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Formulae for the Forced Sound Transmission Coefficients

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Summary
Sound transmission through a single leaf panel via the forced vibration has been associated with the non-resonant response of a stiffened panel and has often been analyzed with a so called limp panel for which the bending stiffness is very low. Several different formulae for calculating the diffuse field sound transmission coefficients, either based on numerical integration or with closed-form analytical expressions, have been suggested. To facilitate their practical applications, the differences and rationalities of these formulae are reviewed in this note.

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1. Introduction

In analyzing sound transmitting through a single leaf panel, it is helpful to distinguish the resonant and non-resonant panel responses such that their contributions in sound transmission loss performance can be dealt with separately. It is acknowledged that while panel resonances govern the overall sound transmission performance above the coincidence frequency, the non-resonant panel response would be more dominant below the coincidence frequency. The non-resonant panel response has been associated with the panel being in forced vibration, for which the behavior can be represented by a limp panel, i.e. a panel with a very low bending stiffness such that the coincidence effect can be ignored.

Sound transmission through a single leaf panel via the forced vibration is widely assumed to follow the mass law. The mass law derived from an infinite panel model is well known today, and the corresponding analytical expressions for the normal incidence and the diffuse field sound transmission coefficients/loss have been discussed and used extensively. In order to address the effect associated with a finite size panel in reality, a model of a limp panel driven by acoustic incident waves is usually adopted in the analysis. By introducing the radiation impedance of a finite sized panel, corrections to the infinite panel transmission coefficient may be applied and the diffuse field sound transmission coefficient of a finite size panel can be calculated based on the Paris formula. In several reported studies [1, 2, 3, 4, 5, 6, 7], however, different corrections have been suggested. Although they are somewhat similar and were all claimed valid in the respective studies, a discussion on the rationalities of each formula seems to be necessary.

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Analytical expressions for the forced transmission coefficient of a finite size panel have also been pursued in the past. Sewell's work [8] on the diffuse field transmission coefficient has been regarded as the earliest effort. Later, Davy [5, 9] presented two different sets of equations, one of which was developed to work in the full frequency range. Recently, new sets of equations were suggested by the author [10] for both the normal incidence and the diffuse field sound transmission coefficients of the panels with low bending stiffnesses. Differences in these expressions and their validity conditions relative to the numerical integration formulae should be assessed as well.

The purpose of this short note is to review these numerical integration formulae and closed-form analytical expressions. Based on three panel setups, the relative differences in calculated results and the fidelity of these models are discussed. It is worth noting that the practicality of these formulae for specific applications is not the focus of this note. The intent is to evaluate the numerical consistencies between these models. In order to facilitate the discussion, the term "low frequency" referred in this note generally means $k\sqrt{S} < \sqrt{e}$, and the "high frequency" implies $k\sqrt{S} > \sqrt{e}$. Both frequency ranges are actually for diffuse field sound transmission performance, as illustrated in [10]. Here, k is the acoustic wavenumber, S is the surface area of the panel, and $e \approx 2.72$, ($\sqrt{e} \approx 1.65$), is Euler's number. For normal incidence, the condition becomes $k\sqrt{S} < 3$, and $k\sqrt{S} > 3$ respectively [10].

2. Numerical integration formulae

The numerical calculation of the diffuse field sound transmission coefficient $\langle \tau_{\text{fin}} \rangle$ is usually based on the Paris formula. For a finite size panel, there is

$$\left\langle \tau_{\text{fin}} \right\rangle = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\pi/2} \tau_{\text{fin}}(\theta, \varphi) \sin\theta \cos\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi. \qquad (1)$$

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Since the oblique incidence sound transmission coefficient of a finite size panel $\tau_{\text{fin}}(\theta, \varphi)$ may also involve multiple integrals, in order to reduce the computation cost, it was assumed in many studies [1, 6] that the dependency of $\tau_{\text{fin}}(\theta, \varphi)$ on the azimuth angle φ is insignificant such that equation (1) can be rewritten as

$$\left\langle \tau_{\text{fin}} \right\rangle \approx \int_0^{\pi/2} \tilde{\tau}_{\text{fin}}(\theta) \sin 2\theta \, \mathrm{d}\theta, \tag{2}$$

where $\tilde{\tau}_{\text{fin}}(\theta)$ can be an average value over several discrete azimuth angles, or simply takes $\tau_{\text{fin}}(\theta, \varphi)$ with a fixed azimuth angle φ.

