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The Influence of Shape Design on the Acoustic Performance of Concert Halls from the Viewpoint of Acoustic Potential of Shapes

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Summary
Shape design is critical to the acoustic performance of concert halls, and room shapes obtained in the shape design stage should have the potential to be acoustically desirable when appropriate materials are assigned in the late design stage. However, the importance of shape design to acoustics, as well as the relationships between shape designs and acoustic performance, has not been thoroughly explored to meet the requirements of design practice. This research employs a method combining acoustic simulation, Multivariate Adaptive Regression Splines and Monte Carlo Simulation, to investigate the influence of shape design on the acoustic performance of shoe-box and fan-shaped concert halls from the viewpoint of acoustic potential of shapes. The acoustic functionalities of materials and the multiplicity of acoustic parameters are taken into account in order to meet the real situations of shape design. The results first quantify and prove the importance of shape designs in achieving desirable acoustic performance for shoe-box and fan-shaped concert halls, and then reveal the relationships between the probability of acoustically desirable shapes and shape parameters, which could help architects stay in an appropriate design direction for acoustics. Finally, a method to evaluate the acoustic potential of shapes in real-time is provided, which can help architects make decisions in shape designs.

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1. Introduction

1.1. Research background

The design of room shape is usually the starting point of auditorium design. It not only largely determines the spacial effect and aesthetic value of auditoriums, but also plays an important role in room acoustic performance [1]. Since the acoustic performance of auditoriums can also be adjusted by materials with acoustic functionalities in the late design stage, shape designs in the early design stage can have a certain degree of freedom so that architects can pursue the spacial effect and aesthetic value they desire. However, it has to be ensured that the shapes can have satisfactory acoustic performance when appropriate compositions of materials are assigned, because the adjustments that materials can offer are not unlimited. Some improper shapes could fail to meet acoustical requirements no matter what materials are assigned. In other words, the shapes should at least have the "potential" to be acoustically "remedied" by materials. Therefore, in order to achieve desirable acoustic performance of auditoriums, it is necessary for architects and acousticians to involve acoustic issues in the early design stage, when only the ideas of shapes are developed while material properties have not been deeply considered yet [2]. However, the acoustic "potential" of shapes is difficult to evaluate. The existing acoustic evaluation methods (scaled model test, acoustic simulation, etc.) can only estimate the acoustic performance of a shape with one given composition of materials. They cannot judge whether the shape has the potential to meet the acoustic requirements when appropriate materials are assigned in detailed acoustic design stage. This could cause conflicts between architects and acousticians, who should work together to design auditorium shapes but think and judge differently [3, 4, 5, 6]: Architects always want their auditoriums to be unique and inspiring, therefore, they might insist on the shape they proposed and argue that the current acoustic problems can be fixed by acousticians in the late design stage. This could lead to the risk that designs with acoustic fatal defects are accepted. Acousticians, on the other hand, tend to learn from the shapes of highly successful halls and regard the acoustic performance as the most important issue, then the novel and acoustically promising shapes proposed by ar-

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chitects could be abandoned regrettably. Therefore, in order to help architects and acousticians find the desirable shape at the balancing point of aesthetic value and acoustic performance in the early design stage, it is important to investigate the influence of shape designs on acoustics performance form the viewpoint of acoustic potential of shapes.

1.2. State of the art

The influence of room shape on acoustic performance has been an important research area for a long period of time. Initially analytical methods are employed. Marshall [7, 8] explored the dependence of the masking effect of early sounds on shape proportions by tracing sound paths geometrically. Vorländer [9] investigated the lateral sound ratio as a function of room characteristics by image source method. Later, more researches with statistical methods are conducted. Gade [10, 11, 12], Bradley [13] and Barron [14] revealed the relationships between acoustic indices and shape parameters using linear regression methods based on the data acquired by measurements of built concert halls. Chan [1] and West [15] conducted similar but more straightforward researches, which skipped the intermediate link of objective acoustic indices. They tried to establish relationships directly between the subjective evaluations of sound qualities among touring musicians and shape parameters. Klosaka [16] also conducted similar research but using acoustic simulation instead of built auditoriums. The simulation results of 24 shoe-box concert halls with the same material composition but different shapes were compared, which proved that room shape is an important factor influencing acoustic parameters including Clarity (C80), Strength (G), Early Lateral Energy Fraction (LF80), etc. Bayazit [17] proposed a more ambitious method that could calculate the "optimized" geometry dimensions of shoe-box concert halls according to seat numbers based on the similarities of 15 existing "successful" halls. Besides, there are also several researches that showed the influence of shapes on acoustics in specific projects [2, 18, 19, 20], and provided useful ideas for architects and acousticians to consider the issues of shapes and acoustics in comparable projects.

These researches have made significant contributions to reveal the influence of room shape on acoustic performance of auditoriums, but there are still some aspects that could be improved when focusing on the actual situations of shape designs in the early design stage. First, in existing researches, the acoustic performance of a certain shape was measured or calculated with only one material composition. The concept of acoustic potential, which is important for shape design, was not adopted. As a result, the influence of shapes on acoustics could be overestimated as the functionalities of materials in adjusting acoustic performance were omitted. Second, the existing researches investigated the influence of shapes on different acoustic parameters separately. However, in real design practice, several acoustic parameters are required to be able to reach desirable ranges at the same time. If only one acoustic parameter is considered, materials could be very effective in adjusting acoustic performance. However, when multiple acoustic parameters are included, the problem could turn more complicated because different acoustic parameters might have contradictory requirements of materials. For example, the reverberation time (RT) and the strength factor (G) can both be largely adjusted by adding or reducing absorption materials. However, when RT is too long while G is too low (e.g., a shape with too large volume), materials no longer have the functionality to adjust RT and G into desirable ranges at the same time. Therefore, it is necessary to consider multiple acoustic parameters at the same time instead of investigating them separately when evaluating the acoustic performance of shapes from the viewpoint of acoustic potential.

1.3. Objective of this research

This research aims to explore the influence of room shape on acoustic performance of shoe-box and fan-shaped concert halls from the viewpoint of acoustic potential of shapes. In order to match the conditions of shape design in the early design stage, the functionalities of materials in adjusting acoustic performance and the multiplicity of acoustic parameters are taken into account. Three goals are specifically pursued here: 1) to prove the importance of shape designs in achieving desirable acoustic performance; 2) to reveal the patterns of shape parameters that are more likely to lead to acoustically desirable shapes; 3) to provide a method to evaluate the acoustic potential of shapes in the early design stage.

2. Methods

In order to achieve these three goals above, a methodology consisting two stages is adopted in this research (Figure 1). *Stage 1* aims to acquire statistical models that can predict acoustic parameters of a certain design precisely and quickly. Concert hall designs (including shape and material information) are generated by a parametric model in *Rhinoceros* platform. Designs are then simulated by *CATT Acoustics* with a set of averaged room acoustic parameters derived. Based on the simulation results, statistical models of acoustic parameters and design parameters are established using Multivariate Adaptive Regression Splines. The reason for using acoustic data from simulation instead of that from actual concert halls is that actual built concert halls are limited in number and are unlikely to be selected randomly or comprehensively [1]. Moreover, results from commercial room simulation programs have been proven to be reliable and have good agreement with measurements [21, 22]. *Stage 2* investigates the influence of room shape on acoustic performance by Monte Carlo Simulation. A vast number of random shapes are first generated. Then the acoustic parameters of each shape are predicted by statistical models (acquired in *Stage 1*) with all possible material compositions. Based on the prediction results, those shapes are divided into two groups

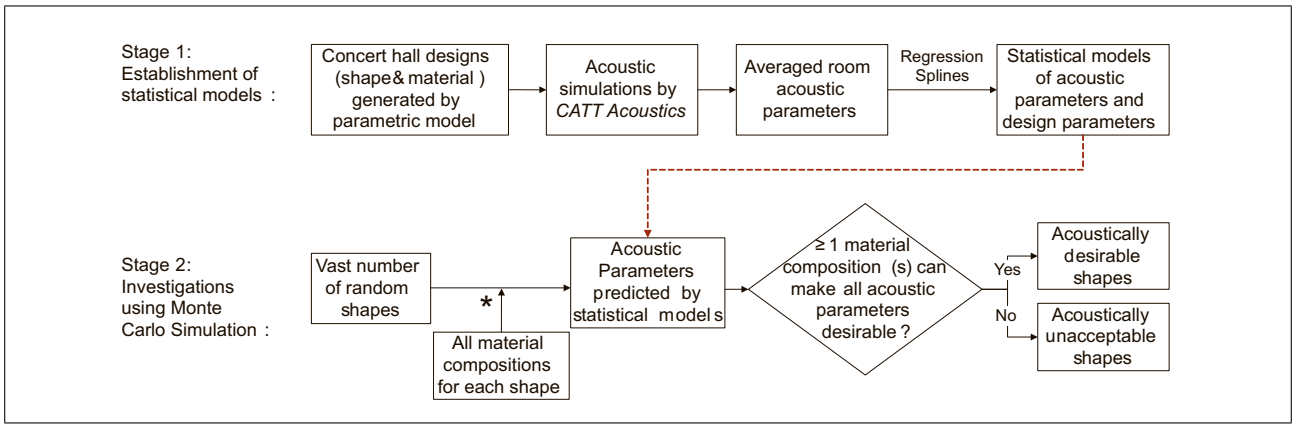

Figure 1. Methodology of this research.

(acoustically desirable shapes and acoustically unacceptable shape) according to whether there is at least one material composition that can make all acoustic parameters of the shape desirable. After finishing these two stages, the importance of shape design and the pattern of shape parameters that are more likely to lead to acoustically desirable shapes can be derived by analyzing the ratio of acoustically desirable shapes out of all shapes. Meanwhile, a method to evaluate the acoustic potential of shapes can be achieved by supplementing the statistical models. More detailed information about the methods employed in this research are presented later in this section.

2.1. Boundary conditions

2.1.1. Definition of acoustically desirable shapes

As indicated in Section 1.1, room shapes of auditoriums should have the potential to be acoustically "remedied" by proper material compositions in the late design stage. Therefore, in the early design stage when materials are not decided, "acoustically desirable shape" should imply that there are at least one possible material composition that can make the acoustic performance of the shape desirable. In order to judge if a certain shape is acoustically desirable, its acoustic parameters can be calculated with all possible material compositions. If there is at least one material composition that can make all acoustic parameters of the shape into desirable ranges, the shape can be regarded as "acoustically desirable"; otherwise, it must be defined as "acoustically unacceptable". This is the definition of "acoustically desirable shape" in this research. In this way, the functionalities of materials in adjusting acoustic performance are fully exploited, so that shape design is given the maximum freedom on the premise that the acoustic performance has the potential to be desirable.

