


**COVER PAGE**

***Document downloaded by @DAEL***

***Sat Nov 28 04:33:31 2026***
***For personal use***

When automatic English translation is provided, only the original document is authentic.
The EAA cannot be held responsible of any translation error

Bibliographical reference

*Effects of the Casimir Force and Surface Elasticity on the Natural Frequencies of Cantilever AFM Probes*, J. Zou and X.-F. Li, Acta Acustica **vol. 104** (Number 1), 2018, pp. 87-93

DOI
https://doi.org/10.3813/AAA.919148

**Acta Acustica united with Acustica**
**Vol. 104 (2018) 87 – 93**

DOI 10.3813/AAA.919148

# Effects of the Casimir Force and Surface Elasticity on the Natural Frequencies of Cantilever AFM Probes

J. Zou[1)], X.-F. Li[1,2)]

[1)] School of Civil Engineering, Central South University, Changsha 410075, China. xfli@csu.edu.cn

[2)] State Key Laboratory of Powder Metallurgy, Central South University, Changsha 410083, PR China

**Summary**

Natural frequencies of cantilever probes are one of the most significant parameters in optimizing the performance of an atomic force microscope (AFM). In this article, a theoretical model based on a continuum mechanics approach is presented towards investigation of the natural frequencies for transverse vibration of a cantilever beam when taking the Casimir force and surface properties into account. The characteristic equation is derived. The effects of surface elasticity, residual surface tension and Casimir force on the natural frequencies of cantilever beams are examined. Obtained results might be helpful for the design of cantilever probes in AFM.

**PACS no. 43.40.Cw**

## 1. Introduction

Atomic force microscopes (AFMs) have been a very powerful instrument at producing high resolution topographic images in 3D on an atomic scale [1, 2, 3]. Extensive efforts have been dedicated to optimizing AFM with high sensitivity, high consistency, as well as fast response [4, 5]. For the purpose of fulfilling these requirements, it is crucial to design the mechanical components of the AFM to meet certain requirements. Because they are required to have resonant frequencies higher than both the external and internal vibration frequencies [6], the first frequency of AFM probes is one of the most significant parameters for their performances, attracting much attention of researchers [7]. The desire of increasing the resolution of surface images drives researchers to investigate various possible factors which could possibly impact the resonance frequency of cantilever probes.

Nano/micro-scale structural elements lead to a large surface area to volume ratio, which is recognized as a significant factor accounting for their size-dependent properties. In recent years, small-scale structures are broadly used in engineering applications and are also studied through the theory of surface elasticity. In particular, the influences of some factors such as flow damping, van der Waals force, Casimir force, surface energy, etc. become quite remarkable in affecting mechanical behavior including static bending, free vibration, and structural stability of nanowire-fabricated NEMS [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. All of the above have demonstrated that the utilization of classical continuum mechanics for nanostructures with tiny sizes is not suitable.

Theoretical modeling is a significant way of understanding the nanoscale mechanical behaviors of materials besides experimental methods, which might be technically difficult and financially expensive [23, 24]. Among various theoretical approaches, continuum mechanics modeling is very popular due to its low computational cost with simple formulations [25, 26]. In the past few decades, a considerable amount of work has been done to explore the dynamic behavior of an AFM [27, 28, 29]. Because of high surface to volume ratio, for micro- and nano-scale structures, their dynamic behavior is hardly satisfactorily explained according to classical linear theory of elasticity. As a consequence, the consideration of surface effects utilizing surface elasticity has been well developed based on a continuum mechanics approach [30, 31]. Based on the surface elasticity theory, Wang and Feng [32] proposed an analytical relation to include both the effect of residual surface tension and surface elasticity in vibration analysis of cantilever beams with thickness falling to microns or nanometers. They proposed an explicit solution for the calculation of natural frequencies of microbeams, and demonstrated that both surface elasticity and residual surface tension are significant effects on vibration frequency of micro/nanobeams. Later, their method was applied to resonant frequency and flutter instability of nanocantilever shaped sensor with attached mass [33]. It was found that resonant frequencies, divergence-type as well as flutter-type instability are all strongly affected by surface effects. Consideration of surface effects enhances apparent fracture toughness for crack growth at nanoscale [34]. Hu et al. recently gave a

Received 27 April 2017,
accepted 31 December 2017.

