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# Physical Modelling of Trombone Mutes, the Pedal Note Issue

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## Summary

Brass players use a variety of mutes to change the sound of their instrument for artistic expression. However, mutes can also modify the intonation and the playability of the muted instrument. An example is the use of a straight mute on a trombone, which makes it very difficult to play stable pedal notes. Previous studies have shown that using a straight mute establishes a subsidiary acoustic resonance in the trombone. To cancel this modification, an active control device was developed and integrated into a mute, with satisfying experimental results [1]. With this device, the perturbed pedal notes can easily be played again. This paper investigates the ability of a physical model of brass instrument to reproduce the behaviour of the trombone pedal  $B\flat$  without mute, or with an “active” or a “passive” straight mute. Linear stability analysis and time-domain simulations are used to analyse the behaviour of the model in the parameter range corresponding to the pedal note. Numerical results are compared for different models of instruments: a trombone, a trombone with a straight (passive) mute, an a trombone with an active mute. It is shown that the simple physical model considered behaves rather qualitatively similarly to what is experienced with real instruments: the playing of the pedal note is perturbed with a passive mute, whereas the model of trombone with the experimental active mute gives results very similar to those obtained with an open trombone.

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## 1. Introduction

A usual solution for changing the timbre of a brass instrument consists in using a mute, which is a device plugged to the opening of the instrument bell, or held by hand just in front of the bell. The shape and the material of the mute affect the pressure radiation of the bell, therefore modifying the emitted sound [2, p.398]. As a side effect, introducing an obstacle in the bell, or close to it, also modifies the acoustical properties of the instrument [3]. This has various consequences, including modification of the instrument tuning and of reported instrument-player interaction.

Pedal notes are the lowest playable notes on a trombone. When the slide is fully closed, the note played is a  $B\flat$ , corresponding to a playing frequency of 58 Hz in equal temperament. In brass instruments, most regimes of oscillation of the instrument have a playing frequency slightly above the acoustical resonance supporting the oscillation. However, the pedal note is an unusual case. Its oscillation frequency is harmonic with the other notes, although it is considerably above the resonance frequency of the first

mode, which supports this oscillation as shown [4, 5]. The inharmonicity of this first mode can be observed in Figure 2. When a straight mute is inserted in the trombone, playing stable pedal notes on the three first slides positions –  $B\flat$ ,  $A$  and  $A\flat$  – is uneasy and results in a rolling, unstable sound [6, 1]. Measurements of the input impedance of a trombone with a mute [1, 7] show the occurrence of a subsidiary acoustical mode between the first and the second modes.

This paper will particularly focus on the pedal  $B\flat$ , corresponding to the slide in the shortest position. It will hereinafter be referred to as “the pedal note”. An active control device has been previously developed to remove this subsidiary mode [1], which makes it possible to play the pedal note with a straight mute. It consists of an “active mute”, a commercial straight mute equipped with an active control device which cancels the aforesaid subsidiary resonance mode.

The purpose of this paper is to investigate to what extent a simple trombone physical model can predict the effect of a trombone straight mute on the pedal note and the effectiveness of the active mute. The physical model of a brass instrument is first presented. Then, linear stability analysis (LSA) and time-domain simulations are used to analyse the behaviour of the pedal note in this model. Anal-

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yses are conducted on an “open trombone” configuration (Courtois 155B tenor trombone without any mute), a “passive mute” configuration (the same trombone with a Denis Wick straight mute) and an “active mute” configuration (the same trombone with an identical straight mute and the described active control loop). Results of this model are compared with the experimental results from [1].

## 2. Tools

### 2.1. Brass instrument model

A physical model of trombone, suitable for a large class of music instruments, is presented in this article. Following Helmholtz pioneering work [8], the trombone is modelled as a closed-loop system consisting of an exciter and a resonator which are coupled, as illustrated in Figure 1. Such a system can produce auto-oscillation on different regimes.

For a brass instrument, the exciter is the lips of the musician, which act as a valve: the section of the channel between the lips depends on the pressure difference through these lips as well as on their mechanical characteristics. Multiple models of the lip reed have been proposed and used, with one degree of freedom [9, 10, 11, 12] or 2 DOF [13, 14, 15, 16]. The model used for this paper is the one-DOF valve model, usually referred to as the “outward-striking” model, also called (+, −) swinging-door model in the literature,

$$\frac{d^2h}{dt^2} + \frac{\omega_l}{Q_l} \frac{dh}{dt} + \omega_l^2(h - h_0) = \frac{1}{\mu}(p_b - p(t)), \quad (1)$$

where  $h$  is the height of the lip channel (m);  $p$  is the pressure at the input of the instrument, in the mouthpiece (Pa);  $p_b$  is the constant blowing pressure in the mouth (Pa);  $\omega_l = 2\pi f_l$  (rad · s<sup>−1</sup>) is the lip resonance angular frequency;  $Q_l$  is the (dimensionless) quality factor of the lips;  $h_0$  is the value of  $h(t)$  at rest;  $\mu$  is an equivalent surface mass (kg · m<sup>−2</sup>).