In deriving the oblique incidence sound transmission coefficient of a finite size panel, several formulae have been developed on the basis of the transmission coefficient of an infinite panel. As a reference, the infinite panel sound transmission coefficient is given here,

$$\tau_{\text{inf}}(\theta) = \left| 1 + \frac{\mathrm{j}\omega m}{2\rho_0 c_0} \cos\theta \right|^{-2} \tag{3}$$

where $\mathrm{j} = \sqrt{-1}$, ω is the angular frequency, $\rho_0 c_0$ is the characteristic impedance of air, and m is the surface weight of the panel.

The model presented by Villot *et al.* [1] was to apply a window function to the incidence wave and the vibration response of an infinite size panel. The finite size transmission coefficient was therefore suggested as

$$\tilde{\tau}_{\text{fin}}(\theta) = \tau_{\text{inf}}(\theta) \left[\tilde{\sigma}(\theta) \cos\theta \right]^2. \tag{4}$$

Here, $\tilde{\sigma}(\theta)$ is the radiation efficiency of a finite sized limp panel with its dependency on the azimuth angle being removed or ignored. However, it was claimed later by Villot *et al.* [2] that equation (4) generally over-accounts the size effect and the window function may only need to apply once for a better correlation with testing results. This has been further confirmed by Vigran [3] in discussing the simplification of radiation efficiency calculations. The corrected formula then becomes identical to that developed by Ghinet and Atalla [4], which is given as

$$\begin{aligned} \tilde{\tau}_{\text{fin}}(\theta) &= \tau_{\text{inf}}(\theta) \tilde{\sigma}(\theta) \cos\theta \\ &= \frac{\tilde{\sigma}(\theta)}{\cos\theta \left| \dfrac{1}{\cos\theta} + \dfrac{\mathrm{j}\omega m}{2\rho_0 c_0} \right|^2} \end{aligned} \tag{5}$$

In a separate study, Davy [5] basically used the radiation efficiency of a finite size panel $\tilde{\sigma}(\theta)$ to replace $1/\cos\theta$, i.e. the radiation efficiency for an infinite limp panel, and obtained

$$\tilde{\tau}_{\text{fin}}(\theta) = \frac{\tilde{\sigma}(\theta)}{\cos\theta \left| \tilde{\sigma}(\theta) + \dfrac{\mathrm{j}\omega m}{2\rho_0 c_0} \right|^2}. \tag{6}$$

Recently, Brunskog [6] employed a variational technique and expressed the transmission coefficient in terms of the radiation impedance of a finite limp panel as

$$\tilde{\tau}_{\text{fin}}(\theta) = \frac{\Re e\{\tilde{z}_f\}}{\cos\theta \left| \tilde{z}_f + \dfrac{\mathrm{j}\omega m}{2\rho_0 c_0} \right|^2}, \tag{7}$$

where z_f is the specific impedance ratio, and $\Re e\{\tilde{z}_f\} = \tilde{\sigma}(\theta)$. It can be seen that if the imaginary part of the impedance is ignored, equations (6) and (7) are identical. Note that the reactive effect of the radiation impedance, such as the mass loading, is generally indispensable especially at low frequencies. Equation (7) with both real and imaginary part of the radiation impedance accounted appears to be more rigorous. In a recent and separate study on sound insulation of plywood panels [7], the same equation can actually be derived for limp panels by ignoring the stiffness term. It should be noted that the impedance and equation (2) have to be solved numerically.