© S. Hirzel Verlag · EAA

theoretical analysis of a cracked thin plate with surface effects [35]. Moreover, surface effects have been proven to be critical in mechanical behaviors like buckling, bending and vibration [36, 37, 38, 39]. Sensitivity analysis of free vibration of AFM cantilever probes has been made as well [40, 41].

In practice, the vibration behavior of the AFM cantilever probe is strongly affected by the surrounding environment [3]. The Casimir force arises between two metal plates due to quantum fluctuations of the electromagnetic field, and it becomes significant in micromechanical systems at submicrometer scales [42]. When the distance between the cantilever tip and sample is sufficiently small, falling into nanoscale, the Casimir force cannot be ignored during vibration analysis [43] . During detection, movable elements of AFM-like cantilever probes may become unstable and stick to the sample due to strong attractive the Casimir force [44]. On the other hand, Chumak and coworkers [45] have demonstrated that fluctuating electromagnetic fields between the surface of the cantilever probe and the sample would induce attractive Casimir forces, which have a strong influence on the cantilever natural frequencies. Further, size effects on the buckling and vibration of a nanobeam subjected to external Casimir attractions were investigated [46, 47, 48, 49]. To date, however, there is little information on the effects of surface elasticity and residual surface tension incorporating the Casimir force on the natural frequencies. In this regard, surface effects and Casimir force-induced instability are both crucial for further improving the performance of AFM.

The present study focuses on the transverse vibration of the cantilever probe in AFM, by taking both surface effects and the Casimir forces into account. The vibration model is based on the Euler-Bernoulli beam theory. The effects of surface elasticity, residual surface tension, and the Casimir forces induced by the attraction between the surfaces of the cantilever probe tip and of the sample on the natural frequencies are analyzed in detail.

## 2. Statement of the problem

Here we consider an AFM probe, which is modeled as a cantilever of length $L$. Since the cantilever used in this article is a simulation of an AFM probe, the cantilever is supposed to be perpendicular to the surface of a flat sample and the cantilever tip is spherical or a section of cylindrical surface, as shown in Figure 1. Usually, if the oscillation amplitudes are small enough, the displacements of the cantilever tip are supposed to be just about parallel to surface of the sample.

For consideration of surface effects, we employ the theoretical model presented by Wang [32] to our cantilever probes. The cantilever is assumed to be divided into surface layers and internal block, and the surface layer is much smaller than the bulk block. Letting the thickness of the surface layer approach zero while keeping the surface elasticity modulus $E^S$ as a constant, the sandwich-beam model reduces to the theory of surface elasticity in

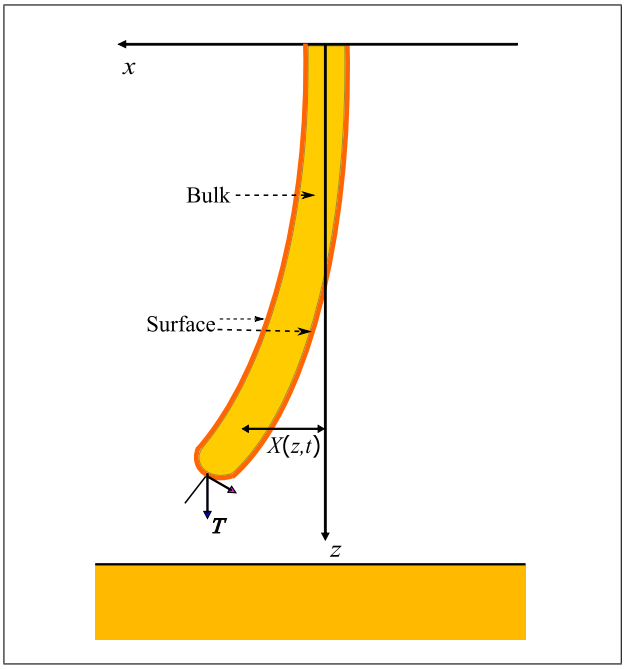


Figure 1. Diagram of a cantilever with attractive Casimir force acts along the $z$-axis.

which the surface possesses zero thickness [30]. In general, the effective bending moment of the cantilever beam is expressed as $EI + E^S J$, where $EI$ is the classical flexural rigidity ($E$: Young's modulus, $I$: the rotational moment of inertia of cross-sectional area), $E^S J$ is surface flexural rigidity induced by surface elasticity ($E^S$: surface elasticity modulus [50], $J$: the rotational moment of inertia of cross-sectional boundary [25]).

