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Sound Field Prediction and Separation in a Moving Medium Using the Time-Domain Equivalent Source Method

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Summary

An alternative computational method based on time domain formulations of the convective Ffowcs Williams–Hawkings equation is proposed to acquire the source terms on the data surface used for sound field prediction. The proposed computational model is constructed based on the time-domain equivalent source method, which supposes that the source terms on the data surface are represented by a series of time-domain equivalent sources of monopole and dipole types. The strengths of those equivalent sources are determined by fitting the pressure data of collocation points outside the source region. The obtained source terms can not only be used to predict the total pressure at the observer, but also be used to separate the pressure component contributed by monopole source term and that contributed by dipole source term from the total pressure. Simulation results of stationary monopole and dipole sources as well as a rotating monopole source in a moving medium validate the effectiveness of the proposed method for both stationary and moving sources.

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1. Introduction

With the rapid development of fluid machinery and aviation industry as well as the continuous pursuit of silent machines, both theoretical and experimental studies in aeroacoustics have been conducted in order to understand and predict the noise generation and radiation in recent years.

Since Lighthill's acoustic analogy [1] was proposed as a milestone in aeroacoustics, it has been widely used and reinterpreted in various forms, for example, the Ffowcs Williams–Hawkings (FW–H) equation for sound generation by surfaces in arbitrary motion. The FW–H equation [2] has formed the basis of a number of noise prediction methods, such as the best known time domain formulations derived by Farassat and Brentner [3, 4, 5] and the frequency domain formulations derived by Hanson [6], Khelladi *et al.* [7] and Tang *et al.* [8]. As aerodynamic sources are generated from the interaction between fluid and structure or the interaction inside the fluid like jet noise problems, the fluid has an important effect on noise generation and radiation. Taking into account the effect of the fluid, for instance, the uniform flow, the classical FW–H equation could be converted to the convective FW–H equation [9]. Recently, Najafi-Yazdi *et al.* [10] and Ghorbaniasl and Lacor [11] developed the time domain formulations by solving the convective FW–H equation for sound radiated from moving bodies in a uniform flow while the

corresponding frequency domain formulations in the uniform flow were proposed by Xu *et al.* [12]. Here it should be noted that the time domain formulations and the frequency domain formulations aforementioned are theoretically equivalent since the frequency domain formulations predict sound through the strength of each harmonic of noise while the time domain formulations present a time history, usually in one or two periods.

When solving practical engineering problems of aeroacoustics, the time domain or frequency domain formulations are often combined with the computational fluid dynamics (CFD) to predict sound field as it could save a lot of computational cost and time compared with the direct noise computation [13]. In the combined method, the noise prediction process is divided into two phases, solution of source terms on the data surface in the near field via CFD and propagation of noise waves via the integral formulations from the data surface to the observer in the far field. Instead of the complicated CFD, an alternative inverse method can also be used to obtain the solution of source terms in the first phase. Li *et al.* [14] and Trabelsi *et al.* [15, 16] developed the inverse methods based on the classical FW–H equation to reconstruct aerodynamic loading on the blade surface by using the measured field pressure in the frequency domain. In this paper, an alternative time domain inverse method based on the time domain formulations of the convective FW–H equation is proposed to acquire the source terms on the data surface by using the field pressure.

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The proposed time domain inverse model is constructed based on the time-domain equivalent source method (TESM). TESM was first proposed by Kropp and Svensson [17] to calculate the sound radiation and scattering. The idea of this method is to locate a series of equivalent sources approximating a sound field outside a given enclosure by fulfilling the given boundary conditions on the enclosed surface. The revival of TESM was made by significant improvements on numerical algorithms by Lee *et al.* [18, 19].

They used time interpolation functions for TESM and introduced the singular value decomposition to stabilize the solutions [18]. They also investigated the parameter sensitivity to the TESM in detail and employed a time-averaging scheme to further stabilize the solution [19]. Lately the TESM has been widely used in sound radiation and scattering problems [20, 21, 22, 23, 24, 25] as well as acoustic field reconstruction [26] and sound source identification [27]. In the present study, the TESM is further developed to make it applicable in predicting sound field with both monopole and dipole source terms included and to take the uniform flow effect into account. The source terms on the data surface are replaced by a series of time-domain equivalent sources of monopole and dipole types. The strengths of those equivalent sources are determined by fitting the pressure data of collocation points on the surface enclosing the equivalent sources. The reconstructed source terms can not only be used to predict the total pressure at the far field observer, but also be used to separate the pressure component contributed by monopole source term and that contributed by dipole source term from the total pressure based on the fact that those two source terms are independent of each other.

This paper is organized as follows. The convective FW-H equation and its time domain formulations in a moving medium are first briefly reviewed in Section 2.1, and then the detailed procedure of the proposed method including solving equivalent source strengths, predicting the total pressure and separating monopole and dipole components from the total pressure are described in Section 2.2. Finally, three numerical test cases are used to examine the performance of the proposed method in Section 3.

2. Theory

2.1. The convective FW-H equation and its time domain formulations in a moving medium

Suppose that a uniform flow moves at a constant velocity $\mathbf{U}_\infty$ and the direction of the velocity is arbitrary. Reorganizing the continuity and momentum equations that include the constant convective velocity term, the sound pressure at the observer $\mathbf{x}$ at time t could be described by the convective FW-H equation [9, 10, 11] as

$$\left[\frac{1}{c_0^2} \frac{D^2}{Dt^2} - \nabla^2 \right] \{ p'(\mathbf{x}, t) H(f) \} = \frac{D}{Dt} [Q\delta(f)] - \frac{\partial}{\partial x_i} [L_i\delta(f)] + \frac{\partial^2}{\partial x_i \partial x_j} [T_{ij}H(f)], \quad (1)$$

with

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + U_{\infty i} \frac{\partial}{\partial x_i}, \quad (2)$$

$$Q = \rho_0(v_n - U_{\infty n}) + \rho[u_n - (v_n - U_{\infty n})], \quad (3)$$

$$L_i = [(p - p_0)\delta_{ij} - \sigma_{ij}]n_j + \rho u_i[u_n - (v_n - U_{\infty n})], \quad (4)$$

$$T_{ij} = \rho u_i u_j + [(p - p_0) - c_0^2(\rho - \rho_0)]\delta_{ij} - \sigma_{ij}, \quad (5)$$

where $\delta(f)$ is the Dirac delta function and $H(f)$ is the Heaviside function; $f = 0$ denotes the data surface; $U_{\infty n} = U_{\infty i} n_i$ with the local unit outer normal $n_i = \partial f / \partial x_i$ in the direction x_i ($i=1,2,3$); δ_{ij} is the Kronecker delta; ρ_0 , p_0 and c_0 are the density, pressure and sound speed in the undisturbed medium, respectively; p is the local fluid pressure, ρ is the local fluid density, p' is the sound pressure; the local fluid velocity component is denoted by u_i ; the local normal components to the data surface of fluid and the body velocities are u_n and v_n , respectively; T_{ij} is the Lighthill stress tensor; σ_{ij} is the viscous stress tensor. The schematic of the flow field and the data surface according to acoustic analogy theory is shown in Figure 1.