Although it does not fully reproduce all the observed behaviours of human or artificial lips, this model is sufficient for reproducing the normal playing situations [17], including the pedal note of the trombone [5] and multiphonic sounds [18]. As a limitation, this model is known to oscillate at higher frequencies than those at which a musician would play on the same acoustical mode. Even for this relatively simple model, choosing the lip parameters is challenging and requires a thorough bibliographical review. This was conducted in [5]. The resulting set of parameters is given in Table I.

The resonator is the air column contained in the bore of the instrument. Given the low playing amplitude considered in this article, the brassiness phenomenon, related to non-linear propagation in the instrument [19] is not taken into account. Under this hypothesis, the resonator can be fully described by its input impedance, which is by definition the ratio of the pressure  $P(\omega)$  to the flow  $U(\omega)$  at the input of the instrument, in the frequency domain,

$$Z(\omega) = \frac{P(\omega)}{U(\omega)}. \quad (2)$$

Table I. Lip parameters retained in this study.

$h_0$ (m)	$W$ (m)	$1/\mu$ (m <sup>2</sup> · kg <sup>−1</sup> )	$Q_l$
$5 \times 10^{-4}$	$12 \times 10^{-3}$	0.11	7

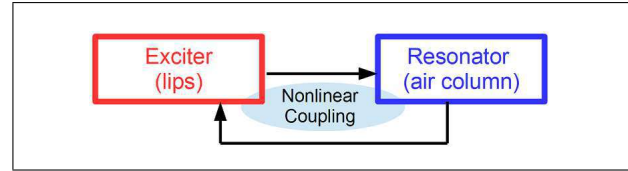


Figure 1. Closed-loop model in free oscillation, suitable for the description of most self-sustained musical instruments, including trombones. Self-sustained oscillations are generated by the localised non-linear coupling (here the airflow between lips) between a linear exciter (here the lips) and a linear resonator (here the air column inside the instrument bore).

This value can be measured using the sensor described in [20]. In this paper, three input impedance measurements are used: the impedance of an open trombone (without any mute), the impedance of the same trombone with a “passive mute” (mute without active control) and the impedance of this trombone with an “active mute”, with the feedback active control device enabled.

The input impedance can be considered as a sum of peaks, each peak corresponding to a resonance mode of the air column inside the instrument. Thus, it can be fitted with a sum of complex modes, corresponding to a sum of poles-residues functions,

$$Z(\omega) = Z_c \cdot \sum_{n=1}^{N_m} \left[ \frac{C_n}{j\omega - s_n} + \frac{C_n^*}{j\omega - s_n^*} \right], \quad (3)$$

$C_n$  and  $s_n$  being the dimensionless complex residues and poles of the complex modes of the fitted impedance, respectively.  $Z_c = \rho c / (\pi r^2)$  is the characteristic impedance of the resonator,  $\rho$  is the air density,  $c$  the celerity of acoustic waves in the air and  $r$  the input radius of the mouthpiece.  $N_m$  is the number of modes used to fit the impedance, fixed to  $N_m = 13$  in this article. Translation of Equation (3) in the time domain leads to an ordinary differential equation for each complex modal component  $p_n$  of the pressure  $p(t)$ ,

$$\frac{dp_n}{dt} = s_n p_n(t) + Z_c C_n u(t) \quad \forall n \in [1 \dots N_m], \quad (4)$$

where  $u(t)$  is the time-domain expression of the flow at the input of the instrument. Furthermore,  $p(t) = 2 \sum_{n=1}^{N_m} \Re[p_n(t)]$ . Details of this modal formulation of the pressure, already used in [5], can be found in [21].

The fit is optimised by a least mean squares algorithm. This results in a very good match between the measured impedance and the fit, as shown in Figure 2.