The residual surface tension induces a distributed traction on the beam surfaces under bending. The relationship between the curvature tensor $\kappa_{\alpha\beta}$ of a surface and the stress jump $\left\langle \sigma_{ij}^{+} - \sigma_{ij}^{-} \right\rangle$ across the surface is determined by

$$\left\langle \sigma_{ij}^{+} - \sigma_{ij}^{-} \right\rangle n_i n_j = \sigma_{\alpha\beta}^{S} \kappa_{\alpha\beta}, \tag{1}$$

where $\sigma_{ij}^{+}$ and $\sigma_{ij}^{-}$ denote the stresses on each side of the bending surfaces ($i, j = 1, 2, 3$), respectively, $n_i$ and $n_j$ represent the unit vector normal to each surface and $\sigma_{\alpha\beta}^{S}$ is the surface stress ($\alpha, \beta = 1, 2$). We assume that the two bending surfaces have the same residual surface tensions $\tau^0$. Thus, in the presence of the residual surface force, we apply the Young–Laplace equation and, omitting details, we get the distributed force $p$, namely

$$p = H \frac{\partial^2 X}{\partial z^2}, \tag{2}$$

where $X(z, t)$ represents deflection of the cantilever beam in the transverse direction along the $x$ axis, and

$$H = \begin{cases} 2a\tau^0 \text{ for rectangular cantilevers with width } a, \\ 2D\tau^0 \text{ for circular cantilevers with diameter } D. \end{cases} \tag{3}$$

From the above, we observe that the influence of residual surface tension is equivalent to a stretching axial loading at the free end of the cantilever.

As for the Casimir force, Chumak and coworkers [45] employed the "proximity approximation" method to derive the Casimir force under the condition of small separation $d$. Generally speaking, for good metals, it requires $d \ll 30nm$. For a cantilever tip the Casimir force takes the form

$$T(d) = \frac{R\hbar\omega_p}{32\sqrt{2}d^2} \tag{4}$$

for a spherically shaped tip, and

$$T(d) = \frac{3bR^{1/2}\hbar\omega_p}{2^8 d^{5/2}} \tag{5}$$

for a cylindrically shaped tip, where $R$ represents the radius of spherical tip or the radius of curvature of cylindrical tip, $\hbar$ is the Planck constant, and $\omega_p \approx 10^{16}s^{-1}$ is the plasma frequency for good metals. It is clear from equations (4) and (5) that the dependence of the Casimir force on separation $d$ is dominant, especially at small values of $d$. In the following analysis, it is reasonable that both $T$ and $H$ are taken as constants for small deformation analysis. The Casimir force between the cantilever tip and the sample is an attractive one, meaning the force acted on the probe tip has a direction perpendicular to and towards the sample (see Figure 1).

From above analysis, combining with the well–known Euler–Bernoulli theory of beams with the surface effects [51], including surface elasticity and residual surface tension, we get the basic differential equation for the vibration of a slender cantilever beam under an axial tensile Casimir force $T$,

$$\left(EI + E^S J\right)\frac{\partial^4 X}{\partial z^4} + \left(\rho S + \rho^S C\right)\frac{\partial^2 X}{\partial t^2} - T\frac{\partial^2 X}{\partial z^2} = p, \tag{6}$$

or in another form

$$\left(EI+E^S J\right)\frac{\partial^4 X}{\partial z^4}+\left(\rho S+\rho^S C\right)\frac{\partial^2 X}{\partial t^2}-\left(T+H\right)\frac{\partial^2 X}{\partial z^2} = 0, \tag{7}$$

where $\rho S + \rho^S C$ is the distributed mass per unit length ($\rho$: the mass density of the cantilever probe material, $S$: the cross-sectional area, $\rho^S$: the mass density of surface material, $C$: the boundary of cross-section). Note that the surface flexural rigidity could be either positive or negative, depending on the crystallographic structure of cantilever materials [52, 53].