In the right side of Equation (1), the first two terms are the monopole source term, also known as the thickness source, and the dipole source term which is also called the loading source. The last term is the quadrupole source term, which is typically small compared to the other terms when the fluid and moving body's velocities are both small [2]. By neglecting the quadrupole source term, the numerical solution of the convective FW-H equation can be described by the following time domain formulations [11]

$$p'(\mathbf{x}, t, \mathbf{M}_\infty) = p'_T(\mathbf{x}, t, \mathbf{M}_\infty) + p'_L(\mathbf{x}, t, \mathbf{M}_\infty), \quad (6)$$

$$4\pi p'_T(\mathbf{x}, t, \mathbf{M}_\infty) = \int_S \left[\frac{(1 - M_{\infty R}) \dot{Q}}{R^*(1 - M_R)^2} \right]_e dS - \int_S \left[\frac{c_0 M_{\infty R^*} Q}{R^{*2}(1 - M_R)} \right]_e dS + \int_S \left[(1 - M_{\infty R}) Q \frac{R^* \dot{M}_{R^*} + c_0(M_{R^*} - |\mathbf{M}|^2)}{R^{*2}(1 - M_R)^3} \right]_e dS - \int_S \left[(1 - M_{\infty R}) Q \frac{c_0 M_{R^*} M_R + c_0 \gamma^2 (M_{\infty M}^2 - M_{R^*}^2)}{R^{*2}(1 - M_R)^3} \right]_e dS - \int_S \left[\frac{c_0 \gamma^2 (M_{\infty R^*} M_{R^*} - M_{\infty M}) Q}{R^{*2}(1 - M_R)^2} \right]_e dS, \quad (7)$$

$$4\pi p'_L(\mathbf{x}, t, \mathbf{M}_\infty) = \frac{1}{c_0} \int_S \left[\frac{\dot{L}_R}{R^*(1 - M_R)^2} \right]_e dS + \int_S \left[\frac{L_{R^*} - L_M}{R^{*2}(1 - M_R)^2} \right]_e dS + \frac{1}{c_0} \int_S \left[L_R \frac{R^* \dot{M}_R + c_0(M_{R^*} - |\mathbf{M}|^2)}{R^{*2}(1 - M_R)^3} \right]_e dS - \int_S \left[L_R \frac{M_{R^*} M_R + \gamma^2 (M_{\infty M}^2 - M_{R^*}^2)}{R^{*2}(1 - M_R)^3} \right]_e dS - \int_S \left[\frac{L_{R^*} M_R + \gamma^2 (M_{\infty M} M_{\infty L} - L_{R^*} M_{R^*})}{R^{*2}(1 - M_R)^2} \right]_e dS, \quad (8)$$

with the quantities

$$R^* = \frac{1}{\gamma} \sqrt{r^2 + \gamma^2 (\mathbf{M}_\infty \cdot \mathbf{r})^2}, \quad (9)$$

$$\mathbf{R} = \gamma^2 (R^* - \mathbf{M}_\infty \cdot \mathbf{r}), \quad (10)$$

$$\gamma = \sqrt{1/(1 - M_\infty^2)}, \quad (11)$$

where $\mathbf{M}_\infty$ is the flow Mach number vector with $M_{\infty i} = U_{\infty i}/c_0$ and $|\mathbf{M}|$ is the amplitude of $\mathbf{M}$ that is the body Mach number vector with $M_i = v_i/c_0$; r is the distance between the observer and the source position with $r = |\mathbf{r}| = |\mathbf{x} - \mathbf{y}|$. In Equations (7) and (8), the quantities in brackets should be evaluated at the retarded time

$$\tau = t - R/c_0, \quad (12)$$

the other nomenclatures are defined as $M_{\infty R} = M_{\infty i} \tilde{R}_i$, $M_{\infty R^*} = M_{\infty i} \tilde{R}_i^*$, $\tilde{R}_i^* = \partial R^*/\partial x_i$, $\tilde{R}_i = \partial R/\partial x_i$, $M_{\infty M} = M_{\infty i} M_i$, $M_R = M_i \tilde{R}_i$, $M_{R^*} = M_i \tilde{R}_i^*$, $M_{\infty L} = M_{\infty i} L_i$, $L_M = L_i M_i$, $L_{R^*} = L_i \tilde{R}_i^*$, $L_R = L_i \tilde{R}_i$.

It should be noted that the data surface S in Equations (7) and (8) is often given as a stationary surface enclosing the body source. Especially, when the body source is moving, the surface should enclose its whole moving region. This simple case of a stationary data surface is commonly used because such case is indicative of practical and industrial application. In this way $v_i = 0$ and Equations (6)–(8) reduce, respectively, to

$$p'(\mathbf{x}, t, \mathbf{M}_\infty) = p'_T(\mathbf{x}, t, \mathbf{M}_\infty) + p'_L(\mathbf{x}, t, \mathbf{M}_\infty), \quad (13)$$

$$4\pi p'_T(\mathbf{x}, t, \mathbf{M}_\infty) = \int_S \left[\frac{(1 - M_{\infty R}) \dot{Q}}{R^*} \right]_e dS - \int_S \left[\frac{Q c_0 M_{\infty R^*}}{R^{*2}} \right]_e dS, \quad (14)$$

$$4\pi p'_L(\mathbf{x}, t, \mathbf{M}_\infty) = \frac{1}{c_0} \int_S \left[\frac{\dot{L}_R}{R^*} \right]_e dS + \int_S \left[\frac{L_{R^*}}{R^{*2}} \right]_e dS. \quad (15)$$

If the data surface is not stationary but encloses a moving body source and moves with it, acquiring the retarded time τ is rather difficult because the iteration process is usually needed to solve the transcendental equation relating t and τ . On the contrary, if the data surface is stationary, the acoustic radius R in Equations (14) and (15) is constant thus the retarded time τ could be obtained directly with an explicit solution and this process saves computational time and resources to a great extent. In fact, as long as the sound source region is not too large and the integration on the data surface is not so difficult to handle with, this simplified case with no iterative process to solve the transcendental equation is effective and efficient. The proposed TESM based on the ease-to-use Equations (13)–(15) for a stationary data surface will be demonstrated in Section 2.2.

2.2. Derivation of TESM for sound field prediction and separation in a moving medium

According to the acoustic analogy theory, the sound field could be divided into two parts with source region

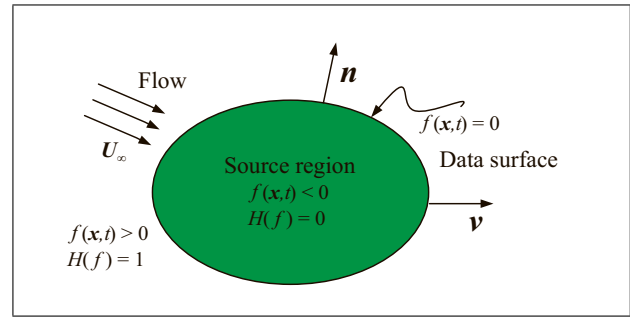


Figure 1. The schematic of the flow field and the data surface according to acoustic analogy theory.