In Figures 1 and 2, sound transmission losses, i.e. $-10\log[\langle\tau_{\text{fin}}\rangle]$, calculated from each model are shown respectively for three different panel setups, one being small and light, the other two about 3 times bigger in surface area but with two different surface weights. For simplicity, equation (4) is excluded from the comparison. The normal incidence sound transmission losses obtained from equa-

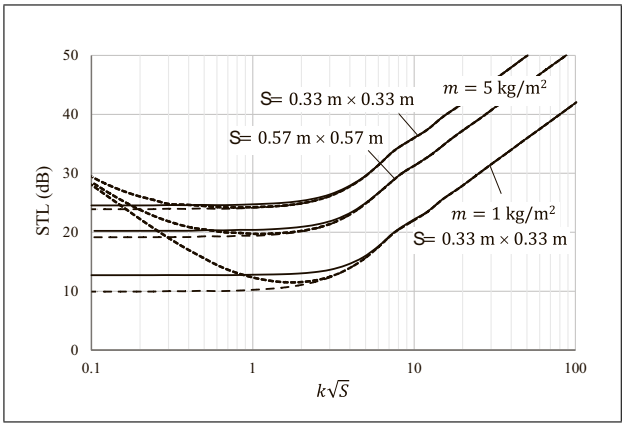


Figure 1. Normal incidence sound transmission loss of a limp panel: · · · · Equation (5), – – – Equation (6), —— Equation (7).

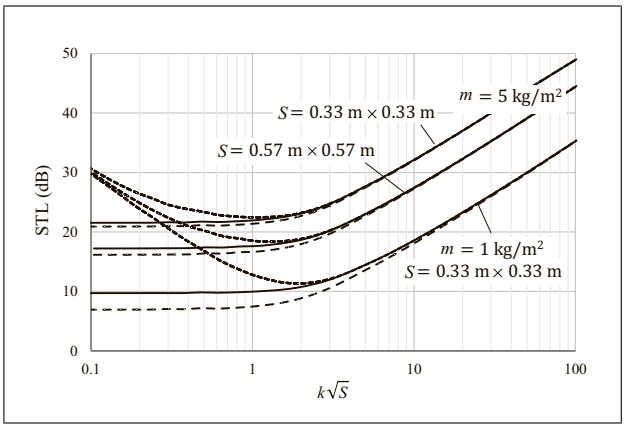


Figure 2. Diffuse field sound transmission loss of a limp panel: · · · · Equation (5), – – – Equation (6), —— Equation (7).

tions (5) to (7) directly by setting $\theta = 0$ are shown in Figure 1, and the diffuse field sound transmission losses based on equations (5) to (7) and equation (2) are shown in Figure 2. All sound transmission loss curves are plotted against the non-dimensional metric $k\sqrt{S}$. It can be seen that two typical frequency ranges exist for each of the calculated sound transmission loss. At high frequencies, results from all three models converge respectively for normal and diffuse field transmission losses. This can be explained by the fact that when $\omega m/2\rho_0 c_0$ dominates the denominator in equations (5) to (7), three equations becomes identical. The same effect can be expected for a heavy panel. In addition, as the panel size increases, $\tilde{\sigma}(\theta)$ may approach $1/\cos\theta$, and the imaginary part of $\tilde{z}_f$ may become a negligible small number. This is another scenario that equations (5) to (7) converge to the same results. This is probably the reason that all these models were found working reasonably well in each of their studies. However, differences can be clearly seen in the low frequency range where the panel size effect becomes important, especially for a lighter weight panel. Specifically, equation (5) yields a sound transmission loss that inversely increases as frequency, which seems to be practically unreasonable. This is primarily due to the radiation efficiency of an infinite limp panel, i.e. $1/\cos\theta$, being used in equation (5). Moreover, equation (6) consistently returns lower values than equation (7), and the difference enlarges as the panel is getting lighter and maybe bigger. This is because the mass loading is ignored in this equation as described above, which generally leads to a lower prediction. Also in this frequency range, the bigger the panel, more effects associated with the mass loading would be expected. It is therefore not difficult to realize that equation (6) only works for heavy panels.

It is worth noting that both equations (6) and (7) present 3 dB difference between the normal and diffuse field sound transmission loss in the low frequency range. This is consistent with the discussion in [10].

