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Fractional Modeling on Moving-coil Loudspeaker Linear System

Kaifeng Han[1)], Xiaopeng Kong[2)]
[1)] Institute of Marine Science and Engineering, National University of Defense Technology, Changsha 410073, P. R. China.
[2)] Naval Aeronautical University, Yantai 264001, P. R. China. kongxp@nudt.edu.cn

Summary
The fractional order derivatives have been used in modeling the lossy inductance as well as the creep effect for moving-coil loudspeakers. Based on the fractional lossy inductance model and the fractional standard linear solid model, this work presented a fractional equivalent electrical circuit (FEEC) for modeling the linear behaviors of moving-coil loudspeakers. The fractional transfer function between the displacement and the input voltage was approximated by the modified Oustaloup filter method, and then the displacements of the FEEC in time domain were numerical calculated. Comparisons between the measured data and the predicted ones illustrate that, the FEEC presented by this paper can do a good job in modeling the linear behaviors of loudspeakers and hand accurate transfer functions in frequency domain as well as the output displacements in time domain.

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1. Introduction

A moving-coil loudspeaker is a transducer which transforms the electrical power to acoustic one via mechanical vibrations [1]. Loudspeaker is widely used for music reproduction, and it should be designed with good linear behaviors under small signal inputs as well as low level distortions under large signal inputs. When operating in low level and in the frequency range where the size of loudspeaker is small compared to the wavelength, the linear behaviors of loudspeakers can be predicted by an equivalent electrical circuit [2, 3] via a small number of parameters. Due to the eddy currents [2, 4] of voice coil as well as the creep effect [5, 6] of suspension, the simple equivalent circuit must be developed by adding additional parameters to give a more accurate prediction both in high frequency range and in low frequency range. The equivalent circuits are quiet developed in the last 30 years, resulting in some advanced lossy inductance models accounting for eddy currents, as well as the creep models predicting the viscoelastic behaviors of suspensions.

Just as Leach model [7], the semi-inductance most used in lossy inductance models can be described by fractional derivatives [2]. Schafer *et al.* [8] gave the first try to describe the inductance losses of voice coil by applying the fractional derivatives in time domain, and then Brunet *et al.* [9] modeled the lossy inductance in the equivalent electrical circuit both in frequency domain and in time domain.

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However, both of the two studies did not consider the creep effects of suspensions.

The suspension of a loudspeaker usually composed two parts – the outer surround and the inner spider. Intuitively, either the surround or the spider are made of viscoelastic materials, which behave somewhere in between purely elastic behaviors and purely viscous behaviors. Some relevant attributes of such viscoelastic materials like creep effect is significant. The creep effect can be observed as an increase in displacement with frequency at low frequencies. It can be described by the viscoelastic constitutive models like the Standard Linear Solid model (SLS), in terms of frequency dependent stiffness and frequency dependent damping [6]. However, the majority of the loudspeaker suspensions in use today have memory effects, and the constitutive models cannot predict such memory effects. But it can be improved by transplanting the fractional order derivatives to the constitutive equations of viscoelastic materials. Introducing fractional order derivatives into the constitutive models can yield an accurate description, while reducing the number of material parameters. The methodology of using fractional order derivatives (FO) to model the suspension creep should be date back to 1993, Knudsen and Jensen [10] gave the first attempt to model the mechanical part of a loudspeaker by FO. Then Thorborg and Futtrup [11] ameliorated this model in [10] without adding unnecessary complexity to the problem. Then Novak [12] presented a work in which FO derivatives are directly use to model the viscoelastic behaviors of the loudspeaker suspensions such as creep at very low frequencies. Brunet [13] has deeply investigated FO and their successful application in loudspeaker modeling. By substituting the Abel dashpot for Newton dashpot in SLS model, Kong *et al.* [14] presented the Fractional SLS model. By fitting the measured electrical impedance curve, the results find that this fractional SLS model can rightly predict the frequency dependent compliance loss factor, and yield higher accuracy for modeling the creep effect, especially in higher loss suspensions. King *et al.* [14] presented the fractional lumped loudspeaker model by using a fractional order viscoelastic suspension with a fractional order semi-inductive voice coil, and the parameters were identified by fitting the fractional model to the measured data in frequency domain. Then the responses of loudspeakers in time domain were calculated by incorporating the identified parameters in a fractional order state-space.