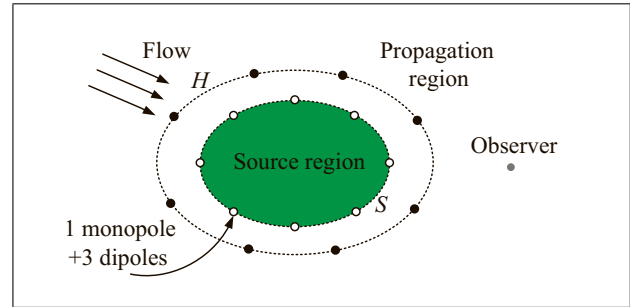


Figure 2. The schematic of the sound field in a moving medium according to acoustic analogy theory and the proposed TESM.

and propagation region. The sound propagation from the source region to the receiver will be described by the time-domain equivalent source model in the following.

Figure 2 shows the sound field schematic in a moving medium with the flow velocity $\mathbf{U}_\infty$. The data surface S encloses the source region and it is inside a collocation points surface H . M collocation points are distributed on H and marked by small solid circles in black. $4N$ equivalent sources are distributed on the surface S and marked by small hollow circles. For simplicity, N small hollow circles are plotted on S and each small hollow circle represents four equivalent sources with one monopole strength Q and three dipole strengths L_i ($i = 1, 2, 3$). The $4N$ equivalent sources are assembled as

$$\left(\underbrace{Q^1, Q^2, \dots, Q^N}_N, \underbrace{L_1^1, L_1^2, \dots, L_1^N}_N, \underbrace{L_2^1, L_2^2, \dots, L_2^N}_N, \underbrace{L_3^1, L_3^2, \dots, L_3^N}_N \right),$$

where the superscripts $1, 2, \dots, N$ indicate the locations of equivalent sources and the subscripts $1, 2, 3$ indicate three types of dipoles.

The first step to derive the proposed TESM is to discretize Equations (14) and (15) as

$$4\pi p'_{T_{Hmn}}(t) = \sum_{n=1}^N \left[\frac{(1 - M_{\infty R_{Hmn}})}{R_{Hmn}^*} \right]_e \dot{Q}^n(\tau) - \sum_{n=1}^N \left[\frac{c_0 M_{\infty R_{Hmn}^*}}{R_{Hmn}^{*2}} \right]_e Q^n(\tau), \quad (16)$$

$$4\pi p'_{L_{Hm}}(t) = \frac{1}{c_0} \sum_{n=1}^N \left[\frac{1}{R_{Hmn}^*} \right]_e \dot{L}_{R_{Hmn}}^n(\tau) + \sum_{n=1}^N \left[\frac{1}{R_{Hmn}^{*2}} \right]_e L_{R_{Hmn}}^n(\tau). \quad (17)$$

Here n denotes the location of the equivalent source. In Equation (17), as both L_R and L_{R^*} contain three components L_i ($i = 1, 2, 3$) for three types of dipoles whose axis is aligned with the x_i -axis ($i = 1, 2, 3$), Equation (17) could be further divided into three parts as

$$4\pi p'_{L_{1Hm}}(t) = \frac{1}{c_0} \sum_{n=1}^N \left[\frac{\tilde{R}_{1Hmn}}{R_{Hmn}^*} \right]_e \dot{L}_1^n(\tau) + \sum_{n=1}^N \left[\frac{\tilde{R}_{1Hmn}^*}{R_{Hmn}^{*2}} \right]_e L_1^n(\tau), \quad (18)$$

$$4\pi p'_{L_{2Hm}}(t) = \frac{1}{c_0} \sum_{n=1}^N \left[\frac{\tilde{R}_{2Hmn}}{R_{Hmn}^*} \right]_e \dot{L}_2^n(\tau) + \sum_{n=1}^N \left[\frac{\tilde{R}_{2Hmn}^*}{R_{Hmn}^{*2}} \right]_e L_2^n(\tau), \quad (19)$$

$$4\pi p'_{L_{3Hm}}(t) = \frac{1}{c_0} \sum_{n=1}^N \left[\frac{\tilde{R}_{3Hmn}}{R_{Hmn}^*} \right]_e \dot{L}_3^n(\tau) + \sum_{n=1}^N \left[\frac{\tilde{R}_{3Hmn}^*}{R_{Hmn}^{*2}} \right]_e L_3^n(\tau). \quad (20)$$

All quantities in the brackets are independent of time when the simplified case ($v_i = 0$) is considered. Therefore the subscript e referring to the retarded time would be omitted in the following formulations.

In Equations (16)–(20), $p'_{T_{Hm}}(t)$ and $p'_{L_{Hm}}(t)$ are the monopole and dipole components of the total pressure at the m th collocation point at time t , respectively, R_{Hmn}^* and R_{Hmn} are the acoustic radii between the m th collocation point and the n th equivalent source, which are evaluated by Equations (9) and (10), respectively, and $\tilde{R}_{iHmn}^*$, $\tilde{R}_{iHmn}$, $M_{\infty R_{Hmn}}$ and $M_{\infty R_{Hmn}^*}$ could all be deduced by the definitions in Section 2.1. The retarded time τ represents the emission time of equivalent sources and the two time coordinates t and τ are discretized as

$$t^j = t^0 + j\Delta t \quad (j = 1, 2, \dots, J), \quad (21)$$

$$\tau^j = \tau^0 + j\Delta \tau \quad (j = 1, 2, \dots, J), \quad (22)$$

where Δt and $\Delta \tau$ are the time steps in their corresponding coordinates and J is the total number of time steps. The equivalent source strength is equal to zero when $\tau \leq \tau^0$. In order to establish the relationship between these two time coordinates, some assumptions are given by

$$\tau^0 = t^0 - \min(R_{Hmn})/c_0 \quad (23)$$

$$(m = 1, 2, \dots, M; n = 1, 2, \dots, N),$$

$$\Delta t = \Delta \tau. \quad (24)$$

At the j th time step t^j , Equation (16) and Equations (18)–(20) can be expressed as

$$4\pi p'_{T_{Hm}}(t^j) = \sum_{n=1}^N \left[\frac{(1 - M_{\infty R_{Hmn}})}{R_{Hmn}^*} \dot{Q}^n(\tau_{Hmn}^j) - \frac{c_0 M_{\infty R_{Hmn}^*}}{R_{Hmn}^{*2}} Q^n(\tau_{Hmn}^j) \right], \quad (25)$$

$$4\pi p'_{L_{1Hm}}(t^j) = \sum_{n=1}^N \left[\frac{1}{c_0} \frac{\tilde{R}_{1Hmn}}{R_{Hmn}^*} \dot{L}_1^n(\tau_{Hmn}^j) + \frac{\tilde{R}_{1Hmn}^*}{R_{Hmn}^{*2}} L_1^n(\tau_{Hmn}^j) \right], \quad (26)$$