3. Closed-form expressions

There are several closed-form analytical expressions developed for the diffuse field sound transmission coefficient of a finite size panel. Sewell's equation [8] was developed based on a modal description of a panel but with the intermodal coupling effects ignored. It is now known as

$$\left\langle \tau_{\text{fin}} \right\rangle = \left(\frac{2\rho_0 c_0}{\omega m} \right)^2 \ln\left(k\sqrt{S} + 0.16 + \frac{1}{4\pi k^2 S} \right). \tag{8}$$

The original expression given by Sewell contained a shape function inside the natural logarithm. It was suggested later by many researchers that the contribution from the shape function is insignificant. At high frequencies, only the leading term $k\sqrt{S}$ will take effects. The last two terms basically work at low frequencies.

Davy's early attempt was to use the infinite panel mass law together with the Paris formula through limiting the integration angle to account for the finite size effect [9]. The limiting angle is linked to the radiation resistance of a finite size panel, which leads to a formula given as

$$\left\langle \tau_{\text{fin}} \right\rangle = \left(\frac{2\rho_0 c_0}{\omega m} \right)^2 \ln\left[\frac{\left(\frac{\omega m}{2\rho_0 c_0} \right)^2 + 1}{\left(\frac{\omega m}{2\rho_0 c_0} \right)^2 \mathcal{K} + 1} \right],$$
$$\mathcal{K} = \begin{cases} 0.9 & \text{for } k\sqrt{S} < 1.1, \\ \dfrac{1}{k\sqrt{S}} & \text{for } k\sqrt{S} > 1.1. \end{cases} \tag{9}$$

Later, Davy presented another expression with the help from a study on the forced radiation efficiency of a finite size panel [5]. The corresponding diffuse field sound transmission coefficient was derived as

$$\left\langle \tau_{\text{fin}} \right\rangle = \left(\frac{2\sqrt{2}\rho_0 c_0}{\omega m} \right)^2 \left[\ln\left(\frac{1 + \sqrt{1+q^2}}{p + \sqrt{p^2+q^2}} \right) + \frac{p}{h-p} \ln\left(\frac{h + \sqrt{h^2+q^2}}{p + \sqrt{p^2+q^2}} \right) \right], \tag{10}$$

where

$$p = \begin{cases} w\sqrt{\frac{\pi}{2kl}} & \text{for } w\sqrt{\frac{\pi}{2kl}} \le 1.1, \\ 1 & \text{for } w\sqrt{\frac{\pi}{2kl}} > 1.1, \end{cases}$$

$$h = \left(\frac{2}{3}\sqrt{\frac{2kl}{\pi}} - \beta \right)^{-1}, \qquad q = \frac{2\pi}{k^2 S}.$$

However, there are three parameters that had to be determined empirically in this equation, i.e. $\beta = 0.124$, $w = 1.3$, and the 2nd root together with the power of 2 appeared inside the natural logarithm. Also in here, $l = 2S/U$, where S is the area of the panel and U is the perimeter of the panel.

In a recent study conducted by the author [11, 10], the modal sound transmission coefficients of each mode were developed with the intermodal coupling being incorporated in the equivalent modal radiation impedances. Benefited from the analytical expressions and the modal superposition, asymptotic solutions for both the normal and diffuse field sound transmission coefficients can be derived for panels with low bending stiffnesses. Specifically for normal incidence, the sound transmission coefficient is

$$\tilde{\tau}_{\text{fin}}(0) = \frac{\rho_0^2 S}{2\left(m + \rho_0\sqrt{S}\right)^2} \qquad \text{for } k\sqrt{S} < 3, \tag{11a}$$

$$\tilde{\tau}_{\text{fin}}(0) = \left[1 + \left(\frac{m\omega}{2\rho_0 c_0} \right)^2 \right]^{-1} \text{for } k\sqrt{S} > 3. \tag{11b}$$