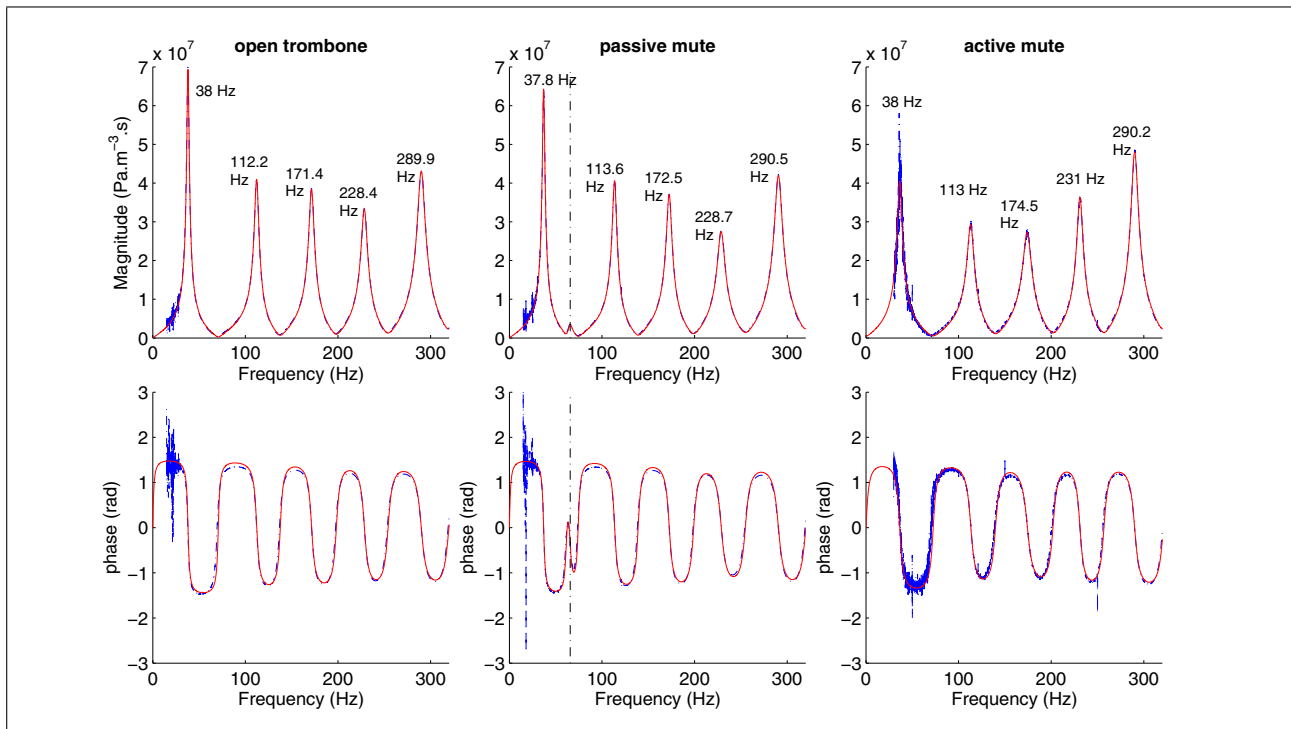


Figure 2. (Colour online) Comparison of the measured impedances (blue, dash-dotted) and their modal fits (red, plain) with 13 complex modes. Magnitudes (top plots) and phases (bottom plots) of the impedances for the three situations – open trombone (left), passive straight mute (middle) and active mute (right) – are displayed. The dash-dotted line at 65.7 Hz indicates the subsidiary resonance. The resonance frequencies of the other modes are written near the amplitude peaks.

The lips and the resonator are coupled through the expression of the flow  $u(t)$  of the air jet through the lip channel,

$$u(t) = W h(t) \sqrt{\frac{2|p_b - p(t)|}{\rho}} \text{sign}(p_b - p(t)) \theta(h), \quad (5)$$

where  $W$  is the width of the lip channel and  $\rho$  the air density, “sign” is the sign function and  $\theta(h)$  is the Heaviside step function. This non-linear expression of the flow was proposed in [22, 9] and has been used in almost every publication about brasswind and woodwind physical models since.

The whole model can therefore be written

$$\begin{cases} \frac{d^2 h}{dt^2} + \frac{\omega_l}{Q_l} \frac{dh}{dt} + \omega_l^2 (h - h_0) = \frac{1}{\mu} (p_b - p(t)), \\ u(t) = W h(t) \sqrt{\frac{2|p_b - p(t)|}{\rho}} \text{sign}(p_b - p(t)) \theta(h), \\ \frac{dp_n}{dt} = s_n p_n(t) + Z_c C_n u(t) \quad \forall n \in [1 \dots N_m], \\ p(t) = 2 \sum_{n=1}^{N_m} \Re [p_n(t)]. \end{cases} \quad (6)$$

## 2.2. Linear stability analysis

The model described above has a variety of possible behaviours. One of them is a static solution, all variables being constant. The stability of this static solution is a useful piece of information, as instability of the static solution indicates possible emergence of oscillating solutions

through Hopf bifurcations. This stability analysis can be carried out on a linearised model: non-linear equations are linearised in the vicinity of the static solution. Then, the stability of this static solution is assessed through computation of the eigenvalues of the Jacobian matrix. If at least one eigenvalue has a positive real part, any perturbation of the static solution will grow exponentially, which by definition means the solution is unstable. Details on the method applied to brass instruments can be found in [5].