In the present study, a fractional equivalent electrical circuit (FEEC) was presented by using the fractional lossy inductance model as well as the fractional standard linear solid model. By fitting the measured electrical impedance and the measured Frequency Response Function (FRF), the model parameters were identified. With the identified fractional model parameters, approximate solutions of loudspeaker displacements in time domain were calculated by using the modified Oustaloup filter method [15, 16].

2. Fractional equivalent electrical circuit

2.1. Simple equivalent electrical circuit

Figure 1a delineates the equivalent circuit of a lumped loudspeaker unit, where e is the input voltage, R_g is the equivalent inner resistance, L_e is the voice coil inductance, B_l is the force factor, i is the current flow through the voice coil, $u = Blv$ is the voltage induced in the voice coil due to its motion through the permanent magnetic field in the gap, v is the velocity of diaphragm, $F_f = Bli$ is the electrical driving force, $C_{\rm ms}$ is the suspension compliance, $M_{\rm md}$ is the total mass of the voice coil, suspension and diaphragm, $R_{\rm ms}$ is the suspension damping, p is the sound pressure, $M_{\rm AR}$ is the equivalent acoustic mass, $R_{\rm AR}$ is the equivalent acoustic resistance. When the loudspeaker working at low frequencies, this linear equivalent circuit can be simplified to Figure 1b by ignoring the eddy currents of the voice coil, where $M_{\rm ms} = M_{\rm md} + 2M_{\rm AR}$ (infinite baffle), Z_E is the electrical impedance of a loudspeaker unit.

Followed by Kirchhoff's voltage law, the integer order differential equations representing

$$\begin{aligned} e(t) &= R_e\, i(t) + L_e \frac{\mathrm{d}i(t)}{\mathrm{d}t} + Bl\frac{\partial x}{\partial t}, \quad R_g = 0, \\ Bli(t) &= M_{\rm ms}\frac{\partial^2 x}{\partial t^2} + R_{\rm ms}\frac{\partial x}{\partial t} + \frac{x(t)}{C_{\rm ms}}, \end{aligned} \tag{1}$$

where t is time, $x(t)$ is the displacement of diaphragm, and in the frequency domain, defining $E(\omega)$, $I(\omega)$ and $X(\omega)$ as the Fourier transforms of $e(t)$, $i(t)$ and $x(t)$, as

$$\begin{aligned} E(\omega) &= R_e I(\omega) + \mathrm{j}\omega L_e I(\omega) + \mathrm{j}\omega Bl X(\omega), \\ BlI(\omega) &= -M_{\rm ms}\omega^2 X(\omega) + \mathrm{j}\omega R_{\rm ms} X(\omega) + X(\omega)/C_{\rm ms}, \end{aligned} \tag{2}$$

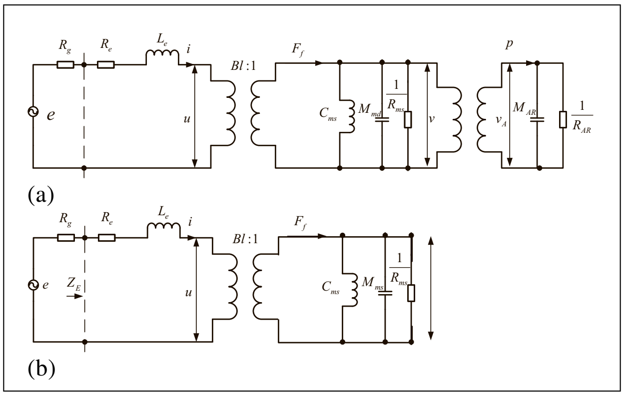

Figure 1. The equivalent circuit of an electro-dynamic loudspeaker. (a) Equivalent circuit, (b) Simplified circuitat low frequencies.