## 3. Vibration of cantilever beams

In practice, of much interest are the natural frequencies of free vibration of cantilever beams. To this end, let us turn our attention to the analysis of this case and figure out the effect of the Casimir force and residual surface tension on the natural frequencies. Thus, appropriate boundary conditions for this case could be expressed as

$$X(0) = 0, \tag{8}$$

$$X'(0) = 0, \tag{9}$$

$$X''(L) = 0, \tag{10}$$

$$\left(EI + E^S J\right) X'''(L) - (T + H) X'(L) = 0, \tag{11}$$

where $X(0)$ and $X(L)$ are the deflection at $z = 0$ and $z = L$, respectively, and the prime denotes differentiation with respect to $z$. The term $(T + H) X'(L)$ represents the effect of restoring force at the condition when there is an angle between the Casimir force $T$ as well as the residual surface tension and the centroidal line at the tip of the cantilever. Note that the assumption is based on the condition of a very small oscillation angle.

By introducing $X = X_0 \cos\omega t$ to equation (7), we have

$$\left(EI + E^S J\right)X_0^{IV} - \omega^2\left(\rho S + \rho^S C\right)X_0 - \left(T + H\right)X_0'' = 0, \tag{12}$$

where $X_0$ is the amplitude, $\omega$ is the circular frequency. Now we introduce dimensionless quantities as

$$\xi = \frac{z}{L}, \qquad X(z) = Y(\xi), \tag{13}$$

$$\Gamma = L\sqrt{\frac{T + H}{EI + E^S J}}, \quad \Omega = L^2\omega\sqrt{\frac{\rho S + \rho^S C}{EI + E^S J}}, \tag{14}$$

from which we can transform equation (12) into a more simplified dimensionless version,

$$Y^{IV} - \Gamma^2 Y'' - \Omega^2 Y = 0. \tag{15}$$

Solving the above equation, one readily finds its solution to be

$$Y = C_1\cos\left(\lambda_1\xi\right) + C_2\sin\left(\lambda_1\xi\right) + C_3\cosh\left(\lambda_2\xi\right) + C_4\sinh\left(\lambda_2\xi\right), \tag{16}$$

where $C_1, C_2, C_3$ and $C_4$ are constants, and

$$\lambda_1 = \sqrt{\frac{\sqrt{\Gamma^4 + 4\Omega^2} - \Gamma^2}{2}}, \qquad \lambda_2 = \sqrt{\frac{\sqrt{\Gamma^4 + 4\Omega^2} + \Gamma^2}{2}}. \tag{17}$$

Next, substituting the general solution into the boundary conditions (8)–(11) gives

$$C_1 + C_3 = 0, \tag{18}$$

$$C_2\lambda_1 + C_4\lambda_2 = 0, \tag{19}$$

$$-C_1\lambda_1^2\cos\lambda_1 - C_2\lambda_1^2\sin\lambda_1 + C_3\lambda_2^2\cosh\lambda_2 + C_4\lambda_2^2\sinh\lambda_2 = 0, \tag{20}$$

$$-C_1\lambda_2\sin\lambda_1 + C_2\lambda_2\cos\lambda_1 - C_3\lambda_1\sinh\lambda_2 - C_4\lambda_1\cosh\lambda_2 = 0. \tag{21}$$

Since the above system admits a nontrivial solution, therefore the determinant of the above coefficient matrix of the system must equal zero. That is, by expanding the above determinant, we get the equation

$$2\Omega^2 + \Omega\Gamma^2\sin\lambda_1\sinh\lambda_2 + \left(\Gamma^4 + 2\Omega^2\right)\cos\lambda_1\cosh\lambda_2 = 0. \tag{22}$$