$$4\pi p'_{L_{2Hm}}(t^j) = \sum_{n=1}^N \left[\frac{1}{c_0} \frac{\tilde{R}_{2Hmn}}{R_{Hmn}^*} \dot{L}_2^n(\tau_{Hmn}^j) + \frac{\tilde{R}_{2Hmn}^*}{R_{Hmn}^{*2}} L_2^n(\tau_{Hmn}^j) \right], \quad (27)$$

$$4\pi p'_{L_{3Hm}}(t^j) = \sum_{n=1}^N \left[\frac{1}{c_0} \frac{\tilde{R}_{3Hmn}}{R_{Hmn}^*} \dot{L}_3^n(\tau_{Hmn}^j) + \frac{\tilde{R}_{3Hmn}^*}{R_{Hmn}^{*2}} L_3^n(\tau_{Hmn}^j) \right]. \quad (28)$$

As the retarded time τ_{Hmn}^j will not generally be an integer multiple of the retarded time step $\Delta \tau$, the equivalent source strengths and their derivatives at τ_{Hmn}^j could be obtained by the following time domain interpolation,

$$Q^n(\tau_{Hmn}^j) = \sum_{k=1}^j \alpha^k(\tau_{Hmn}^j) Q^n(\tau^k), \quad (29)$$

$$L_i^n(\tau_{Hmn}^j) = \sum_{k=1}^j \alpha^k(\tau_{Hmn}^j) L_i^n(\tau^k), \quad i = 1, 2, 3, \quad (30)$$

$$\dot{Q}^n(\tau_{Hmn}^j) = \sum_{k=1}^j \frac{\partial \alpha^k(\tau_{Hmn}^j)}{\partial \tau} Q^n(\tau^k), \quad (31)$$

$$\dot{L}_i^n(\tau_{Hmn}^j) = \sum_{k=1}^j \frac{\partial \alpha^k(\tau_{Hmn}^j)}{\partial \tau} L_i^n(\tau^k), \quad i = 1, 2, 3, \quad (32)$$

where $Q^n(\tau^k)$ and $L_i^n(\tau^k)$ represent the strengths of the n th equivalent monopole and x_i -axis dipole sources at the k th retarded time step, respectively, and $\alpha^k(\tau)$ is the Lagrange linear interpolation function, given by [18]

$$\alpha^k(\tau) = \begin{cases} (\tau - \tau^{k-1})/\Delta \tau, & \tau^{k-1} \leq \tau \leq \tau^k, \\ (\tau^{k+1} - \tau)/\Delta \tau, & \tau^k \leq \tau \leq \tau^{k+1}, \\ 0 & \text{otherwise,} \end{cases} \quad (33)$$

and its derivative is

$$\frac{\partial \alpha^k(\tau)}{\partial \tau} = \begin{cases} 1/\Delta \tau, & \tau^{k-1} \leq \tau \leq \tau^k, \\ -1/\Delta \tau, & \tau^k \leq \tau \leq \tau^{k+1}, \\ 0 & \text{otherwise.} \end{cases} \quad (34)$$

By substituting Equations (29)–(32) into Equations (25)–(28), the equivalent source strengths at τ_{Hmn}^j can be replaced by the equivalent source strengths at the integer

multiple of retarded time steps, $Q^n(\tau^k)$ and $L_i^n(\tau^k)$. It yields

$$p'_{T_{Hm}}(t^j) = \sum_{n=1}^N \sum_{k=1}^j \left[\frac{(1 - M_{\infty R_{Hmn}})}{4\pi R_{Hmn}^*} \frac{\partial \alpha^k(\tau_{Hmn}^j)}{\partial \tau} - \frac{c_0 M_{\infty R_{Hmn}}}{4\pi R_{Hmn}^{*2}} \alpha^k(\tau_{Hmn}^j) \right] Q^n(\tau^k), \quad (35)$$

$$p'_{L_{1Hm}}(t^j) = \sum_{n=1}^N \sum_{k=1}^j \left[\frac{1}{c_0} \frac{\tilde{R}_{1Hmn}}{4\pi R_{Hmn}^*} \frac{\partial \alpha^k(\tau_{Hmn}^j)}{\partial \tau} + \frac{\tilde{R}_{1Hmn}^*}{4\pi R_{Hmn}^{*2}} \alpha^k(\tau_{Hmn}^j) \right] L_1^n(\tau^k), \quad (36)$$

$$p'_{L_{2Hm}}(t^j) = \sum_{n=1}^N \sum_{k=1}^j \left[\frac{1}{c_0} \frac{\tilde{R}_{2Hmn}}{4\pi R_{Hmn}^*} \frac{\partial \alpha^k(\tau_{Hmn}^j)}{\partial \tau} + \frac{\tilde{R}_{2Hmn}^*}{4\pi R_{Hmn}^{*2}} \alpha^k(\tau_{Hmn}^j) \right] L_2^n(\tau^k), \quad (37)$$

$$p'_{L_{3Hm}}(t^j) = \sum_{n=1}^N \sum_{k=1}^j \left[\frac{1}{c_0} \frac{\tilde{R}_{3Hmn}}{4\pi R_{Hmn}^*} \frac{\partial \alpha^k(\tau_{Hmn}^j)}{\partial \tau} + \frac{\tilde{R}_{3Hmn}^*}{4\pi R_{Hmn}^{*2}} \alpha^k(\tau_{Hmn}^j) \right] L_3^n(\tau^k). \quad (38)$$

Let A_{Hmn}^{kj} , B_{Hmn}^{kj} , C_{Hmn}^{kj} , D_{Hmn}^{kj} represent the terms in square brackets of Equations (35)–(38), respectively. And then Equations (35)–(38) could be further rewritten as

$$p'_{T_{Hm}}(t^j) = [A_{Hm1}^{1j} A_{Hm2}^{1j} \dots A_{HmN}^{1j}] \cdot [Q^1(\tau^1) Q^2(\tau^1) \dots Q^N(\tau^1)]^T + \dots + [A_{Hm1}^{jj} A_{Hm2}^{jj} \dots A_{HmN}^{jj}] \cdot [Q^1(\tau^j) Q^2(\tau^j) \dots Q^N(\tau^j)]^T = A_{Hm}^{1j} Q^T(\tau^1) + \dots + A_{Hm}^{jj} Q^T(\tau^j), \quad (39)$$

$$p'_{L_{1Hm}}(t^j) = [B_{Hm1}^{1j} B_{Hm2}^{1j} \dots B_{HmN}^{1j}] \cdot [L_1^1(\tau^1) L_1^2(\tau^1) \dots L_1^N(\tau^1)]^T + \dots + [B_{Hm1}^{jj} B_{Hm2}^{jj} \dots B_{HmN}^{jj}] \cdot [L_1^1(\tau^j) L_1^2(\tau^j) \dots L_1^N(\tau^j)]^T = B_{Hm}^{1j} L_1^T(\tau^1) + \dots + B_{Hm}^{jj} L_1^T(\tau^j), \quad (40)$$