This method is used to find the lowest blowing pressure value leading to an unstable static solution. This  $p_b$  value is hereafter called  $p_{\text{thresh}}$ . The imaginary part of the same eigenvalue indicates the oscillation angular frequency for  $p_b = p_{\text{thresh}}$ , provided that the oscillating solution is periodic. The corresponding frequency is noted  $f_{\text{thresh}}$ .

LSA has been used for flute-like instruments [23, 24] as well as reed woodwinds [22, 25, 26] and brasswinds [11, 5]. This method does not provide information about the stability of the oscillating solution which results from the destabilisation of the static solution. The only piece of information about the resulting waveform is  $f_{\text{thresh}}$ , which is only valid if said solution is periodic.

An example of results is given in Figure 3:  $p_{\text{thresh}}$  (a) and  $f_{\text{thresh}}$  (b) are plotted against the lip resonance frequency  $f_l$ , which is a control parameter used by the musician to change the note played with the trombone. As observed in [5], the plots can be divided in several  $f_l$  ranges corresponding to U-shaped sections of the  $p_{\text{thresh}}$  curves and very lightly growing plateaus of  $f_{\text{thresh}}$  just above the acoustic resonance frequencies of the resonator.

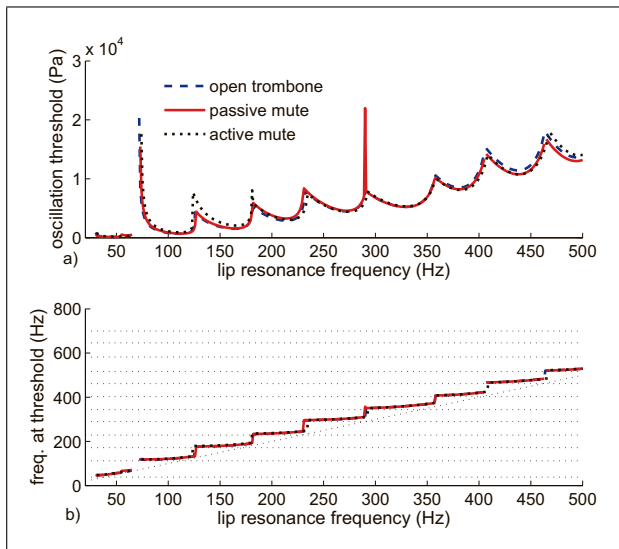


Figure 3. (Colour online). Linear stability analysis results:  $p_{\text{thresh}}$  (a) and  $f_{\text{thresh}}$  (b) are plotted against  $f_l$ . Results for the open trombone (blue, dashed), the passive mute (red, solid) and the active mute (black, dotted) are displayed. Black dotted lines of the bottom plot are the resonance frequencies of the open trombone (horizontal) and the bisector of the axes ( $f_{\text{thresh}} = f_l$ ). The qualitative behaviour of the open trombone, the passive mute and active mute are very similar at this scale.

### 2.3. Time-domain simulation

To get more information about the nature of oscillating solutions of the instrument model, solving the non-linear equation system (6) is required. Numerical differential equation solvers provide simulated values of the system variables. Simulated values of the pressure at the input of the instrument  $p$  have been obtained with the open-source Python library called MoReeSC [27], which has been developed specially for time-domain simulation of self-oscillating reed and lip valve instrument models [28].

To illustrate the additional information provided by time-domain simulation, waveforms and spectra of two simulated pressure signals are given in Figure 4. The simulation in Figure 4a and 4c was computed with  $f_l = 90$  Hz while the one in Figure 4b and 4d was computed with  $f_l = 110$  Hz, each one on an open trombone, with a blowing pressure 10% higher than the oscillation threshold. While LSA results for these two situations are very close to one another, numerical resolution of the complete model shows a difference in the nature of the oscillation: while the oscillation is periodic for  $f_l = 90$  Hz, it appears to be quasi-periodic for  $f_l = 110$  Hz.

The  $f_l$  and  $p_b$  values for simulations are chosen thanks to LSA, avoiding a long and cumbersome search for the oscillation threshold with multiple simulations. The complementarity of these methods quickly provides a lot of information about relevant points of the oscillation regime. For illustration, the Appendix displays spectra and waveforms of time-domain simulation and playing measurements on the trombone.