## 4. Results and discussion

The vibration behavior of beams when considering the Casimir force and surface elasticity could be very different owing to tremendous specific surface area change. To provide a general knowledge of how the vibration behavior differs among cantilever probes with different geometries and materials, three major factors are taken into consideration here, the Casimir force between the cantilever probe and the sample, surface elasticity of the cantilever probe, and the residual surface tension of the probe. As discussed in the previous section, all three factors are greatly dependent on various parameters and properties. As pointed out, the Casimir force is often positive. In the following calculation, the surface mass has been neglected due to its little influence. For convenience, we define the following non-dimensional parameters

$$T_0 = \frac{TL^2}{EI}, \quad E_0^S = \frac{E^S J}{EI}, \quad H_0 = \frac{HL^2}{EI}. \tag{23}$$

and we denote $\omega(0)$ as the fundamental circular frequency at $T_0 = E_0^S = H_0 = 0$ (i.e. the classical fundamental frequency). Then the normalized circular fundamental frequency $\omega(T_0)/\omega(0)$ against the Casimir force parameter $T_0 = TL^2/(EI)$ for various values of residual surface tension parameter $H_0 = HL^2/(EI)$ of a cantilever beam is illustrated in Figure 2. To eliminate the influence of surface elasticity, we assume $E_0^S = E^S J/(EI) = 0$. The graph for $H_0 = 0$ in fact shows the normalized first frequency of the cantilever beam using classical theory and without considering surface effects. The line of $H_0 = 0$ implies that Casimir force alone would give an increase to the cantilever first natural frequency. The larger the Casimir force is, the higher the natural frequency would be. Since the Casimir force is attractive, it would increase the natural frequencies as if the cantilever beam is pulled tightly by a tensile force at the tip. This conclusion is in agreement with experimental observation reported by Bressi [54], who directly measured the frequency shift arising from the effect of the Casimir force between parallel metallic surfaces. Similar theoretical observations have been reported in Ref. [45]. The value of residual surface tension $\tau^0$ could be either positive or negative, and its order of magnitude generally falls into the range $-2N/m$~$2N/m$ for metals [52, 53]. We selected two typical values of residual surface tension for comparison purposes. From Figure 2, it can be observed that as the Casimir force increases, $\omega(T_0)/\omega(0)$ increases correspondingly under all residual surface tension values. It implies that the presence of the Casimir force would increase the natural frequencies of the cantilever beam in all residual surface tension values. By comparing the increment or slope of three lines in Figure 2, we found that when residual surface tension is negative, the normalized fundamental frequency $\omega(T_0)/\omega(0)$ is smaller than that of none residual surface tension. On the contrary, when residual surface tension is positive, the normalized fundamental frequency $\omega(T_0)/\omega(0)$ is larger than that of no residual surface tension. Interestingly, the

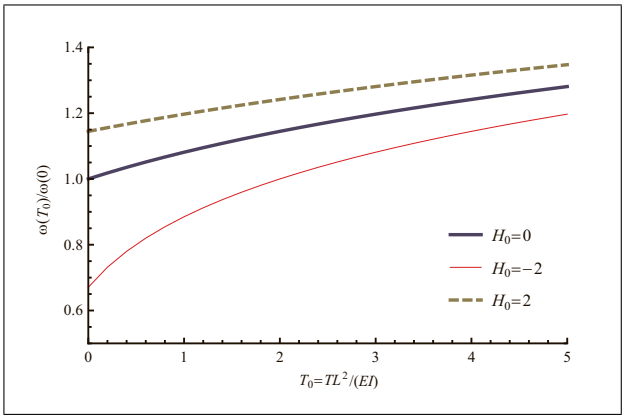


Figure 2. Effect of the Casimir force $T_0 = TL^2/(EI)$ on the normalized circular fundamental frequency $\omega(T_0)/\omega(0)$ for various values of residual surface tension parameter $H_0 = HL^2/(EI)$.

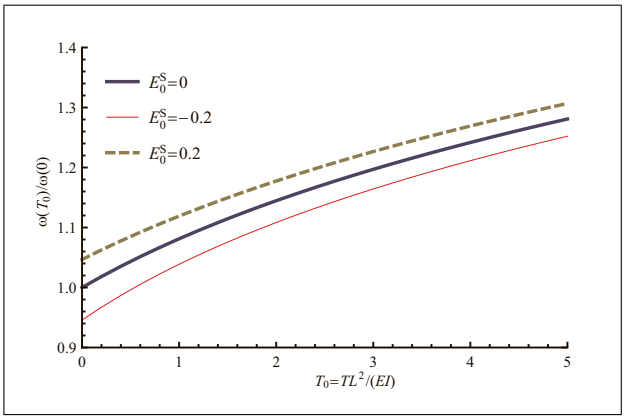


Figure 3. Effect of the Casimir force $T_0 = TL^2/(EI)$ on the normalized circular fundamental frequency $\omega(T_0)/\omega(0)$ for various values of surface elasticity parameter $E_0^S = E^S J/(EI)$.