where ω is the angular frequency, j is the complex operator. Then the transfer function $H(\omega)$ between $X(\omega)$ and $E(\omega)$ can be calculated as

$$\begin{aligned} H(\omega) &= \frac{X(\omega)}{E(\omega)} \\ &= \frac{Bl}{\mathrm{j}\omega Bl^2 + (\mathrm{j}\omega L_e + R_e)(-\omega^2 M_{\rm ms} + \mathrm{j}\omega R_{\rm ms} + 1/C_{\rm ms}}, \end{aligned} \tag{3}$$

the electrical impedance $Z_E(\omega)$ is given by

$$\begin{aligned} Z_E(\omega) &= \frac{E(\omega)}{I(\omega)} \\ &= \mathrm{j}\omega L_e + R_e + \frac{Bl^2}{\mathrm{j}\omega M_{\rm ms} + R_{\rm ms} + 1/\mathrm{j}\omega C_{\rm ms}}. \end{aligned} \tag{4}$$

2.2. Fractional lossy inductance model

At high frequencies, the current flowing in the voice coil wire produces a magnetic field, and the magnetic field circulating the wire induces eddy currents in the conductive center pole, resulting in inductance losses [2]. Even worse, the electrical impedance of loudspeakers is dominated by the voice coil inductance in high frequency range. Therefore, the simple equivalent electrical circuit should be improved by lossy inductance models [4]. Leach introduced a variable exponent to an angular frequency variable [7] to predict the lossy inductance, and it has good correspondences with the measurements at high frequencies [2, 4, 7]. Instead of the no loss inductor L_e, the lossy inductance given by Leach in frequency domain is

$$Z_L(\omega) = L_e(\mathrm{j}\omega)^\alpha, \quad 0 < \alpha < 1. \tag{5}$$

In time domain, the Leach model can be described by the fractional derivatives [2]. For a lossy inductor L_e, assuming the voltage time signal cross L_e is $e(t)$, and the current time signal through this L_e is $i'(t)$. The electrical properties of this lossy inductor can be described by the following Kirchhoff's fractional differential equation,

$$e'(t) = L_e \frac{\mathrm{d}^\alpha i'(t)}{\mathrm{d}t^\alpha}, \quad 0 \le \alpha \le 1, \tag{6}$$

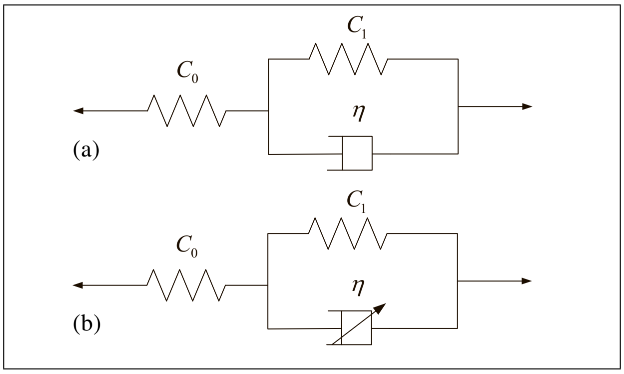

Figure 2. SLS model and FSLS model (a) SLS model (Kelvin-Voigt form).

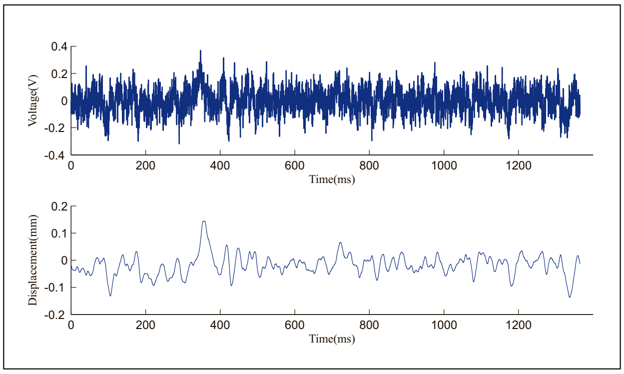

Figure 3. The input voltage time signal and the output displacement time signal (0.1 V).

where α is the fractional order, and the fractional derivative $\mathrm{d}^\alpha/\mathrm{d}t^\alpha$ follows the Caputo definition [17]. Employing the fractional order Fourier Transform, the Leach model can be derived by transforming Equation (6) to frequency domain.