2.1.2. Acoustic parameters

There are several acoustic parameters that can reflect different aspects of the acoustic performance of auditoriums [23]. In this research the parameters are chosen based on the research of Beranek [5, pp.503-552], which summarized important parameters for concert halls including *Reverberation time* (RT), *Early Decay Time* (EDT), *Binaural Quality Index* (BQI), *Strength Factor* (G), *Bass Ratio* (BR), *Initial-Time-Delay Gap* (ITDG), *Lateral Fraction* (LF), *Surface Diffusivity Index* (SDI) and *Clarity (C80)*. Among these parameters, both BQI and LF reflect the spaciousness of the sound. Since BQI is more reliable in judging acoustical quality than LF [5, pp.528], only BQI is included. BR and SDI are fully determined by acoustic properties of materials and have little relationship with shape designs, thus they are excluded. As a result, the number of acoustic parameters is reduced to six in this research: RT, EDT, BQI, G, ITDG, and C80. These parameters respectively represent the fullness of tone, the sequence of early sound, spaciousness, loudness, intimacy and clarity. Therefore, they can reflect the overall acoustic performance of concert halls comprehensively. There are also researches showed that some of these six acoustic parameters are correlated with each other statistically. This provides the possibility of further reducing the number of acoustic parameters included in this research. For example, Barron [13] showed that based on the data of 15 halls, EDT is highly correlated with RT, while G and C80 are also related to RT (as well as room volume and source-receiver distance). However, all six parameters are still included in this research, because each of them represents a different perceptual characteristic of sounds. As a result, all of them have different recommended ranges and are employed in most researches and designs. Another important reason is that there could be some exceptional cases of the correlations of two parameters. For example, in Barron's research [13], the mean EDT and RT of 5 concert halls (out of 15 in total) have statistically significant differences, although others are in good agreement.

For RT, EDT, G and C80, only the value of mid-frequency (500 Hz) is included, because the frequency characteristics of these parameters are determined by the frequency characteristics of materials, which are designed in the late design stage. BQI is calculated by averaging the values of (1-IACC) at 500 Hz, 1000 Hz and 2000 Hz (as its definition). In this way although the absorption coefficients at 500 Hz, 1000 Hz and 2000Hz of a certain material are set to be the same (more details in Section 2.1.4), the difference of *Head-Related Transfer Functions* (HRTF) at

Table I. Acoustic parameters and their desirable ranges used in this research [5, pp.536].

Parameter	RT (s)	EDT (s)	BQI	G (dB)	ITDG (ms)	C80 (dB)
Range	1.8 ∼ 2.1	2.2 ∼ 2.6	0.65 ∼ 0.71	1.5 ∼ 5.5	0 ∼ 25	−3 ∼ 0

Table II. The ranges of shape parameters and the values of constants.

Parameter	room length	room width	room height	total area of balconies
Range/Value	30 m ∼ 45 m	18 m ∼ 30 m	15 m ∼ 25 m	0 ∼ 600 m^2
Parameter	angle of room side walls	angle of room ceiling	number of balconies	stage height
Range/Value	−10° ∼ 10°	−10° ∼ 10°	0 ∼ 3	12 m ∼ 18 m
Parameter	angle of stage side walls	angle of stage ceiling	stage depth	stage width
Range/Value	0° ∼ 25°	0° ∼ 25°	7.6 m	16 m

different frequencies are included to provide a reliable calculation of BQI. ITDG is independent with frequency according to its definition.

The desirable ranges of the acoustic parameters are also decided based on the research of Beranek [5, pp.536] (Table I). It should be noted that these ranges are only suitable for large-scale concert halls with symphonic repertoire. Auditoriums with different repertoire, size, or special client preferences could have divergent optimal values of acoustic parameters (more details about the limitation of the desirable ranges of acoustic parameters are presented in Section 4.1)

2.1.3. Shape parameters

The shape of concert halls could be described by a set of parameters [24]. For more comparable results, two kinds of concert hall shapes are included in this research. One is shoe-box shape, which is described by four parameters: *room length, room width, room height and total area of balconies* (Figure 2). Another is fan shape, which is described by ten parameters: *room length, room width, room height, total area of balconies, angle of room side walls, angle of room ceiling, number of balconies, stage height, angle of stage side walls* and *angle of stage ceiling* (Figure 3). It should be noted that for the angle parameters, positive values stand for inward rotations around the proscenium (or the shape narrowing against the proscenium), while negative values mean outward rotations (or the shape widening against the proscenium). It can be found that the former kind of shapes is a subset of the latter kind. For both kinds, the depth and width of the stage are set to be constants, because they have very limited ranges of variations due to functional requirements. All the shape parameters are given certain ranges, which are based on but broader than the range of common parameters of existing concert halls [5, 24], in order to include more design possibilities. The ranges of shape parameters and the values of constants are shown in Table II.

2.1.4. Material parameters

Three material parameters are employed here: *absorption coefficient of sidewalls, absorption coefficient of ceiling*,

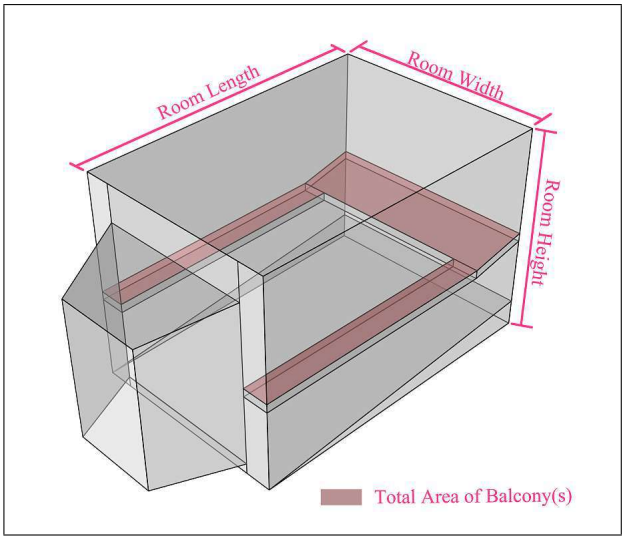

Figure 2. The parameter definition of shoe-box concert hall shapes.

and *absorption coefficient of rear wall*. The absorption coefficient of other surfaces are kept constants: the materials of the stage house for most auditoriums are highly reflective materials with limited range of absorption coefficients. Therefore the absorption coefficient of the stage house is treated as a constant. The absorption coefficients of audience area and orchestra area are similar for most cases, thus are also treated as constants [5, pp.639-640].

Apart from absorption, scattering is another important acoustic functionality of materials. Concert halls need sufficient scattering to reduce "glare" and create "diffusivity of the sound field" [5, pp.521]. Meanwhile, scattering surfaces allow a better blended ensemble sound [25] and a more uniform spatial distribution of sound energy [26]. They also increase the number of peaks in the early fine structures of room impulse responses, and thus enhance the perceptual diffuseness of listeners [27]. Since the aim of this research is to explore the acoustic potential of shapes (or the acoustic performance of shapes with proper material compositions), only the scenarios with sufficient scattering need to be investigated, while other

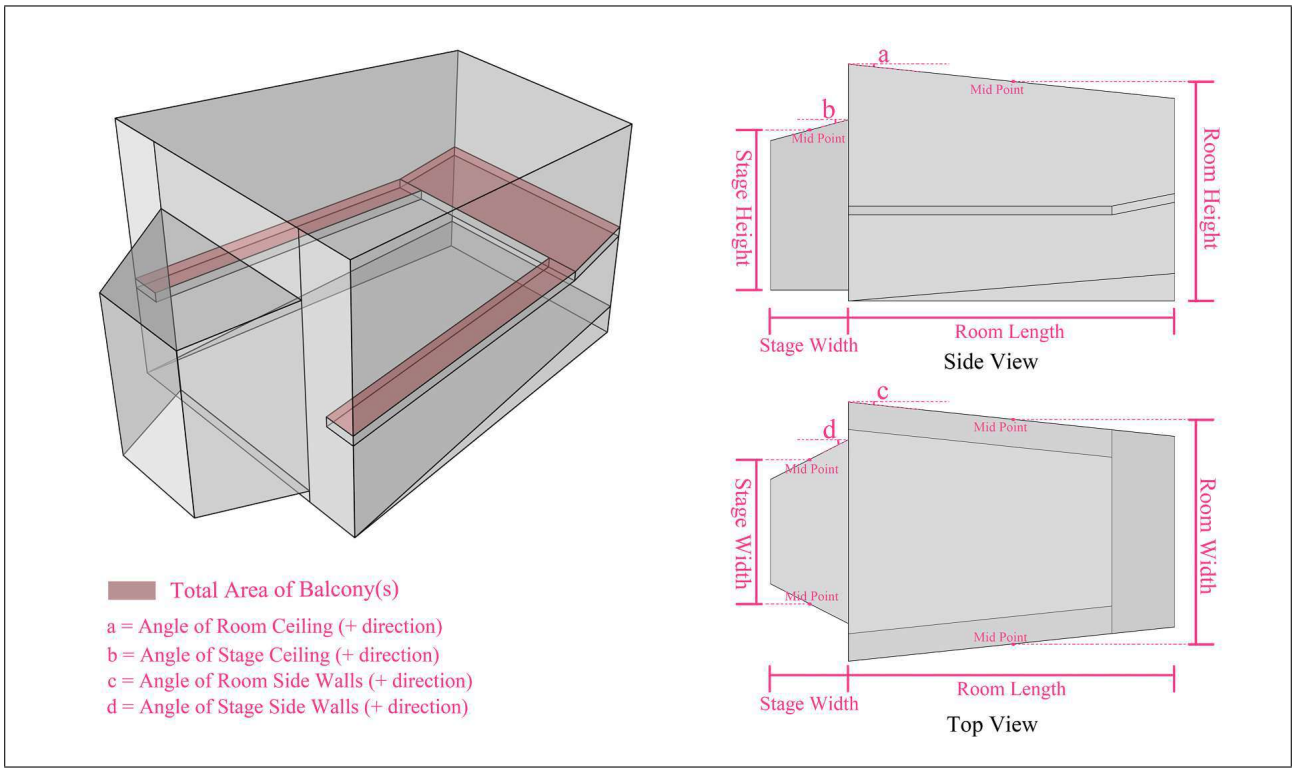

Figure 3. The parameter definition of fan-shaped concert hall shapes.