difference of $\omega(T_0)/\omega(0)$ between $H_0 = 0$ and $H_0 = -2$ is relatively larger than the difference between $H_0 = 2$ and $H_0 = 0$. And the distinction between these two differences decreases as the Casimir force $T_0$ gets larger.

The effect of the Casimir force parameter $T_0$ on the normalized frequency ratio $\omega(T_0)/\omega(0)$ for a cantilever beam for different values of the surface elasticity parameter $E_0^S$ is shown in Figure 3 in the absence of residual surface tension ($H_0 = 0$). Surface elasticity may be either positive or negative, mainly depending on the crystallographic direction, respectively [52]. As shown in Figure 3, fundamental frequency $\omega(T_0)/\omega(0)$ increases as the Casimir force increases, no matter how surface elasticity differs. By comparing the slope of three lines in Figure 3, we found that when surface elasticity is positive, the ratio of $\omega(T_0)/\omega(0)$ is larger than that neglecting surface elasticity. However, when surface elasticity is negative, the ratio of $\omega(T_0)/\omega(0)$ is smaller than that without surface elasticity. This might be a bit trickier than our findings observed in Figure 2, whereas still apprehensible. From a mathematical point of view, from the expression for $\Gamma$ and $\Omega$ in equation (14) we understand that the value of

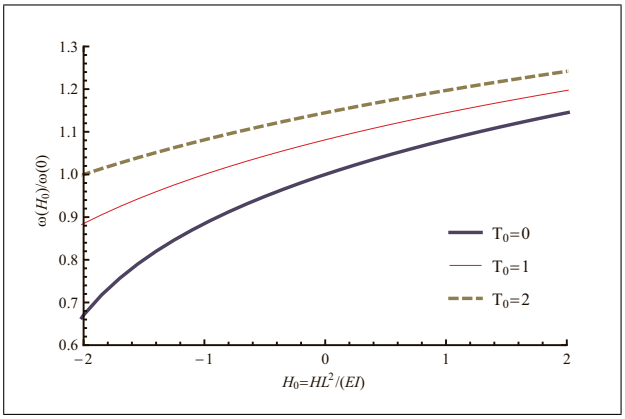


Figure 4. Effect of the residual surface tension parameter $H_0 = HL^2/(EI)$ on the normalized circular fundamental frequency $\omega(H_0)/\omega(0)$ for various values of the Casimir force $T_0 = TL^2/(EI)$.

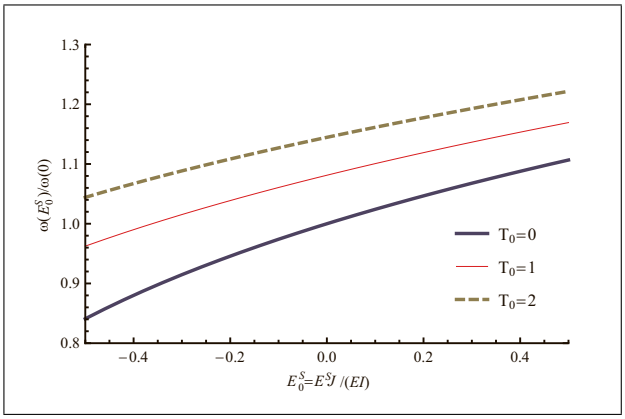


Figure 5. Effect of the surface elasticity parameter $E_0^S = E^S J/(EI)$ on the normalized circular fundamental frequency $\omega\left(E_0^S\right)/\omega(0)$ for various values of the Casimir force $T_0 = TL^2/(EI)$.