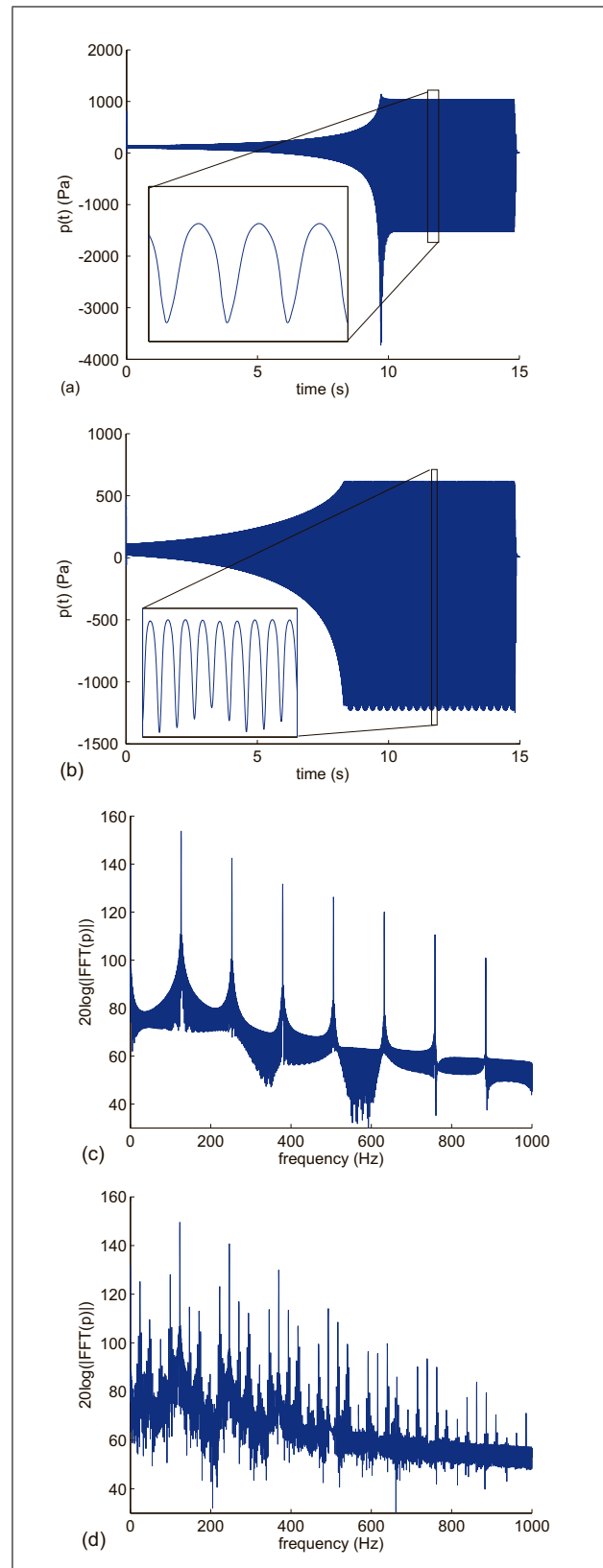


Figure 4. Waveforms of simulated  $p$  signals for  $f_l = 90$  Hz (a) and  $f_l = 110$  Hz (b) with zooms on some periods, along with spectra of their respective sustained regime in c) and d). For each simulation  $p_b$  is set to  $1.1 \cdot p_{\text{thresh}}$ .  $f_l = 90$  Hz results in a periodic oscillation while  $f_l = 110$  Hz results in a quasi-periodic oscillation with well defined secondary peaks.

### 3. Results

#### 3.1. LSA

Linear stability analysis was performed on the three configurations studied: open trombone, passive mute, and active mute. Choosing a configuration involves choosing  $C_n$  and  $s_n$  values among the three sets obtained by fitting, all other parameters of the model remaining the same. Lip parameters were taken from Table I. LSA was performed within the pedal note range, for  $f_l$  from 30 Hz to 65 Hz. This results in  $f_{\text{thresh}}$  values corresponding to an oscillation sustained by the first acoustical mode of the open trombone. Figure 5 is a zoom on Figure 3 in the considered  $f_l$  range. Figure 5a) showing the threshold pressures  $p_{\text{thresh}}$ , while Figure 5b) is the frequency at threshold  $f_{\text{thresh}}$ .

The open trombone and the active mute behaviours are similar: the  $p_{\text{thresh}} = F(f_l)$  plot is U-shaped.  $f_{\text{thresh}}$  is above the trombone's first acoustical resonance frequency (39 Hz) and increases monotonically with  $f_l$ . Within this  $f_l$  range, the oscillation threshold of the active mute trombone is about 75 Pa higher than that of the open trombone, and  $f_{\text{thresh}}$  is also 0.5 to 1.5 Hz higher.

