} \quad (12)$$

where $x_{eq}(n) = \delta(n) * x(n) + \hat{h}(n) * x_{23}(n)$. (13)

Considering the damping nature of real-time systems [21], the upper limit of the variables α_1, α_2 and α_3 in the summations used in Equations (3)–(5) is kept at $L-1$. Further, it is presumed that

$$\begin{aligned} \hat{h}(n) * x_{23}(n) \simeq & \quad (14) \\ & \sum_{\alpha_1=0}^{L-1} \sum_{\alpha_2=\alpha_1}^{L-1} \hat{g}_2(\alpha_1, \alpha_2; n)\, x(n-\alpha_1)\, x(n-\alpha_2) \\ & + \sum_{\alpha_1=0}^{L-1} \sum_{\alpha_2=\alpha_1}^{L-1} \sum_{\alpha_3=\alpha_2}^{L-1} \hat{g}_3(\alpha_1, \alpha_2, \alpha_3; n) \\ & \qquad \cdot\, x(n-\alpha_1)\, x(n-\alpha_2)\, x(n-\alpha_3), \end{aligned}$$

$$\begin{aligned} x_{eq}(n) = x(n) & \quad (15) \\ & + \sum_{\alpha_1=0}^{L-1} \sum_{\alpha_2=\alpha_1}^{L-1} \hat{g}_2(\alpha_1, \alpha_2; n)\, x(n-\alpha_1)\, x(n-\alpha_2) \\ & + \sum_{\alpha_1=0}^{L-1} \sum_{\alpha_2=\alpha_1}^{L-1} \sum_{\alpha_3=\alpha_2}^{L-1} \hat{g}_3(\alpha_1, \alpha_2, \alpha_3; n) \\ & \qquad \cdot\, x(n-\alpha_1)\, x(n-\alpha_2)\, x(n-\alpha_3), \end{aligned}$$

where δ is the impulse signal indicator, $\hat{g}_2$ and $\hat{g}_3$ are the second-order and third-order equivalent Volterra filter kernels. The above equation is rewritten as

$$x_{eq}(n) = x(n) + x_{\hat{g}_2}(n) + x_{\hat{g}_3}(n), \quad (16)$$

where

$$x_{\hat{g}_2}(n) = \vec{S}_2(n)\, \hat{G}_2(n), \quad (17)$$

with

$$\vec{S}_2^T(n) = \begin{bmatrix} x^2(n) & x^2(n-1) & \cdots & x^2(n-N+1) \\ x(n)x(n-1) & x(n-1)x(n-2) & \cdots & \cdots \\ \cdots & & & \\ \cdots & & & \\ x(n)x(n-L+1) & \cdots & & \\ x^2(n-1) & \cdots & & \\ \cdots & & & \\ \cdots & & & \\ x^2(n-L+1) & \cdots & & x^2(n-L-N+2) \end{bmatrix}, \quad (18)$$

$$\hat{G}_2(n) = \begin{bmatrix} \hat{g}_2(0,0;n) \\ \hat{g}_2(0,1;n) \\ \cdots \\ \cdots \\ \hat{g}_2(0,L-1;n) \\ \hat{g}_2(1,1;n) \\ \cdots \\ \cdots \\ \hat{g}_2(L-1,L-1;n) \end{bmatrix}, \quad (19)$$

where the dimension of $\hat{G}_2(n)$ is $N_2 \times 1$ with $N_2 = L(L+1)/2$, and the dimension of $\vec{S}_2(n)$ is $N \times N_2$. Similarly, it is clear for the third-order statistics that

$$x_{\hat{g}_3}(n) = \vec{S}_3(n)\, \hat{G}_3(n), \quad \text{(as in [21])}, \quad (20)$$

where the dimension of $\hat{G}_3(n)$ is $N_3 \times 1$ with $N_3 = L(L+1)(L+2)/6$, and the dimension of $\vec{S}_3(n)$ is $N \times N_3$. To introduce adaptive characteristics in the underlying system, Equation (16) can be modified as

$$\hat{x}_{eq}(n) = x(n) + \hat{x}_2(n) + \hat{x}_3(n), \quad (21)$$

with

$$\hat{x}_2(n) = \vec{S}_2(n)\, \vec{\boldsymbol{w}}_2(n), \quad (22)$$

$$\vec{\boldsymbol{w}}_2(n) = \begin{bmatrix} w_2(0,0;n) \\ w_2(0,1;n) \\ \cdots \\ \cdots \\ w_2(0,L-1;n) \\ w_2(1,1;n) \\ \cdots \\ \cdots \\ w_2(L-1,L-1;n) \end{bmatrix}. \quad (23)$$