surface elasticity parameter $E_0^S$ could influence not only $\Gamma$ but also $\Omega$. This feature makes it different from Casimir force $T_0$ or residual surface tension $H_0$, which only have impact on $\Gamma$. This tends to dampen the overall influence of the Casimir force to $\Gamma$, while negative surface elasticity $E_0^S$ gives an increment influence of the Casimir force, respectively. From a physical point of view, a positive surface elasticity gives rise to the rigidity of the cantilever beam, thus increasing its fundamental frequency. At this point, we can learn from both Figures 2 and 3 that surface elasticity and residual surface tension have different patterns in their influence on the natural frequencies of the cantilever probe. The explanation lies in the difference of their physical meanings. Surface elasticity affects the stiffness of the cantilever probe, while residual surface tension acts more like a distributed force on the cantilever probe.

Figure 4 shows the effect of the residual surface tension parameter $H_0$ to the normalized fundamental frequency $\omega(H_0)/\omega(0)$ of the cantilever beam with or without considering the Casimir force. This graph is drawn in the absence of surface elasticity for clarity. Apparently, from Figure 4, under the conditions of whatever value of the Casimir force, when residual surface tension has a positive value, the natural frequencies are enhanced. In contrast, when residual surface tension has a negative value, the natural frequencies are dampened. Some similar observations are also reported in the literature, i.e. that tensile axial loading gave an increase on the natural frequencies of cantilever beams while compressive axial loading did the opposite [55]. The physical explanation could be explained as follows. A positive residual surface tension is equivalent to exerting a tensile force, while a negative surface tension is equivalent to exerting a compressive force. A comparison of the curves corresponding to $T_0 = 0, 1, 2$ indicates that when Casimir forces are present, they have positive impact on the fundamental frequency of cantilever probes.

Figure 5 shows the effect of surface elasticity parameter $E_0^S$ on the normalized fundamental frequency

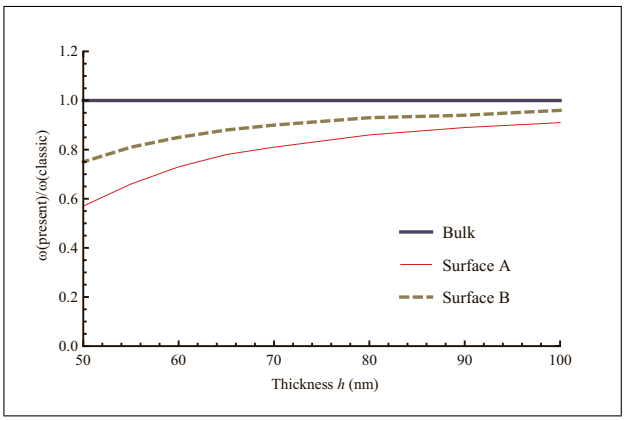


Figure 6. Variation of the present fundamental frequency to the classic one against the thickness $h$ of a rectangular silver nanowire.

$\omega\left(E_0^S\right)/\omega(0)$ of cantilever beams considering the Casimir force, but with no residual surface tension. It can be seen from Figure 5 that within a reasonable range of the Casimir force values, vibrational behavior or frequency of the cantilever beam is retarded, as surface elasticity gets larger. Although the trend seems to be similar compared to which shown in Figure 4, the physical principles behind are different. Let us look at the curve corresponding to $T_0 = 0$ without Casimir force. Obviously, negative surface elasticity reduces the rigidity of a cantilever beam, and positive surface elasticity increases the rigidity. As a result, the cantilever beam has higher frequencies with rising rigidity, but lower frequencies with decreasing rigidity.

For the purpose of verification, the present theoretical numerical model is compared with classical model. As an example, we now consider a silver rectangular cantilever nanowire. The parameters used are as follows: $E = 76$ GPa, $L = 100\,\mu$m, $a = 100$ nm, $\rho = 10.49\,\mathrm{g/cm^3}$. Based on atomic simulations [53], surface properties are taken $E^S = 1.22$ N/m, $\tau^0 = 0.89$ N/m on the surface A of (001) orientation, and $E^S = -1.39$ N/m, $\tau^0 = 0.65$ N/m on the surface B of (111) orientation. As seen in Figure 6, in the

absence of the Casimir force and surface effects, the fundamental frequency ratio of the present model to the classic one is in close proximity to unity. Meanwhile, when surface effect is taken into account, Figure 6 shows the fundamental frequency of a silver cantilever nanowire as a function of the nanowire thickness $h$. Evidently, the influence of surface effects are more prominent for a slender nanowire with a smaller thickness.