2.3. Fractional creep model

Viscoelastic materials will behave somewhere in between purely elastic behaviors and purely viscous behaviors, such as creep, relaxation, and memory effect [17]. Therefore, neither the Hooke spring element nor the Newton dashpot element can predict such viscoelastic behaviors. Shown by Figure 2a, The Kelvin-Voigt form of Standard Linear Solid model (SLS) is created by placing a spring element in series with a Maxwell model. SLS model can give a good prediction for the creep effect in loudspeaker suspensions, but it cannot respect to memory effects. Fortunately, it can be improved by fractional order derivatives. The constitutive relation of the fractional Abel dashpot is given by [14],

$$\sigma(t) = \eta\frac{\mathrm{d}^\beta\varepsilon(t)}{\mathrm{d}t^\beta}, \quad 0 \le \beta \le 1, \tag{7}$$

where $\sigma(t)$ is the stress, $\varepsilon(t)$ is the strain. Transforming Equation (7) to frequency domain by fractional order Fourier Transform [17] of Caputo definition,

$$\Gamma(\omega) = (\mathrm{j}\omega)^\beta \eta E(\omega). \tag{8}$$

The fractional standard linear solid (FSLS) model illustrated by Figure 2b can be realized by substituting the Abel dashpot for the Newton dashpot in Figure 2a. Then the creep compliance $C_{\mathrm{ms},\beta}(\omega)$ is calculated by

$$\begin{aligned} C_{\mathrm{ms},\beta}(\omega) &= C_0 + \frac{1}{(\mathrm{j}\omega)^\beta\eta + 1/C_1} \\ &= C_0 + \frac{C_1}{1+(\mathrm{j}\omega)^\beta\tau}, \quad \tau = C_1\eta. \end{aligned} \tag{9}$$

Setting $Z_{\mathrm{L}}(\omega)$ and $C_{\mathrm{ms},\beta}(\omega)$ in Equation (3) and Equation (4) to replace j$omegaL_e$ and C_{ms} respectively, then the fractional transfer function $H_{\alpha,\beta}(\omega)$ and electrical impedance $Z_{\alpha,\beta}(\omega)$ can be calculated by

$$\begin{aligned} H_{\alpha,\beta}(\omega) &= \frac{X(\omega)}{E(\omega)} \\ &= \frac{Bl}{\left[\mathrm{j}\omega Bl^2 + \left(Z_{\mathrm{L}}(\omega)+R_e\right)\cdot\left(-\omega^2 M_{\mathrm{ms}} + \mathrm{j}\omega R_{\mathrm{ms}} + 1/C_{\mathrm{ms},\beta}(\omega)\right)\right]} \\ &= \frac{Bl}{\left[\mathrm{j}\omega Bl^2 + \left[(\mathrm{j}\omega)^\alpha L_e + R_e\right]\cdot\left(-\omega^2 M_{\mathrm{ms}} + \mathrm{j}\omega R_{\mathrm{ms}} + \left(C_0 + \frac{C_1}{1+(\mathrm{j}\omega)^\beta\tau}\right)^{-1}\right)\right]}, \end{aligned} \tag{10}$$