Table III. The ranges of material parameters and the values of material constants.

Parameter	absorption co. of sidewalls	absorption co. of ceiling	absorption co. of rear wall
Range/Value	0.00 ~ 1.00	0.00 ~ 1.00	0.00 ~ 1.00
Parameter	absorption co. of stage house	absorption co. of audience area	absorption co. of orchestra area
Range/Value	0.05	0.80	0.32
Parameter	scattering co. of audience area and orchestra area		scattering co. of other surfaces
Range/Value	0.65		0.40

scenarios are not worth testing because they must have acoustic defects such as glare or insufficient diffusivity of sound field. In acoustic simulation software, the random incidence scattering coefficient rather than the diffusion coefficient is employed, because it can separate out specular and diffuse components of sound, and can better consider incoherent broadband sound fields [28]. There are several researches that investigated the influence of scattering on the subjective perception of listeners [25, 27, 29], and other researches revealed the influence of the scattering coefficient on the objective acoustic parameters calculated by acoustic simulation software [29, 30, 31]. Among them, the research by Shtrepi [29] offers the most direct guidance for this research. It employed three acoustic software packages (including *CATT Acoustics*, while the other two researches only employed *ODEON*) to conduct acoustic simulations of a shoe-box concert hall. In different simulations, the scattering coefficient of the ceiling, the side walls and the rear wall was assigned to be 0.1, 0.3, 0.5, 0.6, 0.7 and 0.9 respectively. It was found that among parameters calculated by *CATT*, differences larger than one JND (Just-Noticeable Difference, or Subjective Limen [32]) were only observed for cases with the scattering coefficient of 0.1. In other words, *CATT* yielded consistent results on the premise that the scattering coefficient was no smaller than 0.3. Therefore, in this research the scattering coefficient of surfaces other than the audience area and orchestra area (including walls, ceiling, stage house, etc.) is set to be 0.4, in order to get stable results from *CATT* that represent the scenarios of sufficient scattering. The scattering coefficient of the audience and orchestra area is fixed to be 0.65 according to its nature [16].

The ranges of material parameters and the values of constants are shown in Table III. Empirically surfaces with high absorption coefficients are seldom used in concert halls. For example, Hidaka [33] raised an assumption that the "residual" absorption coefficient (excluding the audience area and air absorption) is around 0.1. However, the range of feasible absorption coefficients used for concert halls has never been defined conclusively in previous researches. Therefore, in this research the full range of absorption coefficients is employed. More discussions on

whether the range of absorption coefficients is influential in this research, and the pattern of absorption coefficients used in acoustically desirable shapes, are presented in Section 4.2.

2.2. Acoustic simulation

A parametric model that can automatically generate concert hall designs within the boundary conditions defined in Section 2.1 is developed in *Rhinoceros* (a 3-D modelling software, V.5 SR11). Additionally, an interface bridging *Rhinoceros* and *CATT* (An acoustic simulation software, V9.0b) is developed, which can call *CATT* to run acoustic simulation using the designs generated by *Rhinoceros* (more details about the parametric model and the interface can be found in another article of the authors [34]). Based on the parametric model and the interface, one group of shoe-box concert halls and another group of fan-shaped halls are generated and simulated. Each group contains 1000 random designs with shape and material information. For more reliable results, three simulations are conducted for one design with different sources: one is in the center front of the stage representing the soloist or the first violin, another one is in the center rear, and the last one is in the frontal side. The sources are located 1.2 meters above the stage and are omni-directional. Their positions are chosen in reference to precedent researches [35, 36, 37]. Sixteen receivers are distributed in the main audience area. Additional receivers are distributed in the balcony, the number of which is determined by the ratio of the total balcony area to the main audience area. Due to the symmetry, all receivers are distributed in one half of the auditorium, and are also 1.2 meters above the floor (Figure 4). For all simulations, the setting of 100000 rays and 2000ms echogram is used. Temperature is set to be 20°C, while relative humidity is 50%. To simplify the simulation results, only the average value of all receives in three simulations is recorded. Although not perfectly precise, average values have been proven to be effective to reflect the overall acoustic features of an auditorium [38, 39].

2.3. Statistic models

Multivariate Adaptive Regression Splines (MARS) is a method of flexible nonparametric regression modelling and is capable to model complex, nonlinear relationships between response and explanatory variables. The range of each explanatory variable is first broken into subsets using "knot". Then the data of each subset is fitted using a basis function (piecewise linear basis functions), which contains several splits and ensures the continuity of the overall fitted function. Model fitting starts with a forward stepwise procedure that identifies many knots and splits to over-fit the data, followed by a backward pruning routine to simplify the model. Additions and deletions of knots and splits are evaluated in terms of changes in residual squared errors (see Appendix A1 for detailed information) [40, 41]. MARS is employed here to deal with the simulation data acquired in Section 2.2. Two groups of nonlinear statistical models representing the relationships between each acoustic parameter and all design parameters (4 shape parameters and 3 material parameters for shoe-box halls; 10 shape parameter and 3 material parameters for fan-shaped halls, as indicated in Section 2.1) are established using the two groups of simulation results. For each group, 800 cases are used to train the regression models while the other 200 cases are employed to test their precision. The process of acquiring statistical models is implemented using R software and its CRS package [42, 43]. Two 3D-plots of the statistical models are shown in Figure 5, which respectively show the fitting surfaces of the regressions of RT (Figure 5 top) and BQI (Figure 5 bottom) on *Room Width* and *Room Heigh* for shoe-box shapes (under the condition that *Room Length* is 40 m and *Total Area of Balcony* is 200 m^2). Table IV shows statistical indices of the statistical models. All the statistical models have correlation coefficients (R^2) larger than 0.85, and their residual standard deviations (SD) are much smaller than JND (Just-Noticeable Difference, or Subjective Limen) [32]. Furthermore, they are better than existing statistical models derived from linear regression method [11] (see Appendix A2 for information about the assessment of model performance). Therefore, these statistical models acquired here can be used to provide reliable predictions of the acoustic parameters of designs.

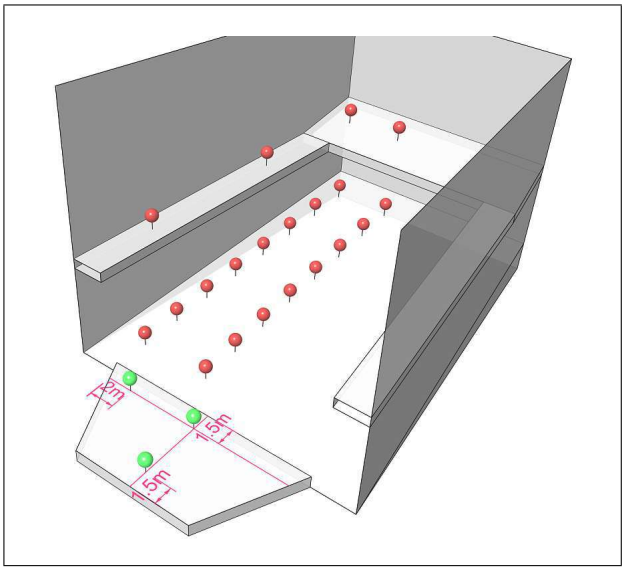

Figure 4. Distribution of the sources (green colour online) and receivers (red colour online).

2.4. Investigations using Monte Carlo Simulation

Monte Carlo Simulation (or *Stochastic Simulation*) is a method that uses repeated sampling to determine the properties of some phenomenon or behavior (e.g., probability distribution) [44]. A set of random values of each variable are used to produce possible outcomes. The outcomes are recorded and the step is repeated with new sets of random values until a sufficient amount of data is gather. Then the distribution of the outcomes are analyzed to get the probabilities of different outcomes occurring [45]. Monte Carlo Simulation is employed here to investigate the importance

Table IV. The statistical indices of the statistical models. Compare with JND: Compare with JND (Subjective Limen) Compare with LR: Compare with Linear Regression [9].

	Shoe-box Shapes		Fan Shapes		Compare with JND		Compare with LR	
	R^2	SD	R^2	SD	JND [32]	Compare	R^2 of LR	SD of LR
RT (s)	0.9692	0.05831	0.9588	0.06016	5% (0.1s when RT=2s)	SD<JND	-	-
EDT (s)	0.9498	0.06169	0.9489	0.06203	5% (0.1s when EDT=2s)	SD<JND	-	-
BQI	0.9180	0.01508	0.9229	0.01476	-	-	-	-
G (dB)	0.9652	0.23950	0.9611	0.24170	1dB	SD<JND	0.94	0.9dB
ITDG (ms)	0.8627	1.41900	0.8575	1.48300	-	-	-	-
C80 (dB)	0.9631	0.31170	0.9609	0.31640	0.5dB	SD<JND	0.86	0.8dB

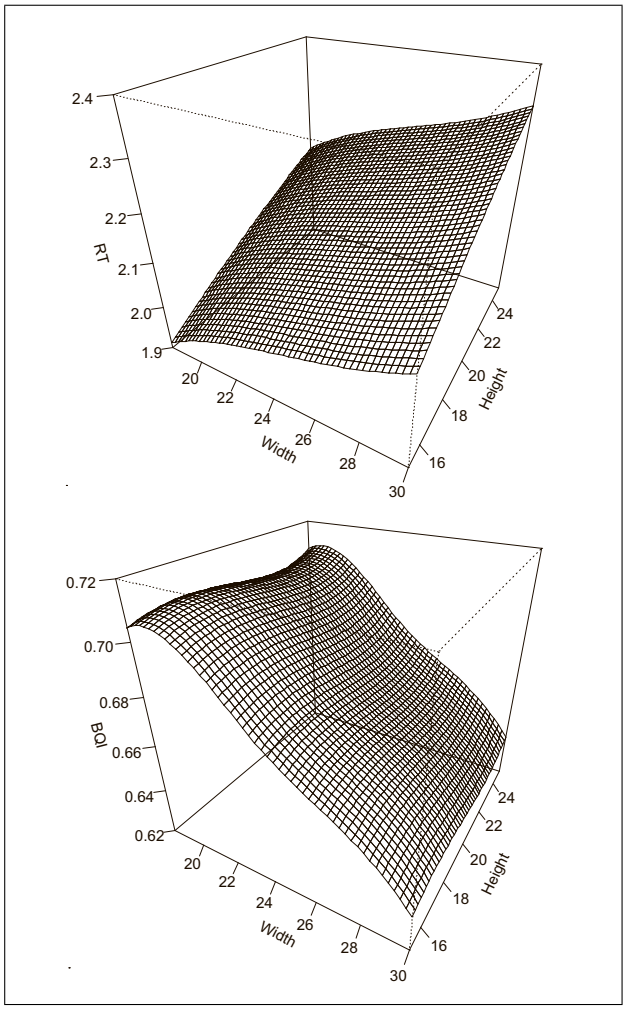

Figure 5. Plots of the fitting surfaces of the regressions of RT (top) and BQI (bottom) on Room Width and Room Height for shoe-box shapes (Room Length is 40 m, Total Area of Balcony is 200 m^2).

of shape design and the pattern of shape parameters that are more likely to lead to acoustically desirable shapes. For each of the two kinds (shoe-box and fan shapes), 100000 shapes are randomly generated. Meanwhile, all possible compositions of materials are enumerated: each of the three material parameters has 10 possible values (from 5 to 95 with a gap of 10, the gap of 10 is chosen because the uncertainty of absorption coefficients at mid frequency is 10 [46]), which results in 1000 possible material compositions. For every shape, its acoustic parameters with all possible material compositions are calculated by the statistical models acquired in Section 2.3. If there is at least one material composition that can make all six acoustic parameters of a certain shape into the desirable ranges at the same time, the shape is judged to be "acoustically desirable"; otherwise it is regarded as "acoustically unacceptable".