$$p'_{L_{2Hm}}(t^j) = [C_{Hm1}^{1j} C_{Hm2}^{1j} \dots C_{HmN}^{1j}] \cdot [L_2^1(\tau^1) L_2^2(\tau^1) \dots L_2^N(\tau^1)]^T + \dots + [C_{Hm1}^{jj} C_{Hm2}^{jj} \dots C_{HmN}^{jj}] \cdot [L_2^1(\tau^j) L_2^2(\tau^j) \dots L_2^N(\tau^j)]^T = C_{Hm}^{1j} L_2^T(\tau^1) + \dots + C_{Hm}^{jj} L_2^T(\tau^j), \quad (41)$$

$$p'_{L_{3Hm}}(t^j) = [D_{Hm1}^{1j} D_{Hm2}^{1j} \dots D_{HmN}^{1j}] \cdot [L_3^1(\tau^1) L_3^2(\tau^1) \dots L_3^N(\tau^1)]^T + \dots + [D_{Hm1}^{jj} D_{Hm2}^{jj} \dots D_{HmN}^{jj}] \cdot [L_3^1(\tau^j) L_3^2(\tau^j) \dots L_3^N(\tau^j)]^T = D_{Hm}^{1j} L_3^T(\tau^1) + \dots + D_{Hm}^{jj} L_3^T(\tau^j). \quad (42)$$

The total pressure $p_{Hm}(t^j)$ at the m th collocation point at time t^j could be obtained by the summation of Equations (39)–(42) in a matrix form as

$$p_{Hm}(t^j) = p'_{T_{Hm}}(t^j) + p'_{L_{1Hm}}(t^j) + p'_{L_{2Hm}}(t^j) + p'_{L_{3Hm}}(t^j) = [A_{Hm}^{1j} B_{Hm}^{1j} C_{Hm}^{1j} D_{Hm}^{1j}] \cdot [Q(\tau^1) L_1(\tau^1) L_2(\tau^1) L_3(\tau^1)]^T + \dots + [A_{Hm}^{jj} B_{Hm}^{jj} C_{Hm}^{jj} D_{Hm}^{jj}] \cdot [Q(\tau^j) L_1(\tau^j) L_2(\tau^j) L_3(\tau^j)]^T. \quad (43)$$

By applying Equation (43) to each collocation point, it yields the following matrix equation:

$$\mathbf{P}_H^j = \boldsymbol{\Psi}_H^{1j} \boldsymbol{\varphi}^1 + \boldsymbol{\Psi}_H^{2j} \boldsymbol{\varphi}^2 + \dots + \boldsymbol{\Psi}_H^{kj} \boldsymbol{\varphi}^k + \dots + \boldsymbol{\Psi}_H^{jj} \boldsymbol{\varphi}^j, \quad (k = 1, 2, \dots, j), \quad (44)$$

where the column vector $\mathbf{P}_H^j$ is composed of pressures at all M collocation points at the j th time step t^j , i.e.,

$$\mathbf{P}_H^j = [p_{H1}^j p_{H2}^j \dots p_{HM}^j]^T, \quad (45)$$

and the column vector $\boldsymbol{\varphi}^k$ consists of equivalent source strengths at the k th retarded time step τ^k , i.e.,

$$\boldsymbol{\varphi}^k = [Q(\tau^k) L_1(\tau^k) L_2(\tau^k) L_3(\tau^k)]^T, \quad (46)$$

and the transfer matrix $\boldsymbol{\Psi}_H^{kj}$ relating the equivalent source strength at the k th retarded time step τ^k to the pressures of the collocation points at the j th time step t^j is

$$\boldsymbol{\Psi}_H^{kj} = \begin{bmatrix} A_{H1}^{kj} & B_{H1}^{kj} & C_{H1}^{kj} & D_{H1}^{kj} \\ A_{H2}^{kj} & B_{H2}^{kj} & C_{H2}^{kj} & D_{H2}^{kj} \\ \vdots & \vdots & \vdots & \vdots \\ A_{HM}^{kj} & B_{HM}^{kj} & C_{HM}^{kj} & D_{HM}^{kj} \end{bmatrix}. \quad (47)$$

Based on Equation (44), an iterative process of solving the equivalent source strengths at an integer multiple of the retarded time step $\Delta\tau$ can be derived in the following. Given that $i = 1$ in Equation (44), the vector $\boldsymbol{\varphi}^1$ can be solved by

$$\boldsymbol{\varphi}^1 = (\boldsymbol{\Psi}_H^{11})^+ \mathbf{P}_H^1, \quad (48)$$

where the superscript “+” indicates the pseudo inverse of a matrix. Next given that $i = 2$, the vector $\boldsymbol{\varphi}^2$ can be obtained by using the vector $\boldsymbol{\varphi}^1$ solved at the last time step as

$$\boldsymbol{\varphi}^2 = (\boldsymbol{\Psi}_H^{22})^+ (\mathbf{P}_H^2 - \boldsymbol{\Psi}_H^{12} \boldsymbol{\varphi}^1). \quad (49)$$

And so on, the rest column vector $\boldsymbol{\varphi}^k$ ($k = 3, 4, \dots, j$) is solved as

$$\boldsymbol{\varphi}^k = (\boldsymbol{\psi}_H^{kk})^+ (\mathbf{P}_H^k - \boldsymbol{\psi}_H^{1k} \boldsymbol{\varphi}^1 - \boldsymbol{\psi}_H^{2k} \boldsymbol{\varphi}^2 - \dots - \boldsymbol{\psi}_H^{(k-1)k} \boldsymbol{\varphi}^{k-1}). \quad (50)$$

In the above iterative solving process, it should be noted that the matrix $\boldsymbol{\psi}_H^{kk}$ is the same at each time step, because all the parameters in $\boldsymbol{\psi}_H^{kk}$ are constant for $k = 1, 2, \dots, j$. Therefore, once the pseudo inverse of $\boldsymbol{\psi}_H^{11}$ is determined, it can be used at each next time step. When the number of collocation points is larger than the number of equivalent sources, each iterative equation above is the over-determined equation which has the unique least-squares solution [28], thus the strengths of the equivalent sources are uniquely determined if their number and locations are set at the beginning.

As is commonly the case in a time domain method, the inverse process aforementioned is often found to be unstable, which means the solution may diverge in the iterative solving process. Lots of methods have been proposed to improve the instability problem such as time averaging [19, 29, 30], space averaging [31] and digital filtering [32]. In this paper the truncated singular value decomposition (TSVD) [18] is adopted to regularize the ill-conditioned matrix by setting very small singular values (below the cut-off singular value) to zero. A parameter $\sigma_p = w_c/w_{\max}$ is introduced to determine the cut-off singular value w_c , and $w_{\max}$ is the maximum singular value.