For  $f_l \leq 54$  Hz, the results for the trombone with a passive mute are similar to those for the other configurations. But from  $f_l = 55$  Hz, both the pressure threshold and the expected playing frequency increase significantly:  $p_{\text{thresh}}$  suddenly jumps from 198.6 to 536.9 Pa, while  $f_{\text{thresh}}$  increases by 8.3 Hz (13%, i.e. slightly more than a tone), to reach 66.8 Hz. This value is just above the resonance frequency of the subsidiary peak induced by the passive mute.

$f_{\text{thresh}}$  covers a range of frequencies around the expected playing frequency of a pedal  $Bb = 58$  Hz. The results for the open trombone and the active mute configurations are very close to one another, the only difference being a rather small offset in  $p_{\text{thresh}}$  and  $f_{\text{thresh}}$ . In contrast, the passive mute results stand out from the two other configurations: for  $f_l$  values above 55 Hz,  $p_{\text{thresh}}$  and  $f_{\text{thresh}}$  increase suddenly. The  $f_{\text{thresh}}$  value obtained is above the acoustic resonance frequency of the subsidiary mode related to the passive mute, and so the regeneration condition [9, 14] is satisfied for an oscillation supported by this subsidiary mode.

These results can account for the difficulty of playing a stable pedal note with a passive mute: the LSA indicates a perturbation of the oscillation frequency at threshold, for parameters which could be those used for the pedal note. However, experimental results shown in [1] suggest a non-periodic oscillation when a musician tries to play a pedal note with a passive mute. As LSA cannot predict the nature of the oscillation, further investigation on the complete non-linear model is needed. This is the purpose of the numerical simulations presented in the following section.

#### 3.2. Time-domain simulations

Time-domain simulations were carried out within the same range of  $f_l$  as for LSA, in 1 Hz steps, for each configuration: trombone alone, trombone with a passive mute and

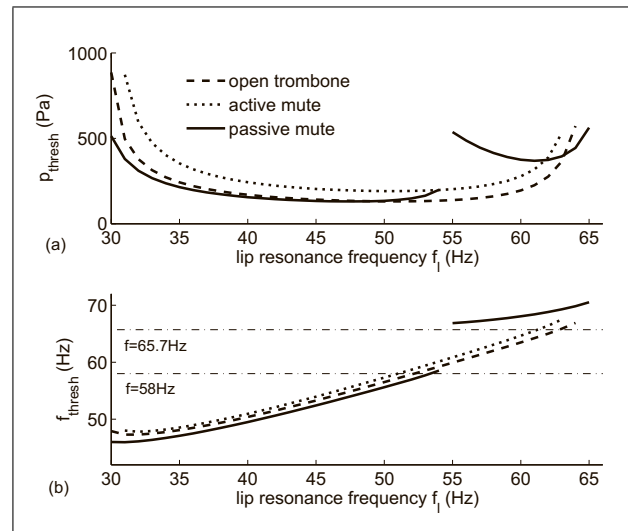


Figure 5. Results of LSA in the vicinity of the pedal note (zoom of Figure 3). Results with an open trombone (dashed line), a passive mute (solid line) and the active mute (dotted) are plotted together. (a) is the oscillation threshold pressure  $p_{\text{thresh}}$ , (b) is the oscillation frequency at threshold  $f_{\text{thresh}}$ , against  $f_l$ . Horizontal dash-dotted lines in (b) indicate 58 Hz (playing frequency of the pedal  $Bb$ ) and 65.7 Hz (resonance frequency of the subsidiary mode of the passive mute). While open trombone and active mute have very similar behaviours, the oscillation regime expected for the trombone with the passive mute becomes different above  $f_l = 55$  Hz with a sudden increase in the  $p_{\text{thresh}}$  and  $f_{\text{thresh}}$  values.

finally trombone with active mute. The blowing pressure was set to  $p_b = 1.1 \cdot p_{\text{thresh}}$  as in [5] in order to keep manageable transient times. This value is close enough to  $p_{\text{thresh}}$  so that cautious comparisons can be carried out between these simulations and LSA.

Simulated pressure signals were separated into a transient and a sustained regime with the help of the "mironsets" function from MIRtoolbox [29]. The spectra of all the sustained regimes were computed. Figure 6 plots spectra of  $p(t)$  for representative values of  $f_l$ .