After all shapes have been divided into two groups (acoustically desirable shapes and acoustically unacceptable shape), the ratios of acoustically desirable shapes to all shapes under different conditions can be calculated. According to Monte Carlo Simulation, these ratios can be regarded as the probabilities of acquiring acoustically desirable shapes under that condition [45]. The results are presented next in Section 3. It should be explained that, the acoustic parameters of these vast shapes are not possible to be acquired directly by simulation, because it could take years to finish all simulations (there are 2 kinds*100000 shapes*1000 material compositions=200,000,000 simulations needed and each simulation takes minutes). Therefore, the acoustic parameters of these shapes are calculated by statistical models in this research, as it only takes hours.

3. Results and Analysis

3.1. The overall importance of shape design in achieving desirable acoustic performance

For shoe-box concert halls, the ratio of acoustically desirable shapes among all 100000 generated shapes is 0.4827, indicating that if an architect randomly designs a shape of shoe-box concert hall without considering acoustic requirements, the probability of that shape to be acoustically desirable is less than 50%, even though the full functionalities of materials in adjusting acoustic performances are included. For fan shapes, the ratio is slightly lower (0.4429), which also indicates that the probability of acquiring acoustically desirable shapes without considering acoustics is less than 50%. Although the ratios could vary if the boundary conditions are changed, these results prove that the functionalities of materials in adjusting acoustic performances are limited. A large number of shapes will fail to meet acoustic requirements no matter what materials are assigned. Therefore, shape design is proven to

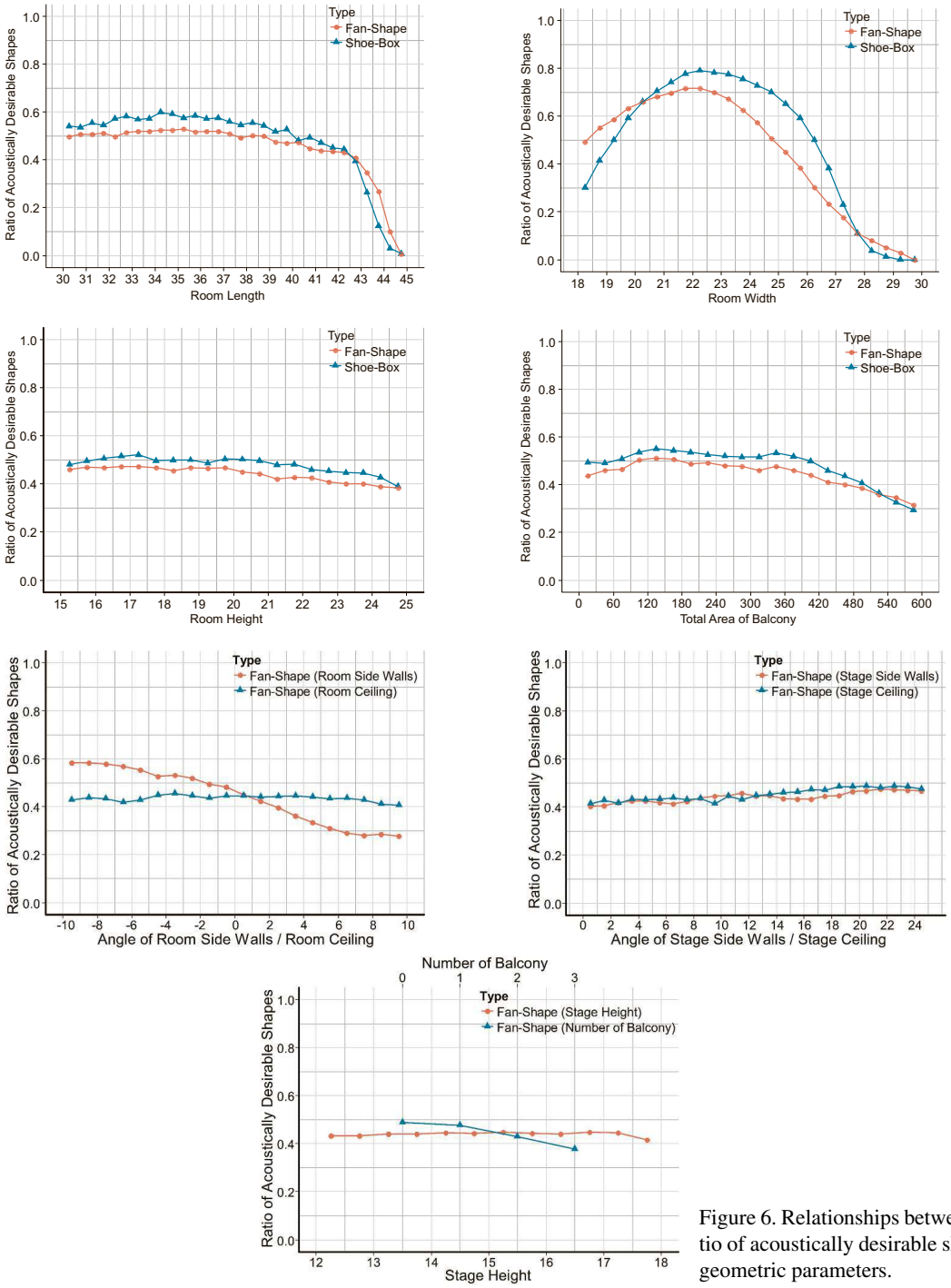

Figure 6. Relationships between the ratio of acoustically desirable shapes and geometric parameters.

have great importance in achieving desirable acoustic performance for concert halls.

3.2. The relationships between the probability of acoustically desirable shapes and shape parameters

As shapes are proven to be critical in achieving desirable acoustic performance, it would greatly help architects in shape design if the patterns of shape parameters that are more likely to lead to acoustically desirable shapes are revealed. This part aims to achieve this goal by showing the relationships between the probability of acoustically desirable shapes and shape parameters. All shape parameters are divided into several intervals (left-open and right-closed). The number of acoustically desirable shapes that lay in a certain interval can be counted, so does the number of all shapes in the same interval. Then the *Ratio of Acoustically Desirable Shapes* of this interval is defined as the proportion of these two numbers. By calculating the

ratios of all intervals, the *Ratio of Acoustically Desirable Shapes* (which is also the probability of acquiring acoustically desirable shapes according to Monte Carlo Simulation) can be expressed as functions of shape parameters. For a clear presentation, shape parameters are divided into four groups: geometric parameters, proportional parameters, seat parameters and volumetric parameters.

3.2.1. Geometric parameters

Geometric parameters here refer to the basic parameters that are used to define the shapes as introduced in Section 2.1.3. Figure 6 shows the relationships between the ratio of acoustically desirable shapes and geometric parameters for shoe-box and fan-shaped concert halls. As can be seen, *Room Width* is the most influential factor of the ratio of acoustically desirable shapes for both kinds. 20.5 m to 25 m seems to be the ideal width for shoe-box concert halls, as the ratio of acoustically desirable shapes in this range stays larger than 0.7. When the room width is larger than 27 m, the ratio of acoustically desirable shapes is very low. Fan shapes share a similar tendency, but the desirable range of *Room Width* (where the ratio is larger than 0.7) is narrower (20.5 m to 23 m). *Room Length* is another influential factor. For both shoe-box shapes and fan shapes, the ratio of acoustically desirable shapes stays around 0.5 when the room length is no larger than 42m, whereas it drops steeply when room length exceeds 42 m. *Angle of Room Side Walls*, which doesn't exist in shoe-box shapes, is influential for fan shapes. The lower its value is (including negative values, the definitions of negative and positive are shown in Section 2.1.3), the larger the ratio of acoustically desirable shapes will be. In other words, shapes with side walls that are widening towards the rear will have a better chance to be acoustically desirable than shapes with side walls narrowing towards the rear. Except for these three influential factors, other geometric parameters are found having no significant effects on the ratio of acoustically desirable shapes.

Based on the results above, it is suggested that architects should pay more attention to the parameters of *Room Width, Room Length* and *Angle of Room Side Walls* when designing shoe-box and fan-shaped concert halls, in order to increase the possibility of achieving acoustically desirable shapes. For other parameters, however, architects may have more freedom.