$$\begin{aligned} Z_{\alpha,\beta}(\omega) &= \frac{E(\omega)}{I(\omega)} \\ &= R_e + (\mathrm{j}\omega)^\alpha L_e + \frac{Bl^2}{\mathrm{j}\omega M_{\mathrm{ms}} + R_{\mathrm{ms}} + 1/\mathrm{j}\omega C_{\mathrm{ms}}(\omega)} \\ &= R_e + (\mathrm{j}\omega)^\alpha L_e \\ &\quad + \frac{Bl^2}{\mathrm{j}\omega M_{\mathrm{ms}} + R_{\mathrm{ms}} + \left[\mathrm{j}\omega\left(C_0 + \frac{C_1}{1+(\mathrm{j}\omega)^\beta\tau}\right)\right]^{-1}} \\ &= R_e + (\mathrm{j}\omega)^\alpha L_e \\ &\quad + \frac{Bl^2}{\mathrm{j}\omega M_{\mathrm{ms}} + R_{\mathrm{ms}} + \frac{1+(\mathrm{j}\omega)^\beta\tau}{\mathrm{j}\omega C_0 + (\mathrm{j}\omega)^{\beta+1}\tau C_1 + \mathrm{j}\omega}}. \end{aligned} \tag{11}$$

3. Model parameters identification

A Scanspeaker woofer (25W/8565) was test and the electrical impedance as well as the transfer function were measured by Klippel analyzer [18]. To get the measured electrical impedance and the response X/I lower than 1 Hz, we use the same experiment method present by [19]. To give a clearly comparison, we have smoothed the measured data when the frequency below 1 Hz by 'Loess method' of Matlab smooth function. The measurements were carried out under a RMS voltage level of 0.1 V in order to ensure all the loudspeakers were working in the linear domain. Figure 3 illustrates the input voltage time signal (RMS 0.1 V) as well as the output displacement time signal. In frequency domain, the electrical impedance $Z_E(\omega)$ can be

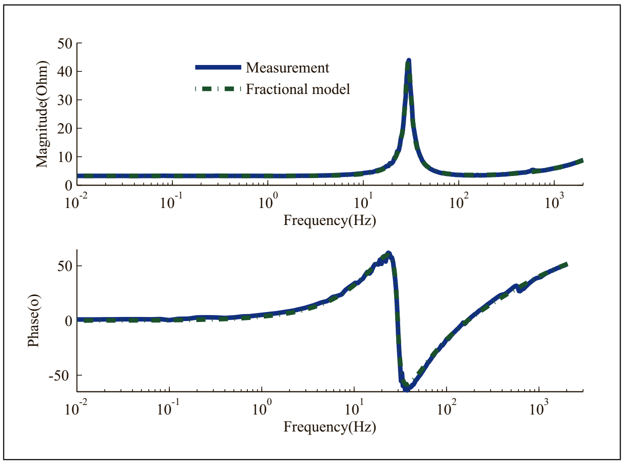

Figure 4. Comparison of the measured electrical imp edance with that predicted by fractional model.

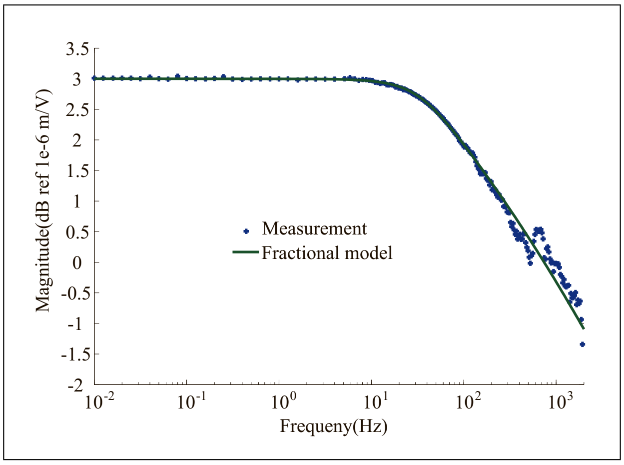

Figure 5. Comparison of the measured transfer funct ion with that predicted by fractional model.

calculated by the input voltage divided by the output current, and the transfer function $H(\omega)$ can be determined by the ratio of the output displacement and the input voltage.