For  $f_l < 55$  Hz, the three configurations - open trombone, passive mute, active mute - lead to a periodic oscillation, as illustrated for  $f_l = 53$  Hz in (Figure 6a). The oscillation frequency is a bit higher than  $f_{\text{thresh}}$ : 7.5% for open trombone and active mute, and 2.5% higher for the passive mute. Oscillation frequencies higher than  $f_{\text{thresh}}$  when  $p_b > p_{\text{thresh}}$  is coherent with the fact that a musician's playing frequency gets higher when the blowing pressure increases. The trombone with a passive mute has a lower oscillation frequency than the open trombone, which has itself a slightly lower oscillation frequency than that of the trombone with the active mute. The oscillation frequencies range from 60 to 64 Hz, a bit higher than  $Bb1 = 58$  Hz. This is sensible since this model is known to oscillate at higher frequencies than those at which a musician plays.

At  $f_l = 55$  Hz (Figure 6b) the oscillation frequency of the passive mute configuration suddenly jumps from 59.4 to 69.6 Hz, making it play sharper (nearly a minor

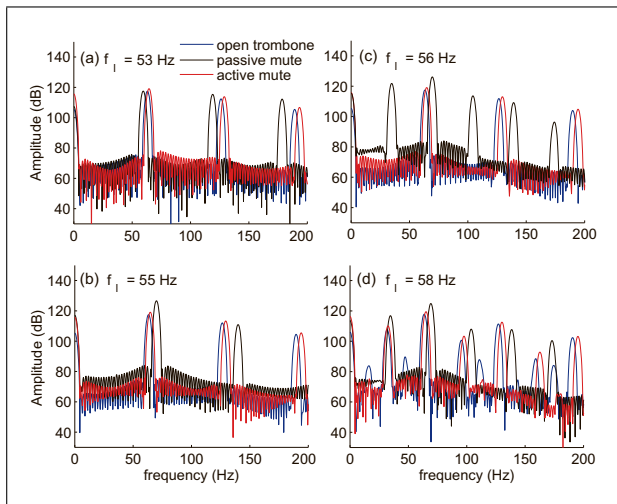


Figure 6. (Colour online) Spectra of the simulated  $p(t)$  signals, for  $f_i = 53$  Hz (a), 55 Hz (b), 56 Hz (c) and 58 Hz (d).  $p_b$  is set to 110% of the oscillation threshold. For each  $f_i$  value, results with the open trombone (blue), the passive mute (black) and the active mute (red) are displayed. The results for the open trombone and the active mute are noticeably similar.

third) than the two other configurations. This is consistent with the LSA results, where  $f_{\text{thresh}}$  suddenly increases by 8 Hz for this  $f_i$  value. The oscillations are still periodic and above the acoustical resonance frequency of the first mode; but for  $f_i = 56$  Hz and above (illustrated by Figure 6c) the fundamental frequency of the passive mute falls to 34.8 Hz. This is nearly half its former value, and under the trombone's first acoustic resonance frequency (39 Hz). Finally, for  $f_i \geq 58$  Hz (Figure 6d), all configurations result in fundamental oscillation frequencies about half, or a quarter, of the oscillation frequencies obtained for lower  $f_i$  values.

Simulation and LSA results are consistent: when  $f_i$  reaches 55 Hz, the oscillation frequency of the trombone with a passive mute suddenly increases. This is related to a regime change in the instrument: for  $f_i < 55$  Hz, the oscillation is mainly supported by the trombone's first acoustical mode whose resonance frequency is 38 Hz. The unusual gap between the trombone's first mode and the pedal note is studied in [5]. For  $f_i = 55$  Hz and above, the subsidiary mode at 65.7 Hz caused by the mute becomes the main supporting mode of the oscillation, which explains the increase in the oscillating frequency.

Above  $f_i = 56$  Hz, however, the oscillation frequency of the passive mute decreases to half of its former value. As the oscillation frequency is under the trombone's first acoustical resonance frequency, the regeneration condition of a model with outward-striking valve is not satisfied [14]. This situation suggests a period-doubling phenomenon [30]. When increasing  $f_i$  again, the three configurations appear to undergo period doubling, which is further doubled for the open trombone with a fundamental frequency of 16.2 Hz. Sub-harmonic cascades have already been observed for trombones [31], and simulated in

a previous study with the very same model and parameters [5].

These results confirm the existence of a subsidiary regime of oscillation for the passive mute configuration, which could explain why musicians experience difficulties when trying to play the  $B\flat$  pedal in this situation. This subsidiary regime is sustained by the subsidiary acoustical mode introduced by the mute. Furthermore, in accordance with the experimental results published in [1], the simulation results are qualitatively the same for the open trombone and the active mute, with very close oscillation frequencies. The range of  $f_i$  leading to periodic oscillations near the pedal note frequency is noticeably wider for the open trombone and the active mute than for the passive mute.