The diffuse field sound transmission coefficient is given as

$$\left\langle \tau_{\text{fin}} \right\rangle = \frac{\rho_0^2 S}{\left(m + \rho_0\sqrt{S}\right)^2} \qquad \text{for } k\sqrt{S} < \sqrt{e}, \tag{12a}$$

$$\left\langle \tau_{\text{fin}} \right\rangle = \left(\frac{2\rho_0 c_0}{\omega m} \right)^2 \ln \left[\frac{\left(\frac{\omega m}{2\rho_0 c_0} \right)^2 + 1}{\left(\frac{\omega m}{2\rho_0 c_0} \right)^2 \frac{1}{k\sqrt{S}} + 1} \right] \quad \text{for } k\sqrt{S} > \sqrt{e}. \tag{12b}$$

It can be seen that at low frequencies, the normal sound transmission loss is 3 dB higher than the diffuse field transmission loss. This is already confirmed by Brunskog's model shown in Figures 1 and 2.

It can also be seen that the high frequency expression in equation (9) and equation (12b) are essentially the same although they were obtained from different approaches. When $\rho h\omega / 2\rho_0 c_0 \gg 1$, both equations would yield equation (8) under the high frequency approximation. This supports the statement made by Sewell that his equation may only work for heavy panels [8]. In Figure 3, calculations based on equations (8) to (10) and (12) are compared together with Brunskog's model (equation 7) for the two panel setups. At high frequencies, equations (8), (9), (10) and (12) all converge to Brunskog's model.

At low frequencies, however, equation (8) starts diverging when $k\sqrt{S} < 1$ and even with the term $1/4\pi k^2 S$ included for low frequency corrections. Equation (9) presents a low frequency expression, but indicating that sound transmission coefficient doesn't depend on the panel size. Actually, it can be shown that the expression has a low frequency asymptotic value of 10 dB irrespective of panel mass. These facts together with the unrealistic peak shown around transition at $k\sqrt{S} = 1.1$ seem to suggest that this expression does not apply for low frequency performance. It can be found in Figure 3 that the condition given for equation (12b), i.e. $k\sqrt{S} > \sqrt{e}$, is more reasonable to use for the validity of equations (8) and (9).

Equation (10), which was developed on the basis of equation (6) and is correlated with equation (6) very well, is able to capture the frequency independent performance at low frequencies. But as discussed before, since equation (6) doesn't have the reactive mass loading considered, equation (10) behaves the same, under-estimating the performance in the low frequency range with more errors seen for panels with a lighter weight. In Figure 3, the difference between equation (10) and Brunskog's model is 2.8 dB at $k\sqrt{S} = 0.5$ for $m = 1\,\text{kg/m}^2$. With a heavier panel, $m = 5\,\text{kg/m}^2$, the difference is 0.75 dB at $k\sqrt{S} = 0.5$. It is not difficult to conclude that equation (10) only works for heavy panels.

Compared with Brunskog's model, a good correlation is obtained not only for equation (12) in the diffuse field sound transmission loss, but also equation (11) for the normal incidence sound transmission loss as shown in Figure 4. However, it appears that both equations (11a) and (12a) consistently over-predict the low frequnecy performance by about 1 dB. Note that equations (11a) and (12a) were developed for a panel with low bending stiffnesses, and that the superposition of participated modes was taken with limited number of terms [10]. Given an extreme case in which a sufficient number of modes are considered, the summation over all odd-odd modes would approach a con-

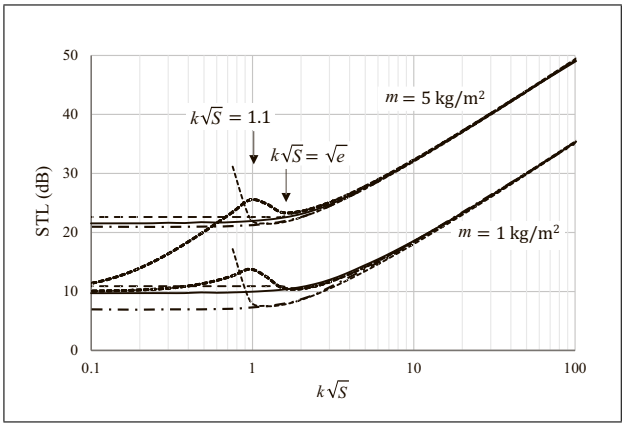


Figure 3. Diffuse field sound transmission loss of a limp panel: · · · Equation (8), ● ● ● Equation (9), – · – · – Equation (10), – – – Equation (12), —— Brunskog's model. Panel size 0.33 m × 0.33 m.