The lumped parameters used in the fractional circuit model of the invested loudspeaker can be identified by fitting the measured electrical impedance as well as the transfer function based on the Least Mean Square method (LMS). The function *fminunc* in Matlab attempts to find a minimum of a cost function based on the difference between measured data and the output of the model in the frequency domain, starting at an initial estimate. The cost function κ to be minimized was defined by

$$\kappa = \sum_n \left(\left\| \frac{H_{\alpha,\beta}(\omega_n) - H_{\text{MEAS}}(\omega_n)}{H_{\text{MEAS}}(\omega_n)} \right\|_2^2 \right) + \sum_n \left(\left\| \frac{Z_{\alpha,\beta}(\omega_n) - Z_{\text{MEAS}}(\omega_n)}{Z_{\text{MEAS}}(\omega_n)} \right\|_2^2 \right). \quad (12)$$

The fitted model parameters are given by Table I, and the fitted responses of the electrical impedance and transfer function are illustrated by Figure 4 and Figure 5 respectively. Both of the two figures show that the fractional model can give a good prediction both for the electrical impedance and the transfer function.

4. Solutions in time domain

Towards to examine the accuracy of the fractional model in time domain, the displacements time signal $x(t)$ should be calculated. Multiplication in frequency domain corresponds to convolution in the time domain, therefore the output displacement of the investigated fractional model is the convolution of the impulse response $h_{\alpha,\beta}(t)$ and the input voltage signal $e(t)$, where $h_{\alpha,\beta}(t)$ is the inverse Fourier transform of $H_{\alpha,\beta}(\omega)$. Consequently, the point is fixed to get this fractional inverse response $h_{\alpha,\beta}(t)$. In this paper, the modified Oustaloup filter method [15, 16, 20] was used to get an approximate transfer function of the investigated fractional model.

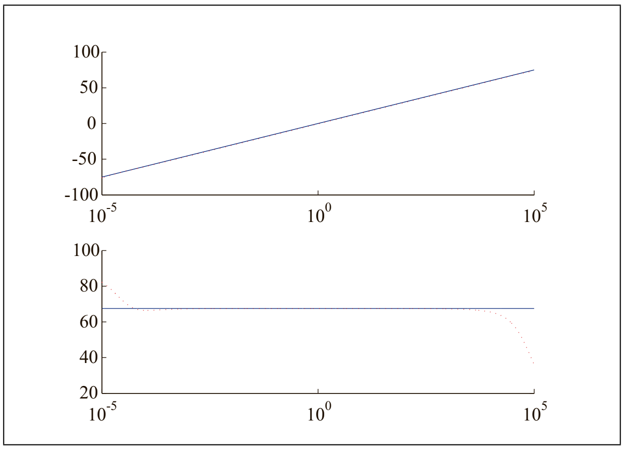

Figure 6. Comparison of the Bode plots between the original fractional order s0.8 (solidline) and the Oustaloup.

4.1. The modified Oustaloup filter method

To give an accurate approximation to a fractional order differential, the main initiative is to construct a bank of integer order filters. By presenting and selecting proper coefficients, Xue *et al.* (2006) [16] modified the Oustaloup's filter method [15] to improve the boundary fitting problems. Suppose the frequency range approximated by the modified Oustaloup's filter method is ω_l, ω_h, then the fractional differentiator s^γ $(s = \mathrm{j}\omega)$ can be approximated as

$$s^\gamma = H_{\gamma,\text{fit}} = \left(\frac{d\omega_h}{b}\right)^\gamma \frac{ds^2 + b\omega_h s}{d(1-\gamma)s^2 + b\omega_h s + d\gamma} \cdot \prod_{i=1}^{N} \frac{s + \omega_i'}{s + \omega_i}, \quad 0 < \gamma < 1, \quad (13)$$

where N is the approximated filter order, $\omega_i' = \omega_l\omega_h \cdot (\omega_h/\omega_l)^{2i-1-\gamma)/2N}$ and $\omega_i = \omega_l\omega_h(\omega_h/\omega_l)^{2i-1+\gamma)/2N}$ are respectively the zeros and poles of rank i. b and d are coefficients, and they are suggested to be 10 and 9 [16].