3.2.2. Proportional parameters

Besides geometric parameters themselves, their proportions could also be influential factors for the ratio of acoustically desirable shapes. Here three proportional parameters are investigated: *Room Width/Room Length, Room Height/Room Length*, and *Room Width/Room Height*. Figure 7 shows the relationships between the ratio of acoustically desirable shapes and these proportional parameters. Similar results are found for both kinds of shapes. *Room Width/Room Length* and *Room Width/Room Height* are highly related to the ratio of acoustically desirable shapes, while *Room Height/Room Length* is less influential. The optimal values of *Room Width/Room Length* for

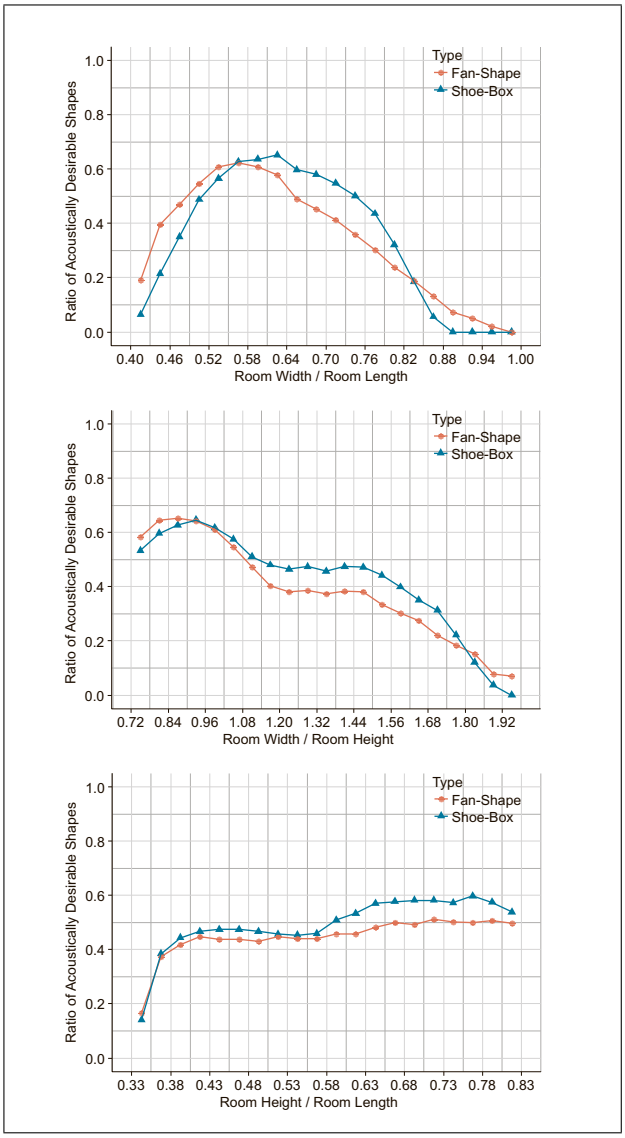

Figure 7. Relationships between the ratio of acoustically desirable shapes and proportional parameters.

shoe-box shapes and fan shapes seem to be 0.6 and 0.55 respectively, which make the ratio of acoustically desirable shapes stays around 0.6 for each kind. The more the value of *Room Width/Room Length* deviates from the optimal value, the lower the ratio will be. *Room Width/Room Height* is generally negatively related to the ratio of acoustically desirable shapes, and the optimal value of it is near 0.9 for both kinds of shapes. *Room Height/Room Length* is a positive factor of the ratio of acoustically desirable shapes, but the change of the ratio caused by *Room Height/Room Length* is not significant when the value of *Room Height/Room Length* is larger than 0.38. When the value of *Room Height/Room Length* is smaller than 0.38, however, the ratio is very low for both kinds of shapes.

The results of this section suggest that *Room Width/Room Length* and *Room Width/Room Height* should be more carefully considered during the shape design of both shoe-box shapes and fan shapes, because they are highly related to the probability of acoustically desirable

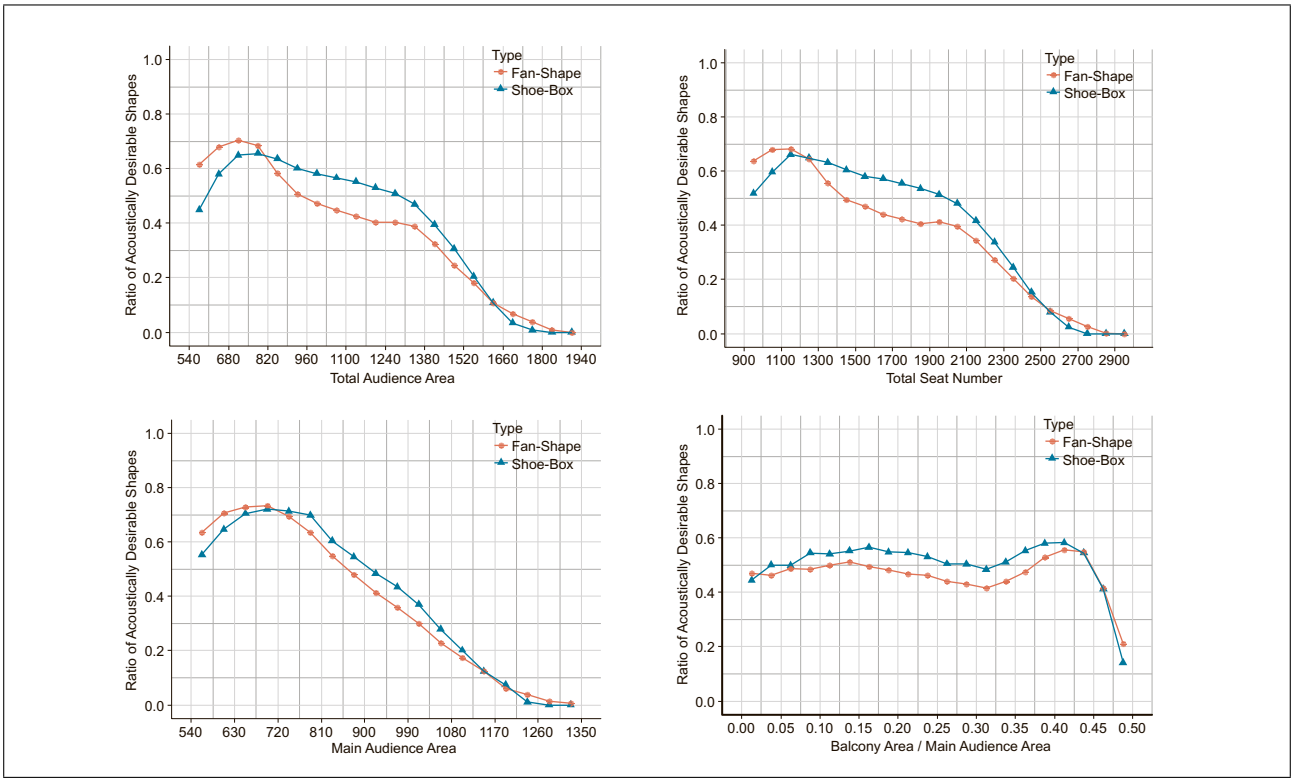

Figure 8. Relationships between the ratio of acoustically desirable shapes and seat parameters.

shapes. *Room Height/Room Length* can be given more freedom as long as it doesn't reach extreme values (smaller than 0.38).

3.2.3. Seat parameters

Audience area and seat number not only are essential functional parameters of concert halls, but also largely determine the total absorption. Here four seat parameters are included for each kind of shapes: *Total Audience Area, Total Seat Number, Main Audience Area*, and *Balcony Area/Total Audience Area*. Among these parameters *Total Seat Number* is very important but cannot be obtained from shape information directly. Since for most cases the ratio of *Total Audience Area* to *Total Seat Number* is within a narrow range (0.63 ± 0.08) [1], here this ratio is assumed to be 0.63, and then *Total Seat Number* can be calculated based on *Total Audience Area*.

Figure 8 shows the relationships between the ratio of acoustically desirable shapes and seat parameters for shoe-box shapes and fan shapes. Similar relations are found for both kinds. *Total Audience Area* and *Total Seat Number* are generally negatively related to the ratio of acoustically desirable shapes. For both kinds, shapes with a total capacity of around 1200 seats are most likely to be acoustically desirable (with the ratio near 0.7), while shapes with the capacity of more than 2500 seats are very unlikely to be acoustically desirable (with the ratio under 0.1). *Main Audience Area* is also generally negatively related to the ratio of acoustically desirable shapes. For each kind, concert hall shapes with main audience area of around $700\,m^2$ (or about 1100 seats) are most likely to be acoustically desirable (with the ratio more than 0.7), while disadvantage emerges when the main audience area is larger than $1200\,m^2$ (or about 1900 seats, with the ratio under 0.1). The dependence of the ratio of acoustically desirable shapes on *Balcony Area/Total Audience Area* is less significant for each kind. The ratio stays around 0.5 when *Balcony Area/Total Audience Area* is no larger than 0.45, and then drops sharply when *Balcony Area/Total Audience Area* is larger than 0.45.

The results shown in Figure 8 indicate that for both kinds of shapes, the larger the scale of an auditorium (*Total Audience Area, Total Seat Number* and *Main Audience Area*) is, the more difficult it will be to acquire acoustically desirable shapes. This is accordant with the empirical knowledge. However, in practice the scale and seat numbers of concert halls are decided considering not only the acoustics but also restrictions of the land, budget, operation, etc. Therefore, it could be more useful to examine the appropriate proportion of the room parameters based on a certain scale. For example, when the total seat number (or the total audience area) is set, how to choose an appropriate proportion to distribute the seats into main audience area and balconies. Moreover, when the area of the main audience is decided, how to determine the proportion of width and length of the auditorium. These two issues are shown in Figure 9 and Figure 10. It can be seen that for both shoe-box shapes and fan shapes, the more the *Total Audience Area* is, the larger the *Balcony Area/Total Audience Area* should be chosen to pursue larger ratio of acoustically desirable shapes. This indicates that architects should consider to distribute more seats in the balconies

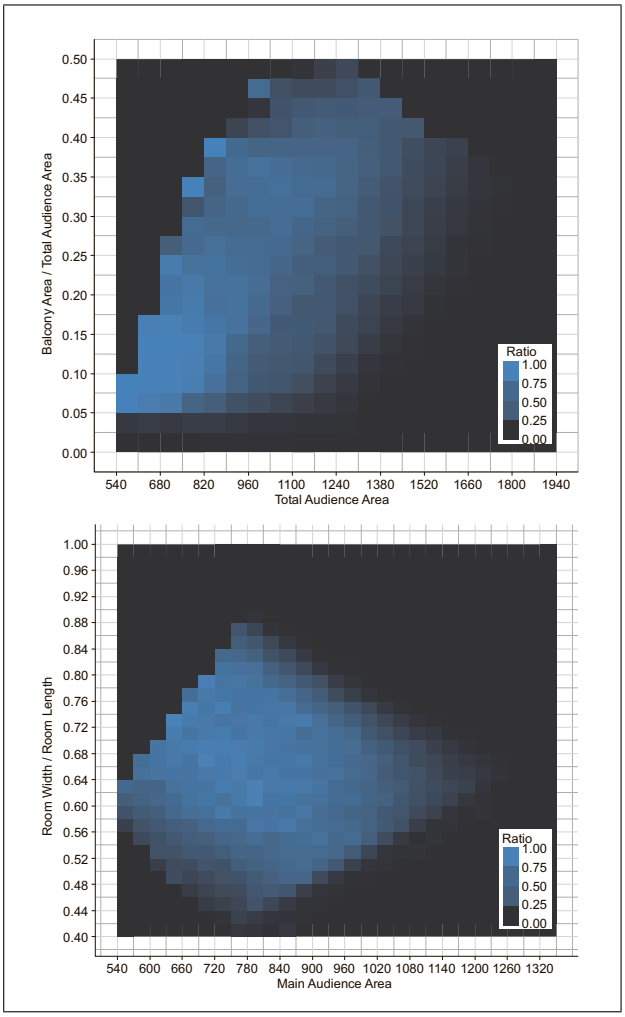

Figure 9. Relationships between the ratio of acoustically desirable shapes and shape parameters in a certain scale of auditoriums for shoe-box shapes.