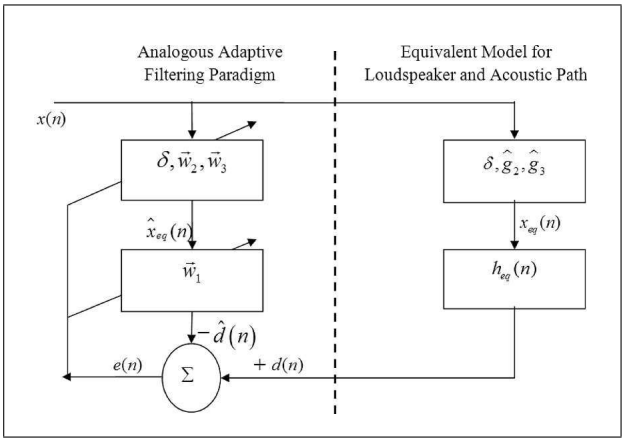

Figure 2. Equivalent loudspeaker, acoustic path and microphone assembly with adaptive system model for AEC.

Similarly,

$$\hat{x}_3(n) = \vec{S}_3(n)\,\vec{\boldsymbol{w}}_3(n), \tag{24}$$

with the adaptive second-order and third-order weight vectors $\vec{\boldsymbol{w}}_2(n)$ and $\vec{\boldsymbol{w}}_3(n)$ of the Volterra filter respectively, as shown in Figure 2. The dimensions of $\vec{\boldsymbol{w}}_2(n)$ and $\vec{\boldsymbol{w}}_3(n)$ are $N_2 \times 1$ and $N_3 \times 1$ respectively.

The error signal between the original received loudspeaker signal at the microphone and the output of the adaptive filtering system is

$$e(n) = d(n) - \hat{d}(n), \tag{25}$$

with

$$\hat{d}(n) = \hat{\boldsymbol{x}}_{eq}^T(n)\,\vec{\boldsymbol{w}}_1(n), \tag{26}$$

$$\vec{\boldsymbol{x}}_{eq}(n) = \left[x_{eq}(n)\, x_{eq}(n-1) \ldots x_{eq}(n-N+1)\right]^T, \tag{27}$$

$$\hat{\boldsymbol{x}}_{eq}(n) = \left[\hat{x}_{eq}(n)\, \hat{x}_{eq}(n-1) \ldots \hat{x}_{eq}(n-N+1)\right]^T, \tag{28}$$

and the first-order adaptive weight vector of the Volterra filter is

$$\vec{\boldsymbol{w}}_1(n) = \left[w_1(0;n)\, w_1(1;n) \ldots w_1(N-1;n)\right]^T, \tag{29}$$

As per the adaptive theory, the cost function is defined as

$$J(n) = E\left[e^2(n)\right], \tag{30}$$

where $E[.]$ is the expectation operator [24] and $(.)^T$ is the matrix transpose operator. In the above scenario, minimization of the above cost function leads to the weight update equations for the conventional LMS filter [21] as

$$\vec{\boldsymbol{w}}_1(n+1) = \vec{\boldsymbol{w}}_1(n) + \mu_v(n)e(n)\nabla_{\vec{\boldsymbol{w}}_1}\left\{\hat{\boldsymbol{x}}_{eq}^T(n)\vec{\boldsymbol{w}}_1(n)\right\}. \tag{31}$$