Given the fitted frequency range $[10^{-3}/3\,\text{Hz}, 3000\,\text{Hz}]$ as well as the finite order $N = 10$, Figure 6 illustrates the comparison of the bode plots between the approximated

Table I. Fitted parameters of the fractional model.

R_e [Ω]	Bl [N/A]	L_e [mH]	α	M_{ms} [g]	R_{ms} [Ns/m]	C_0 [mm/N]	C_1 [mm/N]	τ [ms]	β
3.25	8.85	4.9	0.77	78.9	1.92	0.37	1.21	0.86	0.80

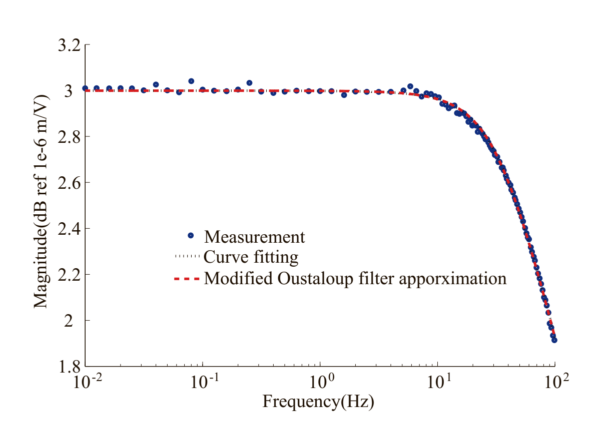

Figure 7. Comparisons of the measured transfer func tion (solid line) with that fitted bythe investig ated fractional model (dotted line), as well as that approximated by the

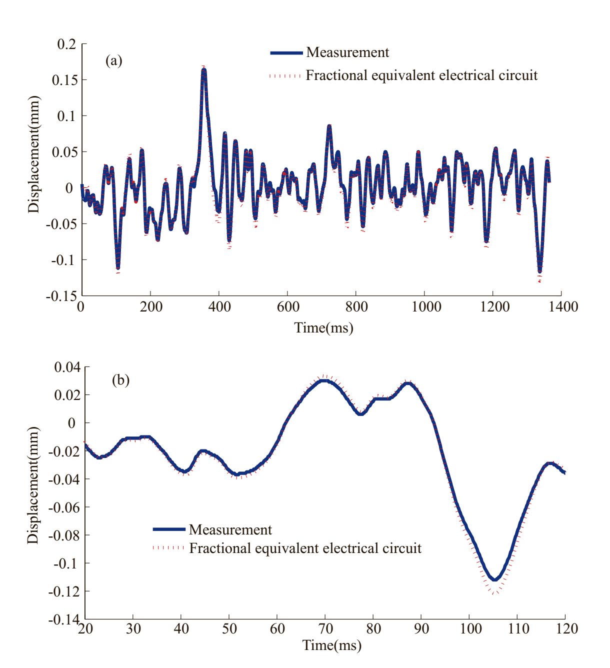

Figure 8. Comparisons of the measured displacement (solid line) with that predictedby the fractiona l equivalent electrical circuit (dotted line).

ones with the original fractional differential $s^{0.8}$. Note that there are significant discrepancies of the phase at very low frequencies as well as at even higher frequencies, and this is because the frequencies are beyond the fitted range. The modified Oustaloup model can be used to realize the fractional transfer function $H_{\alpha,\beta}(\omega)$ of the investigated loudspeaker. Figure 7 gives the comparisons of the transfer functions between the approximated one $H'(\omega)$ and the fractional one $H_{\alpha,\beta}(\omega)$ with the measured one. The approximated transfer function in good agreement with the fitted fractional one as well as the measured one. It should be pointed that there are some ripples at some frequency points and it corresponds for the measurement noise.