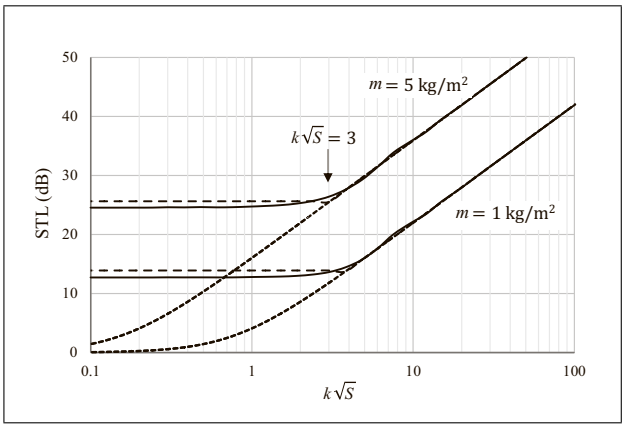


Figure 4. Normal incidence sound transmission loss of a limp panel: · · · Equation (3), – – – Equation (11), —— Brunskog's model. Panel size 0.33 m × 0.33 m.

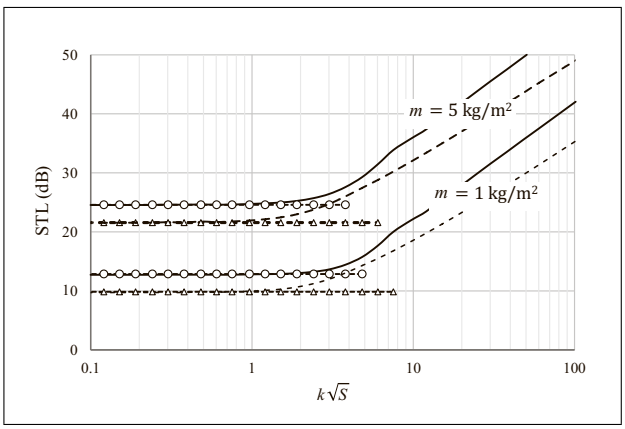


Figure 5. Sound transmission loss of a limp panel: – – – Brunskog's model for diffuse field, —— Brunskog's model for normal incidence, – ○ – ○ – Equation (13), – △ – △ – Equation (14). Panel size 0.33 m × 0.33 m.

stant of $\pi^4/2^6 = 1.52$. Comparing this number to the constant of 1.35 taken for deriving equations (11a) and (12a) in [10], there is a difference of $20\log(1.52/1.35) = 1$ dB. This indicates that for a limp panel the diffuse field sound transmission coefficient at low frequencies can be derived explicitly as,
for normal incidence,

$$\tilde{\tau}_{\text{fin}}(0) = \frac{2}{\pi}\frac{\rho_0^2 S}{\left(m+\rho_0\sqrt{S}\right)^2} \quad \text{for } k\sqrt{S} < 3, \qquad (13)$$

and for diffuse field,

$$\left\langle \tau_{\text{fin}} \right\rangle = \frac{4}{\pi}\frac{\rho_0^2 S}{\pi\left(m+\rho_0\sqrt{S}\right)^2} \text{ for } k\sqrt{S} < \sqrt{e}. \qquad (14)$$

In Figure 5, equations (13) and (14) are compared to Brungskog's model.

4. Conclusions

Though all numerical formulae, except for Villot's original equation, are able to predict the high frequency sound transmission performance well, Brunskog's model appears to be the most rigorous one over the entire frequency range for a limp panel. As for the analytical expressions, equations developed by the author appear to be more explicit than others, especially the low frequency expressions. It has also been shown here that the analytical expressions for both normal incidence and randomized incidence can be extended to limp panels by introducing a new coefficient at low frequencies in order to have an even better correlation with Brunskog's model. However, since the "limp" panel is an ideal case, analytical expressions developed in [10] may still be more appropriate to use in practice for panels with low bending stiffnesses.

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