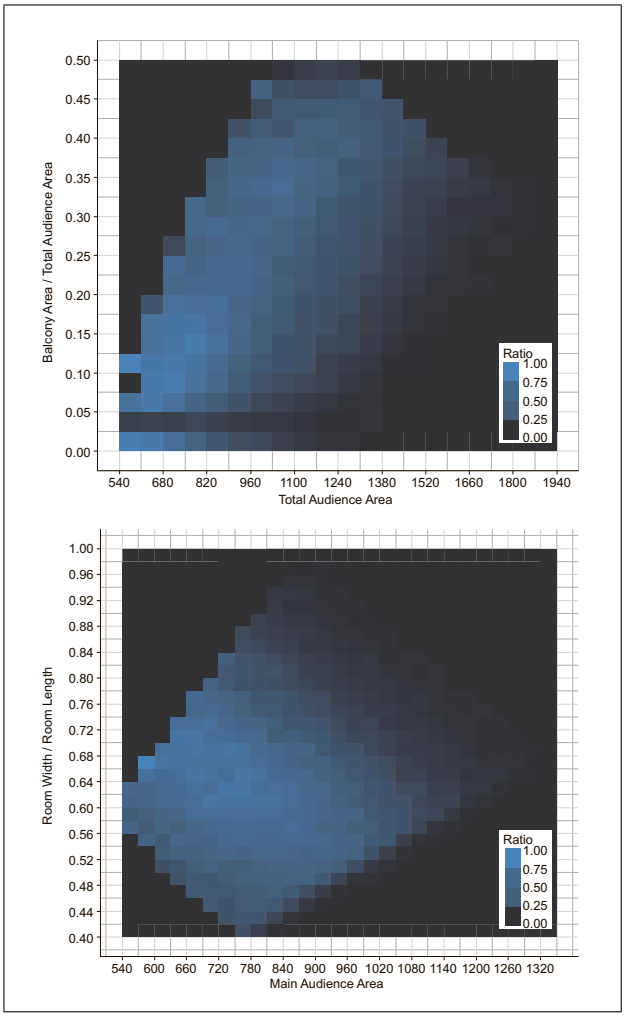

Figure 10. Relationships between the ratio of acoustically desirable shapes and shape parameters in a certain scale of auditoriums for fan shapes.

when dealing with auditoriums with larger scales. The choice of *Room Width/Room Length* should also consider *Main Audience Area*: when the Main Audience Area is around 800 m^2 (or around 1250 seats), the choice of *Room Width/Room Length* is very flexible; when the Main Audience Area is far from 800 m^2, *Room Width/Room Length* should be around 0.6 to 0.64, which is close to the golden ratio accidentally, to pursue larger probability of acoustically desirable shapes.

3.2.4. Volumetric parameters

Sabine Equation shows that the reverberation time of an auditorium is proportional to its volume and inversely proportional to its total absorption. As the total absorption largely depends on the audience, the ratio of volume to audience area (or seat number) has great importance and is widely used in the shape design of auditoriums to control acoustic performance. Here two volumetric parameters are included for each kind: *Volume per Audience Area* and *Volume per Seat*. As same as Section 3.2.3, the seat number is calculated from the total audience area by assuming that the audience area per seat is 0.63.

Figure 11 shows the relationships between the ratio of acoustically desirable shapes and volumetric parameters. For each kind, the ideal range of *Volume per Seat* is between 7 and 10.5 m^3/seat (or *Volume per Audience Area* is between 11 to 16 m), which is accordant with empirical knowledge and makes the ratio of acoustically desirable shapes larger than 0.5. When *Volume per Seat* is larger than 10.5 m3/seat but smaller than 14 m^3/seat (or *Volume per Audience Area* is between 16 to 22 m), the ratio can still stay above 0.4. However, when Volume per Seat is smaller than 7 m3/seat or larger than 14 m^3/seat (or *Volume per Audience Area* is smaller than 11 m or larger than 22 m), the ratio will drop sharply towards 0.

These results show that the probability of acoustically desirable shapes is significantly related to *Volume per Seat* (and also *Volume per Audience Area*), and the recommended range (7 to 10.5 m^3/seat) is similar to that of empirical knowledge (e.g., 8–10 m^3/seat). However, it should be noticed that, *Volume per Seat* is not as strict and powerful as expected from the viewpoint of acoustic potential of shapes. The probability of acoustically desirable shapes stays no higher than 0.6 when *Volume per Seat*

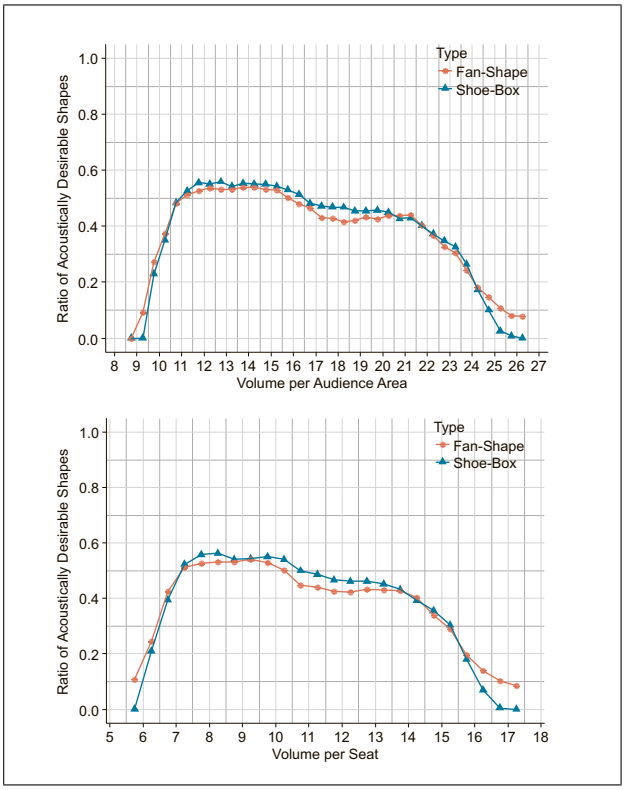

Figure 11. Relationships between the ratio of acoustically desirable shapes and volumetric parameters.

is within the recommended range, and drops only about 0.1 when *Volume per Seat* changes to the range of 10.5 to 14 m^3/seat. This suggests that when designing shoe-box or fan-shaped concert halls, architects could have more freedom to choose the value of *Volume per Seat* than what people would usually expect.

3.3. The method to evaluate the acoustic potential of shapes

Although Section 3.2 shows that the probability of acoustically desirable shapes is highly related to several shape parameters including *Room Width, Total Seat Number, Volume per Seat*, etc., it can be noticed that there is not any value of shape parameters that can ensure the shapes to be acoustically desirable (in other words, make the probability of acoustically desirable shapes approach 1). There are also very few values of certain shape parameters that let all shapes become acoustically unacceptable (or, make the probability of acoustically desirable shapes approach 0). Therefore, designers need to evaluate the acoustic potential of every individual shape obtained in the process of shape design, in order to find out whether the shape is acoustically desirable or not. Here, a method to evaluate the acoustic potential of shapes is raised and implemented.

This method is based on the statistical models acquired in Section 2.3. For a given shape, all its possible compositions of materials are enumerated: each of the three material parameters has 10 possible values (from 5 to 95 with a gap of 10, as same as Section 2.4), which leads to

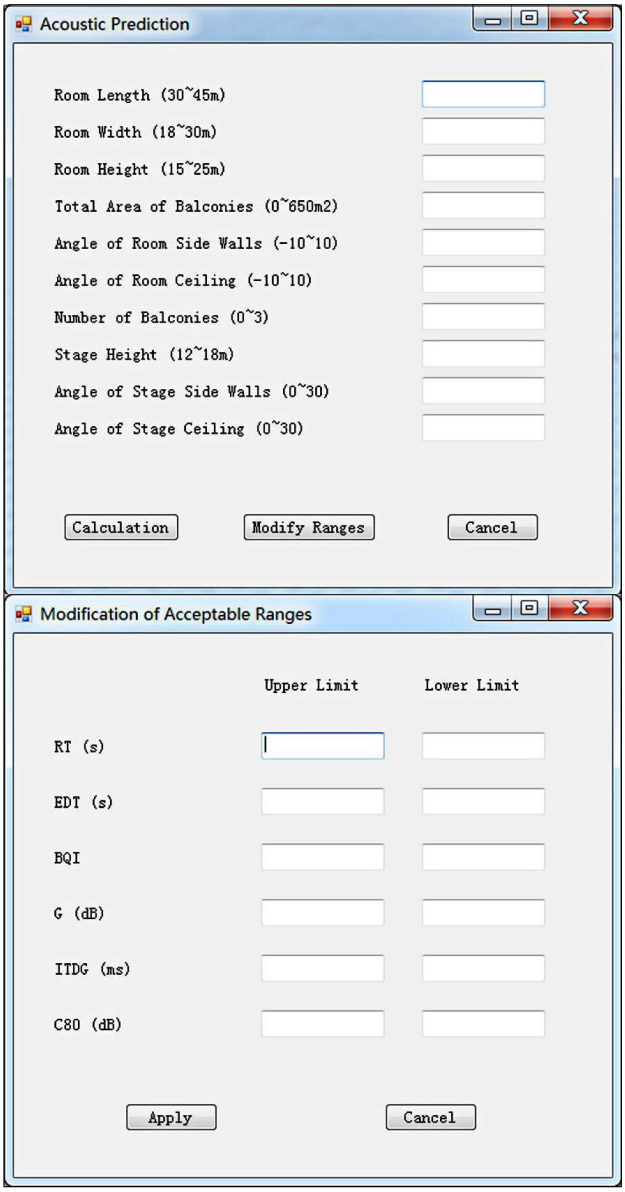

Figure 12. The user interface of the tool (top) and the window for modifying acceptable ranges of acoustic parameters (bottom).