In the above equation, $\nabla_{\vec{\boldsymbol{w}}_1}$ is the gradient with respect to $\vec{\boldsymbol{w}}_1$. Its solution gives

$$\vec{\boldsymbol{w}}_1(n+1) = \vec{\boldsymbol{w}}_1(n) + \mu_v(n)e(n)\hat{\boldsymbol{x}}_{eq}(n), \tag{32}$$

where the variable step-size $\mu_v(n)$ controls the convergence and tracking characteristics of the adaptive algorithm in the linear part of the presented cascaded model. However, the VSS is bounded in the domain $0 < \mu_v(n) \leq 2/3tr(\vec{R}_{xxeq})$; where the autocorrelation matrix is given as $\vec{R}_{xxeq} = E[\hat{\boldsymbol{x}}_{eq}(n)\hat{\boldsymbol{x}}_{eq}^T(n)]$. Similarly, for the second-order adaptive weight vector [21], it is apparent that

$$\vec{\boldsymbol{w}}_2(n+1) = \vec{\boldsymbol{w}}_2(n) + \mu_s e(n)\nabla_{\vec{\boldsymbol{w}}_2}\left\{\hat{\boldsymbol{x}}_{eq}^T(n)\vec{\boldsymbol{w}}_1(n)\right\} \tag{33}$$

In the above equation, $\nabla_{\vec{\boldsymbol{w}}_2}$ is the gradient with respect to $\vec{\boldsymbol{w}}_2$. Its elucidation provides

$$\vec{\boldsymbol{w}}_2(n+1) = \vec{\boldsymbol{w}}_2(n) + \mu_s e(n)\nabla_{\vec{\boldsymbol{w}}_2} \cdot \left\{\vec{\boldsymbol{w}}_1^T(n)\left[x(n) + \vec{S}_2(n)\vec{\boldsymbol{w}}_2(n) + \hat{x}_3(n)\right]\right\}, \tag{34}$$

$$\vec{\boldsymbol{w}}_2(n+1) = \vec{\boldsymbol{w}}_2(n) + \mu_s e(n)\nabla_{\vec{\boldsymbol{w}}_2} \cdot \left\{\vec{\boldsymbol{w}}_1^T(n)\vec{S}_2(n)\vec{\boldsymbol{w}}_2(n)\right]\right\}, \tag{35}$$

$$\vec{\boldsymbol{w}}_2(n+1) = \vec{\boldsymbol{w}}_2(n) + \mu_s e(n)\left\{\vec{S}_2^T(n)\vec{\boldsymbol{w}}_1(n)\right\}. \tag{36}$$

Akin to the above solution, it can be shown for the third-order adaptive weight vector [21] that

$$\vec{\boldsymbol{w}}_3(n+1) = \vec{\boldsymbol{w}}_3(n) + \mu_s e(n)\left\{\vec{S}_3^T(n)\vec{\boldsymbol{w}}_1(n)\right\}, \tag{37}$$

where the fixed step-size μ_s controls the convergence and tracking characteristics of the adaptive algorithm in the nonlinear part of presented cascaded model. However, the FSS is bounded in the range $0 < \mu_s < 2$. To incorporate the stability norm in the above adaptive AEC, Guerin *et al.* [21] have suggested the usage of gradient based adaptive NLMS algorithm [25], which results in

$$\vec{\boldsymbol{w}}_1(n+1) = \vec{\boldsymbol{w}}_1(n) + \mu_v(n)e(n)\frac{\hat{\boldsymbol{x}}_{eq}(n)}{||\hat{\boldsymbol{x}}_{eq}(n)||_2^2}, \tag{38}$$

$$\vec{\boldsymbol{w}}_2(n+1) = \vec{\boldsymbol{w}}_2(n) + \mu_s e(n)\frac{\{\vec{S}_2^T(n)\vec{\boldsymbol{w}}_1(n)\}}{||\vec{S}_2^T(n)\vec{\boldsymbol{w}}_1(n)||_2^2}, \tag{39}$$

$$\vec{\boldsymbol{w}}_3(n+1) = \vec{\boldsymbol{w}}_3(n) + \mu_s e(n)\frac{\{\vec{S}_3^T(n)\vec{\boldsymbol{w}}_1(n)\}}{||\vec{S}_3^T(n)\vec{\boldsymbol{w}}_1(n)||_2^2}, \tag{40}$$