4.2. Comparisons of the loudspeaker displacement

The approximated impulse response $h'(t)$ of the investigated fractional circuit can be calculated as the inverse Fourier transform of the approximated transfer function $H'(\omega)$. Then the displacement time signal of $x'(t)$ the fractional circuit can be gained as the convolution of the $x'(t) = h'(t)*e(t)$. Figure 8 shows the comparison between this predicted displacement with the measured one. It can be seen that the fractional equivalent electrical circuit does a good job in predicting the loudspeaker displacement under small voltage input, but with minor deviations around the peaks of the displacement.

It is interesting to note that the absolute values of displacement predicting by the fractional model is smaller than the measured one. The reason can be discovered by doing Empirical Mode Decomposition (EMD) for both the measured displacement and the predicted one. Figure 9 gives the EMD results from the high frequency component to low frequency component. By comparing with the

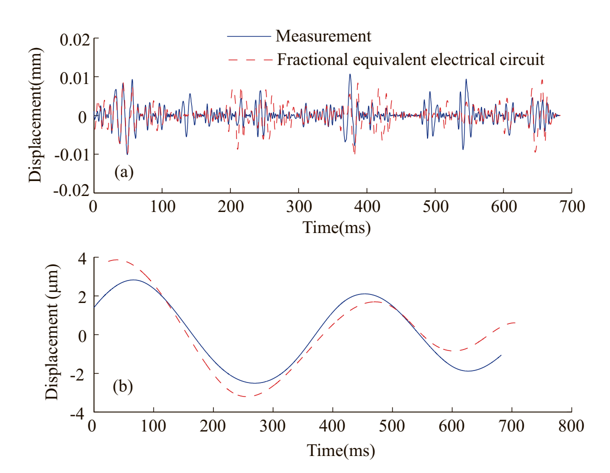

Figure 9. The EMD results of the displacement, (a) is the high frequency component and (b) is the low one.

measured displacement, one can see that for the deviations of the high-frequency component is more significant than the low-frequency one. The discrepancy of the low-frequency displacement component is really small, and it can be ignored. It can be initially concluded that the errors predicting by the investigated fractional circuit mainly comes from the used Leach lossy inductance model. Reference [2] has pointed that for there are only two parameters, the Leach model cannot predict the change of lossy inductance phase along with the frequency.

5. Discussion

t should be pointed out that the essence of the modified Outaloup filter method is to find higher order integer filter for estimating the fractional transfer functions. The higher of the order, the estimation is more accurate. Nevertheless, as the order of the filter increases, the numerical calculation in time domain will become more complicated. Therefore, the order of the filter used by the modified Oustaloup filter method is better not more than 10. Additionally, it is complicated to use the modified Oustaloup filter method in nonlinear modeling of loudspeakers. But the fractional order state-space model presented in [21] can be used as a basis for nonlinear state-space models, and this work should be carried out in future.

Another point should be discussed is that this paper has used 6 parameters (M_{ms}, R_{ms}, C_0, C_1, β, τ) to model the mechanical impedance of the moving coil loudspeaker. One parameter more than FO model [12]. There are two parts of the loudspeaker suspensions, the inner part spider and the outer surround. The spider usually behaves as a spring in a loudspeaker. Therefore we use C0 to predict its elastic behaviors. The behaviors of the outer surround is more complicated, consequently we take a FO model to predict its viscoelastic behaviors as well as its memory effects like time varying behaviors [22]. Additionally, we will make a deep investigation on the two mentioned FO models in future.

6. Conclusion

This paper presented an equivalent electrical circuit for loudspeaker linear modeling based on the fractional lossy inductance model as well as the fractional creep model. The fractional transfer function between the output displacement with the input voltage was estimated by the modified Oustaloup filter method, and then the displacement predicting by the fractional equivalent electrical circuit was numerical calculated. By compared with the measured diaphragm displacement of a *Scanspeaker* woofer, the results find that: the displacement predicting by the fractional circuit model presented in this paper gives good correspondences with the measurements. The minor deviations in high frequency range may come from the lossy inductance predicted by Leach model.

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