1000 possible material compositions. For a certain shape, its acoustic parameters with all possible material compositions are calculated by the statistical models. If there is at least one material composition that let all six acoustic parameters of this shape fall into the desirable ranges, then the shape is judged to have the acoustic potential of been "remedied" by materials (in other words, it is regarded as acoustically desirable); otherwise it is regarded as acoustically unacceptable.

The method is then implemented in *Rhinoceros* (V.5 SR11) with its Iron Python development kit. Since shoe-box shapes are a subset of fan shapes, here only a tool for fan shapes are developed. Architects need to input all shape parameters into the user interface (Figure 12 top), and then the program will calculate if the shape is acoustically desirable after a click of "*Calculation*". Considering that in reality the desirable ranges of acoustic parame-

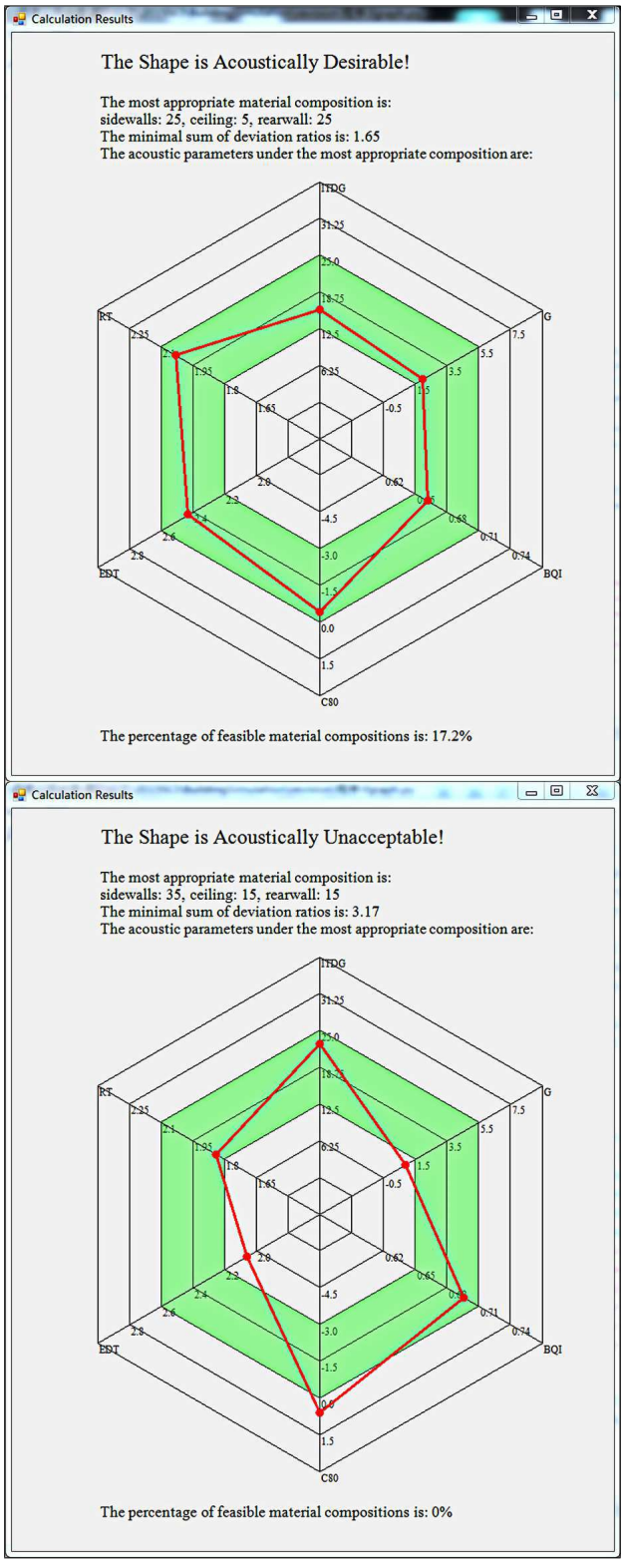

Figure 13. Results of an acoustically desirable shape (top) and an acoustically unacceptable shape (bottom).

ters for a specific project may vary due to different repertoire, size, client preferences, etc., the desirable ranges of acoustic parameters in this tool can be changed by clicking "*Modify Ranges*" button of the user interface and then filling the blanks of the pop-up window (Figure 12 bottom).

The conclusion, whether the shape is acoustically desirable or unacceptable, will be shown after calculation (Figure 13). Besides the conclusion, no matter whether the shape is acoustically desirable or not, two parts of additional information are provided for architects to understand the "degree" of desirability or unacceptability of the shape. One part is about the acoustic performance under the most appropriate composition of materials. For each acoustic parameters, the deviation of its actual value from the center value of its recommended range is calculated. As followed, the ratio of this deviation to the length of its recommended range is adopted as the deviation ratio. Finally, the material composition that leads to the minimal sum of the deviation ratios of all six acoustic parameters is selected as the most appropriate material composition. The information of the most appropriate material composition, the values of the six acoustic parameters under that composition, and the minimal sum of deviation ratios are shown for architects. This information is more reasonable for telling how "unacceptable" an unacceptable shape is. It is clear that the larger the minimal sum of deviation ratios is, the more unacceptable the shape is. However, this information is less useful for comparing different desirable shapes, because there is no evidence showing that the center value of the recommended range of a certain acoustic parameter is better than other values within the recommended range. The other part of information is the percentage of material compositions that can turn all six acoustic parameters into the desirable ranges. The larger the percentage is, the better the shape is judged to be. This information is effective to show the freedom of material design that a certain shape can offer. In other words, it can reflect how difficult a shape is to be acoustically remedied by materials. Therefore, this can be used to evaluate how "desirable" a desirable shape is, and help architects pick out the "best" shape among several "desirable" shapes. Nevertheless, the problem of this information is that it gives us no clue on how "unacceptable" a certain unacceptable shape is, since the percentage for unacceptable shapes is always 0. Briefly, although each of the two parts of information has certain limitations, by combining them together they can provide useful detail information for architects to make design decisions or further modifications, no matter whether the shape is acoustically desirable or unacceptable. Since this whole method is based on statistical models instead of multiple simulations, the total calculation time for one shape is within seconds. Thus it can provide architects with real-time feedback when they are trying different possibilities of shape designs.

4. Discussion

4.1. Limitation of the definition of acoustically desirable shapes

The current definition of acoustically desirable shapes is based on six objective parameters that are widely used and have well-recognized recommended ranges. These quantitative parameters are effective to reflect major aspects of

the acoustic performance of concert halls and can be used to judge if the acoustic performance is desirable straightforwardly (by comparing the actual values and recommended ranges). However, it has to be admitted that there are still some acoustic features that cannot be captured by these parameters. For example, the temporal envelope of early reflections and the distribution uniformity of sound energy are also important in concert halls [47], but there are no reliable quantitative parameters or recommended ranges to reflect these characteristics effectively. Therefore, if these characteristics are included in the definition of acoustically desirable shapes, it could be very difficult to make judgments that whether a shape is acoustically desirable or not. There also have been some existing researches trying to quantify these acoustic characteristics, for example, using the autocorrelation function to represent the distribution uniformity of sound energy [48]. However, these methods have not been widely recognized yet and provided no reliable desirable ranges. As a result, the acoustic characteristics that are unable to be covered by existing room acoustic parameters are not included the current research. This limitation could cause some shapes that are acceptable from the viewpoint of parameters but fail to meet other acoustical characteristics to be judged as acoustically desirable. In other words, the probability of acoustically desirable shapes provided by this research could be higher than reality. Efforts will be made to make the definition of acoustically desirable shapes more comprehensive in the future, but it seems impossible to quantify and include all acoustic characteristics of concert halls, because room acoustics is still regarded as a mixture of art and science at the current stage [48].

Another limitation of the definition of acoustically desirable shapes is that the desirable ranges of acoustic parameters keep constant for all shapes. However, in reality the desirable ranges could vary in different projects due to different repertoire, size, and client preference, etc. There have been some researches trying to quantify the influence of these factors on the ideal values of the acoustic parameters. For example, Smith [49] raised an empirical equation to revise the recommended RT, which suggested that the ideal RT is linearly related to the cubic root of room volume ($t = r \cdot (0.012\sqrt[3]{V} + 0.1070)$, where t is the ideal reverberation time, V is the room volume, and $r = 5$ for orchestra performance). However, this method cannot revise the desirable ranges of other acoustic parameters (EDT, BQI, etc.). Therefore, the method is not included in the current research because the desirable ranges of different parameters will become contradictory with each other if only RT is revised by volume while others remain constant. In the future, more explorations will be made to make the desirable ranges more practical, but the best method available now is to employ the set of widely-recognized desirable ranges raised by Beranek. This is not perfect for all possible shapes but are most representative for large scale concert halls with symphonic repertoire. The desirable ranges used in the tool introduced in Section 3.3, on the other hand, can be adjusted by acousticians according to the requirements of specific projects. Therefore, the tool are applicable for all projects with different repertoire, size, and client preference, etc.

4.2. The pattern of absorption coefficients used in acoustically desirable shapes

As indicated in Section 2.1.4, although empirically surfaces with high absorption coefficients are seldom used in concert halls, in this research the full range of absorption coefficients (0–1) is used. In order to reveal whether the choice of the range of absorption coefficients is influential in this research, and to explore the pattern of absorption coefficients used in acoustically desirable shapes, two more investigations are supplemented.