Once all equivalent source strengths at every time step are determined, the total pressure at the observer at time t^j can be calculated by

$$\begin{aligned} p_O(t^j) = & [\mathbf{A}_O^{1j} \mathbf{B}_O^{1j} \mathbf{C}_O^{1j} \mathbf{D}_O^{1j}] \\ & \cdot [\mathbf{Q}(\tau^1) \mathbf{L}_1(\tau^1) \mathbf{L}_2(\tau^1) \mathbf{L}_3(\tau^1)]^T \\ & + \dots \\ & + [\mathbf{A}_O^{jj} \mathbf{B}_O^{jj} \mathbf{C}_O^{jj} \mathbf{D}_O^{jj}] \\ & \cdot [\mathbf{Q}(\tau^j) \mathbf{L}_1(\tau^j) \mathbf{L}_2(\tau^j) \mathbf{L}_3(\tau^j)]^T, \end{aligned} \quad (51)$$

where the expressions of $\mathbf{A}_O^{kj}$, $\mathbf{B}_O^{kj}$, $\mathbf{C}_O^{kj}$, $\mathbf{D}_O^{kj}$ are similar to those of $\mathbf{A}_{Hm}^{kj}$, $\mathbf{B}_{Hm}^{kj}$, $\mathbf{C}_{Hm}^{kj}$, $\mathbf{D}_{Hm}^{kj}$ in Equations (39)–(42) but they relate the observer to the equivalent sources. By this process, the obtained equivalent source strengths could be used to predict the sound pressure at the observer.

In order to calculate the monopole and dipole components of the total pressure at the observer, the corresponding equivalent source strengths and transfer functions of monopole and dipole are used. Then these two components could be deduced as

$$p_{OT}(t^j) = [\mathbf{A}_O^{1j} [\mathbf{Q}(\tau^1)]^T + \dots + [\mathbf{A}_O^{jj} [\mathbf{Q}(\tau^j)]^T], \quad (52)$$

$$\begin{aligned} p_{OL}(t^j) = & [\mathbf{B}_O^{1j} \mathbf{C}_O^{1j} \mathbf{D}_O^{1j}] [\mathbf{L}_1(\tau^1) \mathbf{L}_2(\tau^1) \mathbf{L}_3(\tau^1)]^T \\ & + \dots \\ & + [\mathbf{B}_O^{jj} \mathbf{C}_O^{jj} \mathbf{D}_O^{jj}] [\mathbf{L}_1(\tau^j) \mathbf{L}_2(\tau^j) \mathbf{L}_3(\tau^j)]^T. \end{aligned} \quad (53)$$

Based on Equations (51)–(53), the total pressure at the observer as well as its monopole and dipole components

could be predicted and calculated at the same time. In consideration of the easy process to obtain the two types of pressure, it is more convenient to study the magnitude and directive properties of each component.

3. Numerical simulations

In this section, numerical simulations with three test cases are carried out to investigate the proposed TESM in a moving medium. The first two test cases are stationary monopole and dipole sources located in a moving medium, while the third test case consists of a rotating monopole for validating the corresponding moving source case. The total pressure at the observer predicted by the proposed TESM are compared with its analytic solution, and the separated results of monopole and dipole components are also shown against the results calculated directly by time domain formulations of the convective FW-H equation with the analytical $\mathbf{Q}$, $\dot{\mathbf{Q}}$, $\mathbf{L}$ and $\dot{\mathbf{L}}$. It should be noted that in the third moving source case a moving data surface is used in the time domain formulations of the convective FW-H equation while the proposed TESM still uses the stationary data surface enclosing the moving region of rotating monopole. In order to consider the disturbing noise in the flowing media, a Gaussian white noise with a signal-to-noise ratio of 30 dB is added in all three test cases.

3.1. Test case 1: Monopole source in a moving medium

The first test case is to consider a single-frequency monopole source located at the origin of a Cartesian coordinate system in a uniform flow with an arbitrary orientation.

The velocity potential for the monopole contains the uniform flow with an arbitrary direction is defined as [33]

$$\varphi(\mathbf{x}, t) = \frac{A}{4\pi R^*} \exp[i\omega(t - R/c_0)], \quad (54)$$

where the acoustic radii R^* and R are calculated from Equations (9) and (10), respectively. The acoustic particle velocity can be obtained from the gradient of the velocity potential

$$\mathbf{u}(\mathbf{x}, t) = \nabla \varphi(\mathbf{x}, t), \quad (55)$$

where the spatial derivatives of R^* and R could be calculated using definitions in Section 2.1. The induced sound pressure and density in a moving medium with uniform flow in an arbitrary direction is described through the unsteady Bernoulli equation

$$p'(\mathbf{x}, t) = -\rho_0 \frac{\partial \varphi(\mathbf{x}, t)}{\partial t} + U_{\infty i} \frac{\partial \varphi(\mathbf{x}, t)}{\partial x_i} \quad (56)$$

and

$$p'(\mathbf{x}, t) = p'(\mathbf{x}, t)/c_0^2. \quad (57)$$

The velocity potential amplitude of the monopole is $A = 1 \text{ m}^2/\text{s}$. The angular frequency is $\omega = 10\pi \text{ rad/s}$. The ambient speed of sound c_0 is chosen as 340.75 m/s. The free

stream flow density ρ_0 is assumed to be 1.234 kg/m^3 . The data surface S and collocation points surface H are the stationary spherical surfaces with radii of 0.1 m and 0.2 m , respectively. In the direct calculation by time domain formulations of the convective FW-H equation, the data surface is meshed finely with 15292 elements uniformly distributed for ensuring enough spatial resolution, while in the TESM, it is discretized into 5 and 4 grids along the azimuthal angle θ and the polar angle φ , and the equivalent sources are located at these grid points. The positions of collocation points (marked as small solid circles in black in Figure 2) on the surface H are described with 11 and 11 grids along with θ and φ angle. The parameter σ_p is 0.001. There are 50 time points used per source period ($T = 2\pi/\omega$) to ensure enough temporal resolution.

Figure 3 shows the total pressure time history at observer $\mathbf{x} = (0.1, 0.5, 0) \text{ m}$ with the flow parameter $\mathbf{M}_\infty = (0.7, 0.1, 0.5)$. It can be seen that the predicted results by the proposed TESM are always in good agreement with the analytic solutions except at the first few points. With the assumption in Section 2.2 that the equivalent source strengths are set to be zero especially when $\tau \leq \tau^0$, the equivalent source strengths contributing to pressures at the first few time steps could be zero. Imagine the case that at the first few time steps, the observer time t is very small while if R is relatively large, the retarded time $\tau = t - R/c_0$ could easily be less than τ^0 . So it is reasonable and acceptable that the amplitude underestimated at the first few time points.

Figure 4 shows the separated results of monopole and dipole components using the proposed TESM and those calculated directly by time domain formulations of the convective FW-H equation at observer $\mathbf{x} = (0.1, 0.5, 0) \text{ m}$ with the flow parameter $\mathbf{M}_\infty = (0.7, 0.1, 0.5)$. The excellent agreement demonstrates that the proposed TESM is valid to separate the monopole and dipole components from the total pressure in moving medium.