where $||.||_2$ is the L_2 norm [9]. We now focus on the incorporation of GVSS in Equation (32). The GVSS criterion is favorable in the tracking of a slowly time-varying environment [22]. The VSS must increase or decrease as the mean square error (MSE) increases or decreases, allowing the adaptive filter to track changes in the underlying system and thus to produce a small MSE. It also reduces the tradeoff between misadjustment and the speed of adaptation, due to its inherent capability of providing both fast tracking as well as small misadjustment. Therefore, the generalized variable step-size (GVSS) criterion is proposed to fine-tune the step-size under the influence of a stationary as well as nonstationary environment, which is

$$\mu_v(n) = \overline{\alpha}\mu_v(n-1) + \overline{\gamma}J_1(n-1) + \overline{\beta}J_2(n), \tag{41}$$

where

$$J_1(n) = \sum_{\bar{p}=0}^{\bar{P}} \lambda_1^{\bar{p}} e(n)e(n-\bar{p}) \quad \text{with } 0 \le \lambda_1 < 1, \tag{42}$$

$$J_2(n) = \left[\sum_{\bar{q}=1}^{\bar{Q}} \lambda_2^{\bar{q}} e(n-\bar{q})\hat{\boldsymbol{x}}_{eq}^T(n-\bar{q})\right] \hat{\boldsymbol{x}}_{eq}(n)e(n) \quad \text{with } 0 \le \lambda_2 < 1, \tag{43}$$

where the parameter values are adjusted in the domain $0 < \overline{\alpha} \le 1$, $0 \le \overline{\gamma} < 1$ and $0 \le \overline{\beta} < 1$. The parameter $\overline{\alpha}$ stimulates the global exponential forgetting of the VSS, the parameter $\overline{\gamma}$ controls the convergence time and the level of misadjustment [22], and the parameter $\overline{\beta}$ adjusts the adaptive behavior of step-size $\mu_v(n)$. However, λ_1 and λ_2 are the local exponential forgetting factors in Equation (42) and Equation (43) respectively. For the appropriate convergence, the value of VSS should be bounded in the range $\mu_{\min} \le \mu_v(n) \le \mu_{\max}$ [22]. The initial step-size is usually taken as $\mu_{\max}$, therefore the MSE of algorithm remains bounded. However, the value of $\mu_{\min}$ is chosen to provide the minimum level of tracking ability. The advantage of cascaded system model (as depicted in Figure 2) is a reduction in the number of filter coefficients due to the segregation of linear and nonlinear coefficients in the presented adaptive method.

4. Simulation results

We shall investigate the behavior of the presented adaptive Volterra filtering scheme using the GVSS- NLMS algorithm for the nonlinear acoustic echo cancellation. We first consider a known second-order and third-order nonlinearity to test the efficacy of the proposed technique. We next take the samples of a real-time loudspeaker signal recorded with a microphone, which is similar to the setup considered in [12]. The major appraisal factor is the echo-return-loss-enhancement (ERLE), which is used to compare the performance of the presented adaptive technique with the existing method given in [21]. Here,

$$\text{ERLE}(n) \text{ in dB} = 10\log_{10}\left[E\left\{d^2(n)\right\}/E\left\{e^2(n)\right\}\right]. \tag{44}$$

According to Monte-Carlo simulations, the performance of adaptive algorithms is compared on the basis of measured minimum mean square error (MMSE) criterion as

$$\hat{J}(n) = \sum_{j=1}^{500} \frac{e^2(n,j)}{500}. \tag{45}$$

The acoustic path is modeled as a linear tapped-delay-line filter, in which the FIR filter coefficients are Gaussian distributed (chosen at random). The number of coefficients is assumed to be equivalent to N. First, we consider the second-order nonlinear system with Volterra kernels as $\vec{\boldsymbol{g}}_s = [0.76, -1, 1, 0.5, 2, -1.6, 1.2, 0.8, 0.05]$ with $N_2 = L(L+1)/2 = 6$ and $N = L = 3$, in which