#### 4. Discussion and Conclusions

Playing a stable  $B\flat$  on a trombone with a straight mute is very difficult. An active mute has been developed [1] to deal with this issue. When applied to a trombone equipped with this active mute and to an open trombone, LSA and time-domain simulation give nearly identical results. The model is therefore able to predict the efficiency of the active control device which makes the pedal note easily playable again. Results of the model of a trombone equipped with a passive mute, however, are clearly different from those of the open trombone model: the pedal note is disturbed by a new oscillation regime, which seems related to the subsidiary acoustical mode added by the mute. Hence, even a "small" perturbation of the input impedance, such as a peak 20 times smaller in amplitude than surrounding peaks, can strongly affect the behaviour of a resonator.

As in a previous paper [5], this study shows a rather good agreement between LSA results and time-domain simulations, within the limits of the LSA method. This study on mutes also shows the relevance of the chosen brass instrument model, which is able to predict a number of behaviours of the trombone, including particular playing regimes [5, 18] and, in the present case, the influence of modifications of the instrument bore.

Beginning a study with LSA very quickly gives an overview of the potential behaviour of the system under given conditions. This fast computation already provides interesting results, which can be interpreted alone. However, if further exploration of the oscillation regime is required, LSA results give hints for choosing  $f_i$  and  $p_b$  values for initialising other analysis methods.

#### Appendix: Comparison of measurements and time-domain simulations

In addition to this article, two examples of measured and simulated mouthpiece pressure waveforms and spectrums are given in Figure A1 and A2 for information and illustration. Experiment and simulation correspond to the pedal

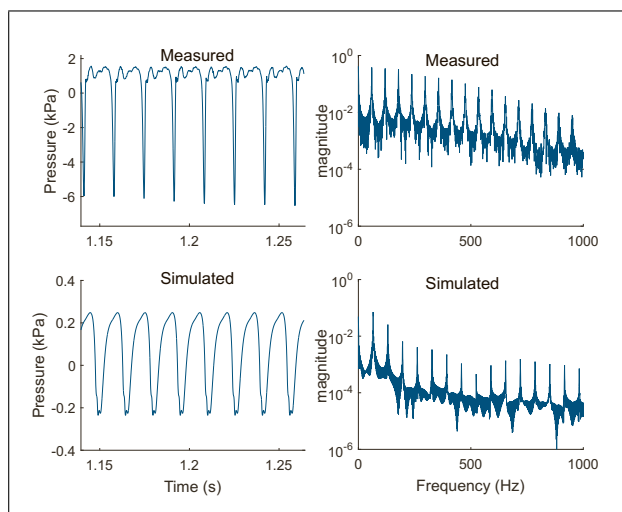


Figure A1. Comparison of a Bb1 played on a trombone (top) and simulated with the model used in this paper (bottom). Waveform (left) and spectrum (right) of the mouthpiece pressure are displayed. Simulation parameters:  $f_i = 50$  Hz,  $p_b = 400$  Pa, other parameters from Table I.

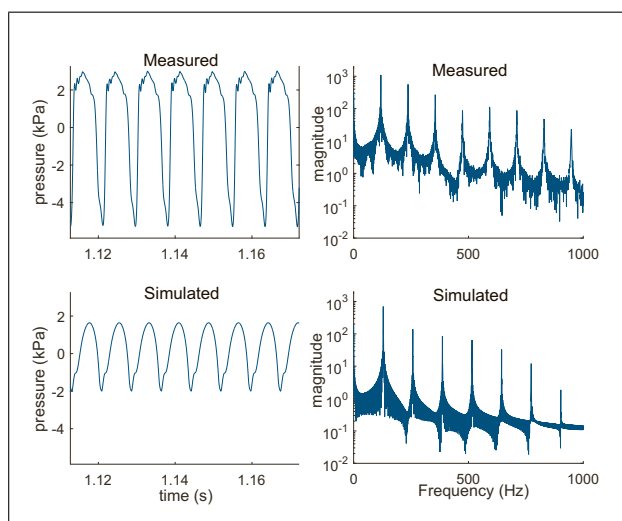


Figure A2. Comparison of a Bb2 played on a trombone (top) and simulated with the model used in this paper (bottom). Waveform (left) and spectrum (right) of the mouthpiece pressure are displayed. Simulation parameters:  $f_i = 90$  Hz,  $p_b = 2$  kPa, other parameters from Table I.

Bb1 note and the Bb2 of a trombone respectively. However, this is not a comparison since none control parameters can be compared.

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