3.2. Test case 2: Dipole source in a moving medium

The second test case is a dipole source in a moving medium. The dipole is located at the origin of coordinates and its axis is aligned with the x_2 -axis. The velocity potential of the dipole could be obtained by

$$\varphi(\mathbf{x}, t) = \frac{\partial}{\partial x_2} \left\{ \frac{A}{4\pi R^*} \exp[i\omega(t - R/c_0)] \right\}. \quad (58)$$

Then, the procedure for obtaining the particle velocity, sound pressure and density is similar to that in the monopole source case. The parameters A , ω , c_0 and ρ_0 are set the same values as those in the monopole case. The data surface S and collocation points surface H are the stationary spherical surfaces with radii of 0.4 m and 0.5 m , respectively. There are 15292 elements uniformly distributed on the data surface used in the time domain formulations of the convective FW-H equation. In the proposed TESM, the data surface is discretized into 5 and 4 grids along the azimuthal angle θ and the polar angle φ , and the equivalent sources are located at these grid points.

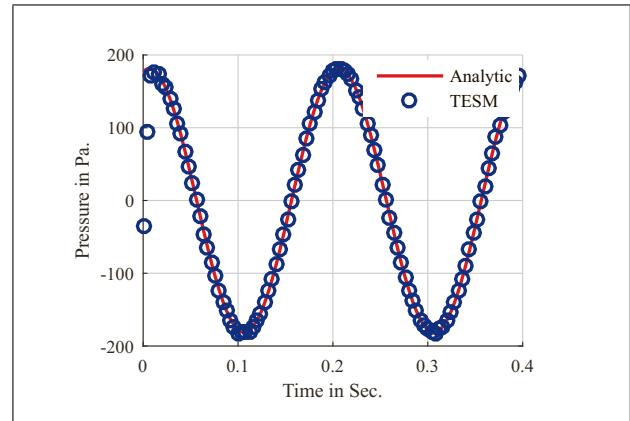


Figure 3. Comparison of the calculated total pressure time history by the proposed TESM and its analytic solution in moving medium.

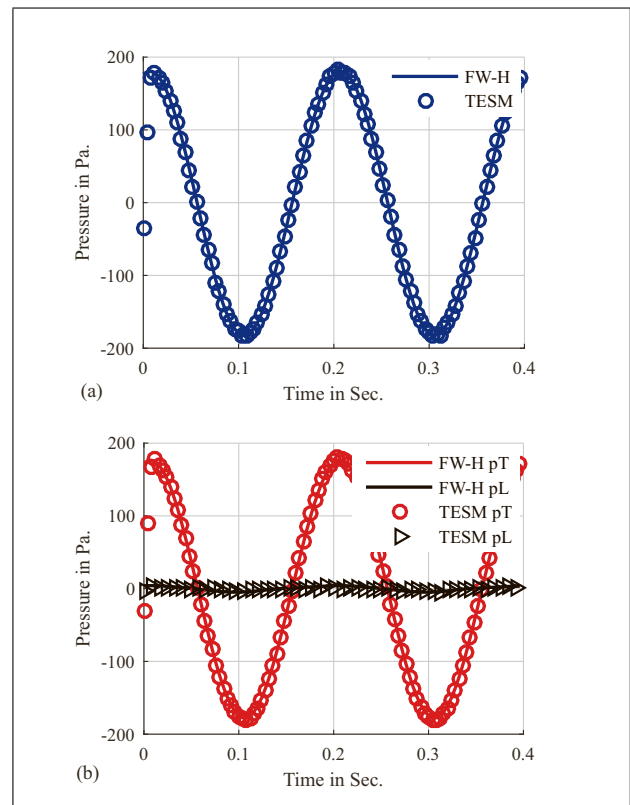


Figure 4. Comparison of the separation results between the proposed TESM and time domain formulations of the convective FW-H equation for monopole source in moving medium: (a) the total noise time history at the observer. (b) the monopole and dipole components time histories at the observer.

The positions of collocation points on the surface H are described with θ and φ discretized into 15 and 15 grids. The truncated parameter σ_p and the time sampling points in one source period are the same as those in the first test case.

Figure 5 depicts the comparison of the total pressure time history calculated from the proposed TESM and the analytic method. The observer is at $\mathbf{x} = (0.3, 0.7, 0.1) \text{ m}$

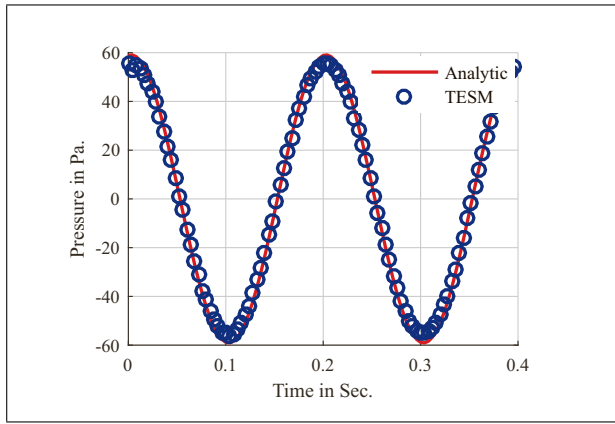


Figure 5. Comparison of the calculated total pressure time history by the proposed method and its analytic solution in moving medium.

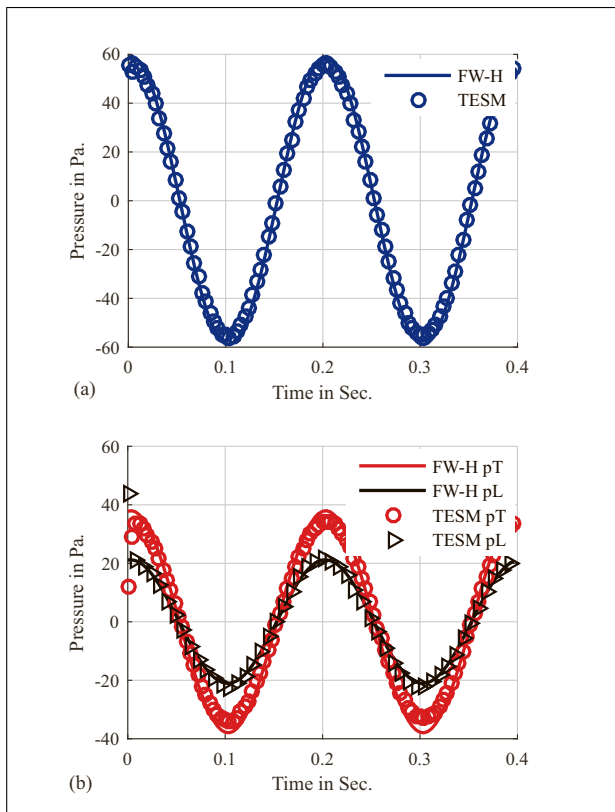


Figure 6. Comparison of the separation results between the proposed TESM and time domain formulations of the convective FW-H equation for dipole source in moving medium: (a) the total noise time history at the observer. (b) the monopole and dipole components time histories at the observer.

and $\mathbf{M}_\infty = (0.3, 0.1, 0.4)$. The predicted results agree very well with the analytic solutions.