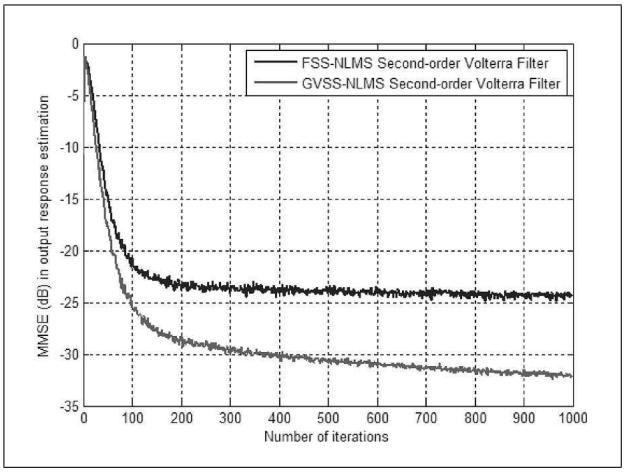

Figure 3. Minimum mean square error in the estimation of output response for SOVFs.

$N + N_2 = 9$. In a second case, the output of the third-order Volterra system is given as

$$\begin{aligned} x_{vf}(n) = {} & -0.78x(n) - 1.48x(n-1) + 1.39x(n-2) \\ & + 0.7x(n-1)x(n-2) + 1.86x(n)x(n-2) \\ & + 3.72x(n)x(n-1) + 1.41x^2(n-2) \\ & - 1.62x^2(n-1) + 0.54x^2(n) \\ & + 0.33x(n)x(n-1)x(n-2) \\ & + 0.15x^2(n-1)x(n-2) \\ & - 0.75x^2(n-2)x(n-1) - 1.52x^2(n)x(n-2) \\ & - 0.23x^2(n-2)x(n) - 0.12x^2(n)x(n-1) \\ & - 0.13x^2(n-1)x(n) + 0.5x^3(n-2) \\ & - 0.76x(n-2) + 0.44x^3(n), \end{aligned}$$

with $N_3 = L(L+1)(L+2)/6 = 10$, $N_2 = L(L+1)/2 = 6$ and $N = L = 3$, in which $N + N_2 + N_3 = 19$. The values of parameters are kept at $\mu_s = 0.05$, $\lambda_1 = 0.66$, $\lambda_2 = 0.66$, $\overline{\alpha} = 0.97$, $\overline{\beta} = \overline{\gamma} = 0.8 \cdot 10^{-4}$ and $\bar{P} = \bar{Q} = 2$. The value of the minimum step-size is $\mu_{\min} = 0.0005$ and the maximum bounded value of step-size $\mu_{\max} = 0.5$. The input $x(n)$ to the underlying system is a white Gaussian random sequence with unit variance and zero-mean [24].

It is apparent from the simulation results presented in Figure 3 – Figure 6 that the second-order Volterra filter exhibits higher convergence rate and lower MMSE in comparison to the third-order Volterra filter using the FSS-NLMS as well as GVSS-NLMS adaptive algorithm. The inferior results of the third-order Volterra filter are due to the large number of Volterra coefficients/kernels that must be adapted. However, these results manifest that the proposed GVSS-NLMS algorithm based approach depicts a higher convergence rate than the FSS-NLMS based scheme.

Further, the input $x(n)$ to the underlying loudspeaker – linear acoustic path – microphone assembly is a real-time sound signal. The corresponding received sound signal at the output of microphone is shown in Figure 7. The acoustic path coefficients are unknown in this case. For memory

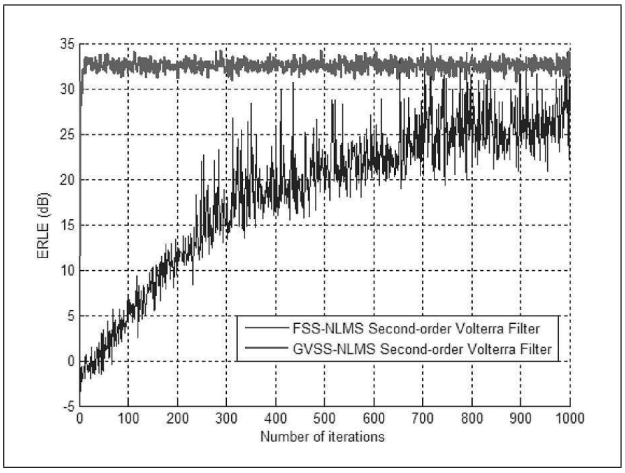

Figure 4. ERLE vs number of iterations for SOVFs.