First, whether the range of absorption coefficients is influential in this research is investigated. As defined in Section 2.1.1, if a shape is acoustically desirable, there will be one or more feasible material compositions that make all its acoustic parameters into the desirable ranges. For a certain feasible material composition, its mean residual absorption coefficient for surfaces excluding audience area and stage area can be calculated. For an acoustically desirable shape, its *minimal residual absorption coefficient* here is defined as the smallest mean residual absorption coefficient among all its feasible material compositions. The *minimal residual absorption coefficient* reflects the lowest amount of absorption that a certain shape needs to fulfill acoustic requirements. Among all acoustically desirable shapes generated here (48272 shoe-box shapes and 44288 fan shapes), the mean value of their *minimal residual absorption coefficients* is 0.0921 for shoe-box shapes and 0.1190 for fan shapes. The standard deviation is 0.0429 for shoe-box shapes and 0.0517 for fan shapes. The largest *minimal residual absorption coefficient* is 0.3865 for shoe-box shapes and 0.4150 for fan shapes. The distribution of acoustically desirable shapes given the *minimal residual absorption coefficient* is shown in Figure 14 top (The Y-axis is defined as the proportion of the acoustically desirable shapes in a certain interval among all acoustically desirable shapes). It can be seen that for both kinds, more than 95% of all acoustically desirable shapes have *minimal residual absorption coefficients* below 0.2. There is no acoustically desirable shape that needs absorption coefficients larger than 0.5, needless to say 0.8 or 0.9, to fulfill acoustic requirements. In other words, the judgment of whether a certain shape is acoustically desirable or not will not be influenced, no matter if the full range of absorption coefficients (0–1) is used, or a specific range that eliminates high values is used. Therefore, using the full range of absorption coefficients is proven to be reasonable in this research.

Second, the pattern of *averaged residual absorption coefficients* is also investigated, which is defined as the average value of the mean residual absorption coefficients of all feasible material compositions for a certain acoustically desirable shape. Among all acoustically desirable shapes generated here, the mean value of their *averaged residual absorption coefficients* is 0.1321 for shoe-box shapes and

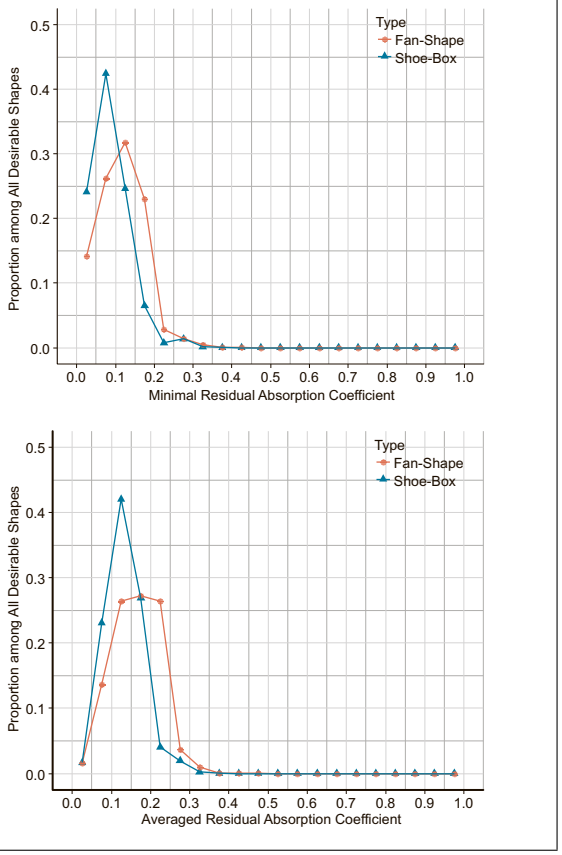

Figure 14. The distribution of acoustically desirable shapes given the minimal residual absorption coefficient (top) and the averaged residual absorption coefficient (bottom).

0.1641 for fan shapes, with a standard deviation of 0.0456 and 0.0583 respectively. The largest *averaged residual absorption coefficient* is 0.3865 for shoe-box shapes and 0.4590 for fan shapes. The distribution of acoustically desirable shapes given the *averaged residual absorption coefficient* is shown in Figure 14 bottom, which is similar to that given the *minimal residual absorption coefficient*. It can be seen that very few acoustically desirable shapes (less than 5%) have *averaged residual absorption coefficients* larger than 0.25. This finding proves that only a small amount of absorption is needed (except for audience area and stage area) for most shoe-box and fan shaped concert halls, and an average absorption coefficient around 0.1 to 0.2 is most commonly used. This result is also close to the assumption raised by Hidaka that the residual absorption coefficient in concert halls is around 0.1 [33].

4.2.1. Interpretation of the results in shape designs of concert halls

This research provides three parts of results, and each of them should be interpreted and used appropriately in shape design of concert halls.

The first part (Section 3.1) indicates that for both shoe-box and fan-shaped concert halls, only less than 50% of all possible shapes are acoustically desirable, even when the functionalities of materials are taken into account. Since shoe-box shapes are believed to be the most favorable type for concert halls to have desirable acoustics and fan shapes are also very commonly used, it is likely that other types (e.g., vineyard, arena) will have even smaller ratios of acoustically desirable shapes. This proves the great importance of shape design in achieving desirable acoustic performance, and shows that architects should take acoustic issues into consideration at the early design stage of concert halls.

The second part (Section 3.2) shows the relationships between the probability of acoustically desirable shapes and shape parameters. This can be used by architects as references on shape design. Architects are suggested to pay more attention to the shape parameters that are highly related to the probability of acoustically desirable shapes. They are also suggested to choose the values that are more likely to lead to acoustically desirable shapes for these shape parameters, as long as the values are not contrary to their design ideas or other design requirements. In this way they are aided to stay in an appropriate direction to pursue acoustically desirable shapes. It should also be noticed that, however, the results should not limit architects' design freedom, since there are very few values of certain shape parameters that cause all shapes to be acoustically unacceptable. Therefore, if architects would like to choose the values of shape parameters that are unlikely to achieve acoustically desirable shapes, they should be allowed. However, the bottom line is that any shape they choose should be tested by the method provided in Section 3.3 in order to check whether the specific shape has the potential to be acoustically remedied by materials.

The third part (Section 3.3) provides a real-time method to test whether a specific shape is acoustically desirable or not. This can help architects make design decisions on whether to keep the current shape or revise it. Besides the extreme judgments of "desirable" or "unacceptable", the method can also provide quantitative details on the degree of desirability or unacceptability of shapes. Therefore, it is a practical design-assisting tool for architects to consider acoustic issues in the early design stage.

5. Conclusion

Herein, the influence of room shapes on the acoustic performance of shoe-box and fan-shaped concert halls are investigated from the viewpoint of acoustic potential of shapes. Several conclusions can be drawn from this research:

1. Shape designs are proven to have great importance in achieving desirable acoustic performance for shoe-box and fan-shaped concert halls. The probability of a random shape within the boundary conditions of this research to be acoustically desirable is below 0.5 (0.4827 for shoe-box and 0.4429 for fan shapes). Acoustic issues should be taken into consideration at the early design stage of concert halls.
2. Several shape parameters are found to have significant influence on the probability of acoustically desirable

shapes. The appropriate range of *Room Width* is 20.5 m to 25 m for shoe-box shapes and 20.5 m to 23 m for fan shapes. *Room Length* should be no longer than 42 m for both kinds. Angle of Room Side Walls is recommended to have a negative value for fan shapes (or the shape is narrowing towards the rear). *Room Width/Room Length* is suggested to be around 0.6 for shoe-box shapes and around 0.55 for fan shapes. *Room Width/Room Height* is recommended to be around 0.9 for both kinds. *Total Seat Number* should not exceed 2500 seats, while *Main Audience Area* is recommended to be no larger than 1200 m^2 for both kinds. The appropriate range of *Volume per Seat* is 7 to 10.5 m^3/seat for both kinds. Architects are suggested to pay more attention on these shape parameters and choose the values corresponding to higher probability of acoustically desirable shapes if possible.

3. A method that can evaluate the acoustic potential of shapes in real-time is proposed and implemented. Shapes of concert halls conceived by architects in the early design stage should be tested by this method in order to ensure acoustic desirability.

Appendix: Details about Multivariate Adaptive Regression Splines

A1. Model fitting

Multivariate Adaptive Regression Splines (MARS) is a method for fitting adaptive non-linear regressions that uses piece-wise basis functions (splines) to define the relationship between a response variable and a set of explanatory variables [41]. The range of each explanatory variable is first broken into subsets using a set of knots, and then an MARS model can be expressed with a set of basic functions:

$$\hat{f}(x) = \sum_{m=1}^{M} a_m B_m(x),$$

where $\hat{f}$ is the predicted vector, x is the vector of explanatory variables, M is the number of subsets, $B_m(x)$ is the basic function, while a_m is a coefficient whose value is adjusted to give the best fit to the data. And then a forward stepwise procedure is conducted, which subdivides each subset into splits. The basic function (multivariate spline) can then be defined as

$$B_m^{(q)}(x) = \prod_{k=1}^{K_m} \left[s_{km}(x - t_{km}) \right]_+^q,$$

where $B_m^{(q)}(x)$ is the basic function with a power of q, x is the vector of explanatory variables, K_m is the number of splits in the subset, $s_{km} = \pm 1$ according to the right $(+1)$ and left (-1) sense of the associated split, t_{km} is the values of explanatory variables at the splitting point, while q is the power of the spline (the most commonly used power is 4, which corresponds to cubic spline). It should also be noted that the function $[a]_+$ equals $\max(0, a)$. Different knots and splits are tested until the lowest residual sum of squares are reached. Then a backwards pruning procedure is applied, in which the splits that contribute least to model fit are progressively removed. At this stage, an explanatory variable can be dropped from the model completely if none of its basis functions contribute meaningfully to predictive performance. The sequence of models generated from this process is then evaluated using generalized cross-validation, and the model with the best predictive fit is selected as the final model.

A2. Assessment of model performance

The nonlinear statistical models obtained by MARS method are assessed by two indices in this research. One is the correlation coefficient (R^2), which is a natural measure of the strength of the relationship between a response variable and a set of explanatory variables. More often it is used for linear relationships, but it is also applicable for relationships that are either a linear or other specific parametric model [50]. Its definition is

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}},$$

where R^2 is the correlation coefficient, SS_{tot} is the total sum of squares, while SS_{res} is the residual sum of squares:

$$SS_{\text{tot}} = \sum_i \left(y_i - \overline{y} \right)^2,$$

$$SS_{\text{res}} = \sum_i \left(y_i - \hat{y}_i \right)^2,$$

where y_i is the observed value of the response variable, $\overline{y}$ is the mean of the observed values, while $\hat{y}_i$ is the predicted value.

Another index to assess the model performance is the residual standard deviation (SD), which is also a widely used measure for linear and other nonparametric relationships. Its definition is

$$s = \frac{1}{n} \sqrt{\sum_{i=1}^{n} \left(y_i - \hat{y}_i \right)^2},$$

where s is the residual standard deviation, y_i is the observed value of the response variable, is the predicted value, while n is the number of observations. It should be noted that in Section 2.3, R^2 is calculated with the 800 cases that are used to train the regression models, while residual standard deviation is calculated with the other 200 cases that are meant to test the precision of statistical models.

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