Figure 6 shows the separated results for dipole source and the agreement for each component between the proposed TESM and time domain formulations of the convective FW-H equation proves the validity of the proposed method.

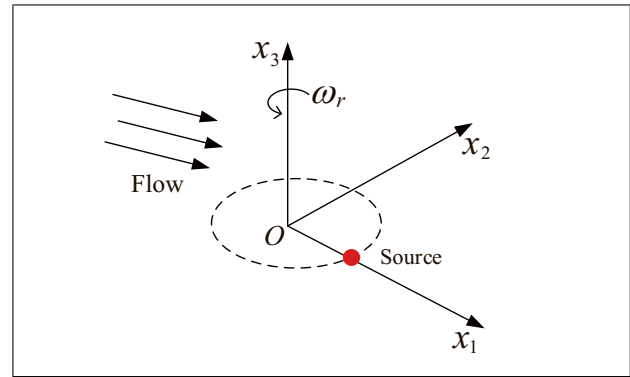


Figure 7. The schematic of a rotating monopole in a moving medium.

3.3. Test case 3: Rotating monopole source in a moving medium

In order to show the feasibility of the proposed TESM for moving sources in a moving medium, a rotating monopole case is considered as shown in Figure 7. The corresponding parameters A , ω , c_0 and ρ_0 are set the same values as those in the stationary monopole case. The monopole rotates counter-clockwise around the x_3 -axis with an angular speed of $\omega_r = 2\pi$ rad/s at radius $r_s = 1$ m on the x_1Ox_2 plane and the initial position is $\mathbf{x}_s = (1, 0, 0)$ m when $\tau = 0$. The ambient medium has a Mach number of $\mathbf{M}_\infty = (0.2, 0.4, 0.3)$. In this case, a spherical data surface moving along with the source is used for the direct calculation by time domain formulations of the convective FW-H equation. Its center coincides with the monopole and the radius is 0.5 m. This surface has a fine mesh with 15292 elements uniformly distributed. In the proposed TESM, a stationary spherical data surface with the radius of 1.6 m is used and it is discretized into 7 and 7 grids along the azimuthal angle θ and the polar angle φ . The collocation points surface H are discretized into 14 and 15 grids along the azimuthal angle θ and the polar angle φ with radius 2 m. The truncated parameter σ_p is 0.01 and the time sampling points in one rotating period is 100.

Figure 8 shows the total pressure time history calculated from the proposed TESM and the analytic method, respectively. The observer is at $\mathbf{x} = (1.5, 1.4, 1.3)$ m. The predicted results agree very well with the analytic solutions.

Figure 9 shows the separated results for rotating monopole source and the agreement for each component between the proposed TESM and time domain formulations of the convective FW-H equation proves the validity of the proposed formulations for moving sources.

4. Conclusions

In this paper, the TESM based on time domain formulations of the convective FW-H equation is presented for predicting the total pressure in a moving medium and separating monopole and dipole components from the total pressure. In this method, the source terms on a data surface could be represented by a series of equivalent sources

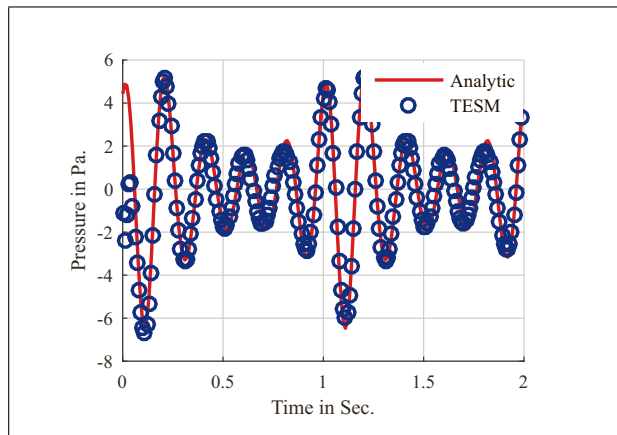


Figure 8. Comparison of the calculated total pressure time history of rotating monopole by the proposed method and its analytic solution in moving medium.

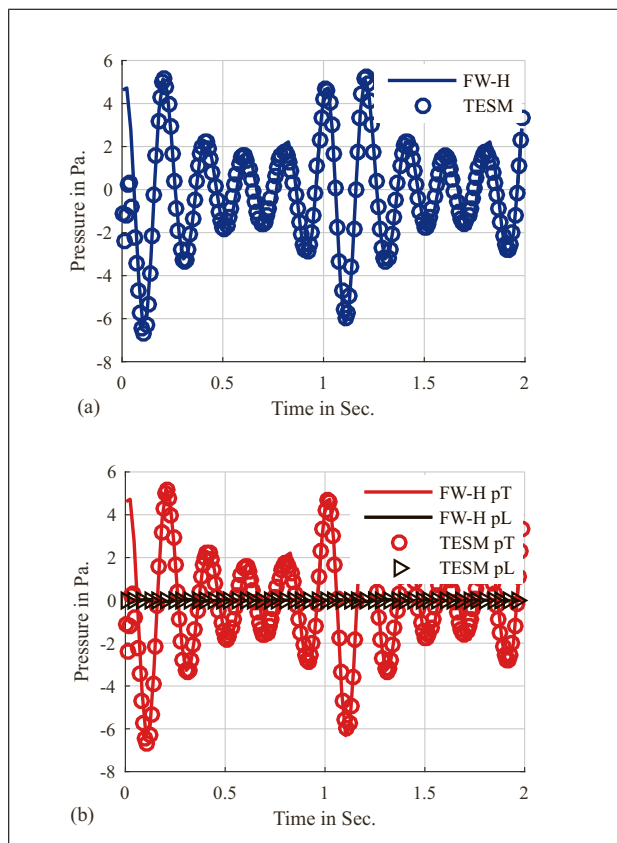


Figure 9. Comparison of the separation results between the proposed TESM and time domain formulations of the convective FW-H equation for rotating monopole source in moving medium: (a) the total noise time history at the observer. (b) the monopole and dipole components time histories at the observer.

whose strengths are determined by the pressure data of collocation points outside the source region, which provides an alternative method of obtaining source terms for sound field prediction. Besides, as there are two types of equivalent source strengths to represent the monopole and dipole sources, the obtained source strengths can not only be used to calculate the total pressure at the observer but

also be used to separate the two types of noises outside the equivalent source surface.

A verification study is realized through three numerical test cases consisting of a stationary monopole source, a stationary dipole source as well as a rotating monopole source. The results obtained by the proposed method are in good agreement with the analytic solutions and the results calculated by time domain formulations of the convective FW-H equation, which proved the effectiveness of the proposed method.

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