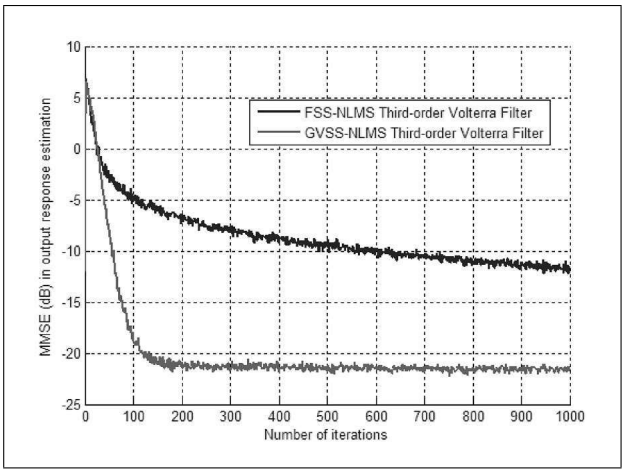

Figure 5. Minimum mean square error in the estimation of output response for third-order Volterra filters.

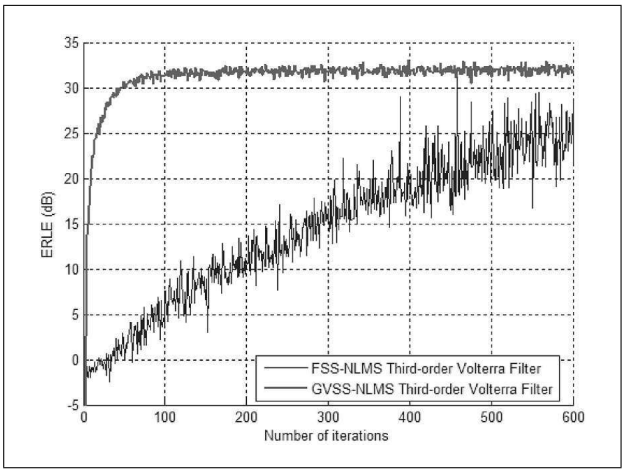

Figure 6. ERLE vs number of iterations for third-order Volterra filters.

length $L = 10$, the first module in the presented adaptive cascaded structure encompasses $N_2 = 55$ and $N_3 = 220$ for the nonlinear filter. Here, the number of linear FIR filter coefficients in the second module is $N = 128$. The values

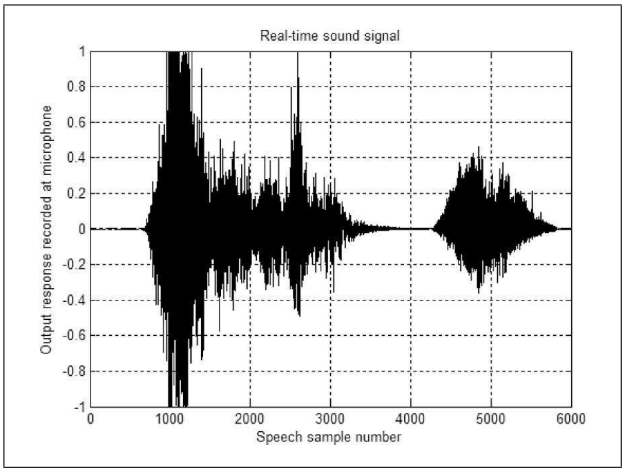

Figure 7. Real-time loudspeaker acoustic signal received at microphone.

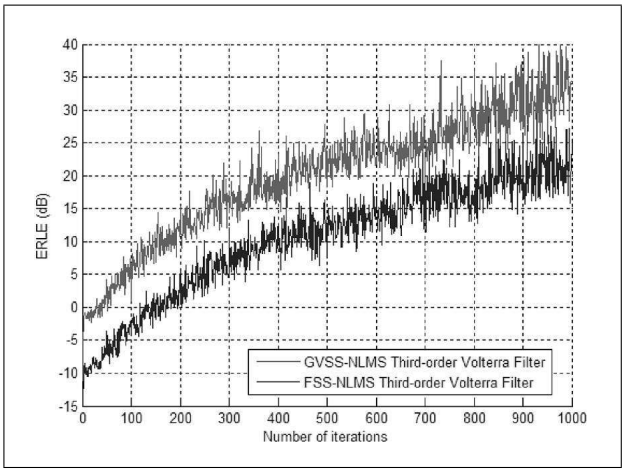

Figure 8. ERLE vs number of iterations for third-order Volterra filters for real-time acoustic signal.

of parameters are kept at $\mu_s = 0.05$, $\lambda_1 = 0.8$, $\lambda_2 = 0.5$, $\overline{\alpha} = 0.99$, $\overline{\beta} = \overline{\gamma} = 0.8 \cdot 10^{-4}$ and $\bar{P} = \bar{Q} = 2$. The value of the minimum step-size is $\mu_{\min} = 0.0005$ and the maximum bounded value of step-size $\mu_{\max} = 0.5$. Under practical conditions, the third-order Volterra filter using GVSS-NLMS performs 5 dB better than the FSS-NLMS based AEC as far as the ERLE is concerned (as shown in Figure 8). Under similar conditions, the second-order Volterra filter using GVSS-NLMS is compared with the third-order Volterra filter using GVSS-NLMS. The simulation results demonstrated in Figure 9 illustrate that the SOVF converges at a faster rate, but it is at the cost of higher MMSE in comparison to the third-order polynomial filter using GVSS-NLMS.

5. Concluding remarks

This correspondence has presented a generalized variable step-size NLMS algorithm based adaptive nonlinear acoustic echo canceller, in which second- and third-order Volterra filtering techniques are incorporated. The first module of the adaptive cascaded AEC updates the higher-

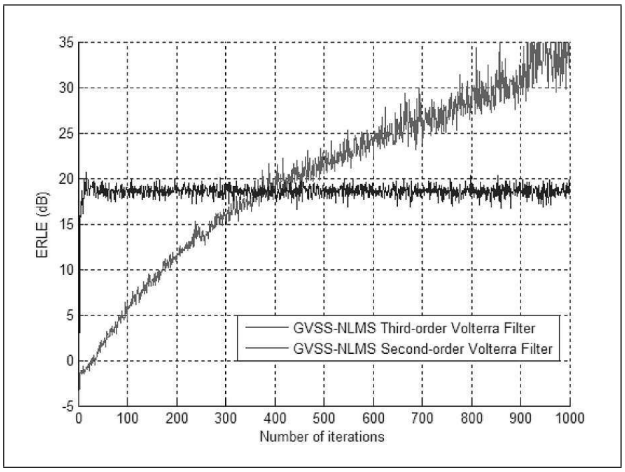

Figure 9. ERLE vs number of iterations for SOVF and third-order Volterra filters for real-time acoustic signal.

order nonlinear components/coefficients of the Volterra filter, and subsequently the second module updates the first-order linear components/coefficients of the Volterra filter as well as the coefficients of the linear acoustic path. It undoubtedly reduces the computational complexity as compared to conventional polynomial Volterra filtering based AECs. The application of GVSS enhances the convergence rate as compared to the FSS based NLMS algorithms for AECs. Moreover, the GVSS-NLMS algorithm also efficiently tracks the acoustic tap- coefficients in a slowly time-varying environment. For a known nonlinear system scenario, the second-order Volterra filter outperforms the third-order Volterra filter as far as convergence rate and MMSE in the output response estimation are concerned. This is due to a smaller number of filter coefficients in SOVFs that need to be updated for the system identification and echo cancellation. However, under practical conditions, loudspeaker nonlinearities are modeled quite well using the third-order Volterra filter paradigm. The simulation results connote that the third-order Volterra filter using GVSS-NLMS performs approximately 5 dB better than the FSS-NLMS algorithm based AECs, in terms of ERLE. It may be inferred that third-order adaptive Volterra filtering is an appropriate choice for acoustic echo cancellation, and for the estimation of the output response of the presented adaptive cascaded structure using GVSS-NLMS algorithm. Future scope includes the development of computational complexity reduction schemes for the proposed AEC method. However, analytical techniques required to cope with the ill-conditioned input signal autocorrelation matrix also need to be investigated further.

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