To recapitulate, the Casimir force in case of small separation would come into play on the mechanical behavior of cantilever beams at nano-micro scale. From a physical point of view, the Casimir force is served as a compressive or pulling axial loading. Surface elasticity acts as if it has the ability of changing the stiffness of the cantilever probe, and surface tension has a similar role as a distributed force on the cantilever probe surface. Evidently, these three factors would interfere with each other, as discussed in our work. In practice, establishing the influence of these nanoscale effects on the natural frequency of nano-micro cantilevers is a necessity for developing and constructing reliable nano-devices.

## 5. Conclusions

In this article, the vibration behavior of the cantilever probe in AFM has been studied based on the Euler-Bernoulli beam theory incorporating surface effects and the Casimir force. The effects of surface elasticity, residual surface tension and the Casimir force on the natural frequencies have been analyzed in detail. According to the results, attractive Casimir force could increase the fundamental natural frequencies of the cantilever probe. Positive residual surface tension increases the natural frequencies and negative does the opposite. Similarly, positive surface elasticity increases the natural frequencies, and vice versa. Obtained results might be helpful to aid the design of cantilever probes for AFM and related measuring equipment. In order to optimize the fundamental natural frequency of a cantilever probe in nanoscale, the influence of residual surface tension, surface elasticity, and the Casimir force should be taken into account.

### Acknowledgement

This work was supported by the National Natural Science Foundation of China (No. 11672336) and by State Key Laboratory of Powder Metallurgy, Central South University, China.

### References

[1] M. Ashhab, M. V. Salapaka, M. Dahleh, I. Mezic: Dynamical analysis and control of microcantilevers. Automatica **35** (1999) 1663–1670.

[2] H. J. Butt, B. Cappella, M. Kappl: Force measurements with the atomic force microscope: Technique, interpretation and applications. Surf. Sci. Rep. **59** (2005) 1–152.

[3] N. Gadegaard: Atomic force microscopy in biology: technology and techniques. Biotech. Histochem. **81** (2006) 87–97.

[4] J. R. Lozano, R. Garcia: Theory of multifrequency atomic force microscopy. Phys. Rev. Lett. **100** (2008) 076102.

[5] B. Arash, J.-W. Jiang, T. Rabczuk: A review on nanomechanical resonators and their applications in sensors and molecular transportation. Appl. Phys. Rev. **2** (2015) 021301.

[6] C. Kim, J. Jung, W. Youm, K. Park: Design of mechanical components for vibration reduction in an atomic force microscope. Rev. Sci. Instru. **82** (2011) 035102.

[7] T. E. Schaffer, H. Fuchs: Optimized detection of normal vibration modes of atomic force microscope cantilevers with the optical beam deflection method. J. Appl. Phys. **97** (2005) 083524.

[8] Y. Tadi Beni, A. Koochi, A.S. Kazemi, M. Abadyan: Modeling the influence of surface effect and molecular force on pull-in voltage of rotational nano–micro mirror using 2-DOF model. Can. J. Phys. **90** (2012) 963–974.

[9] M. Keivani, A. Koochi, N. Abadian, M. Rezaei, M. Abadyan: Static and dynamic instability of nanowire-fabricated NEMS: effects of flow damping, van der Waals force, surface energy and microstructure. Can. J. Phys. **94** (6) (2016) 594–603.

[10] M Keivani, M. Mardaneh, A. Koochi, M Rezaei, M. Abadyan: On the dynamic instability of nanowire-fabricated electromechanical actuators in the Casimir regime: coupled effects of surface energy and size dependency. Physica E **76** (2016) 60–69.

[11] A. Koochi, H. Hosseini-Toudeshky, M. Abadyan: Nonlinear beam formulation incorporating surface energy and size effect: application in nano-bridges. Appl. Math. Mech. **37** (2016) 583–600.

[12] R. Ansari, R. Gholami: Size-dependent modeling of the free vibration characteristics of postbuckled third-order shear deformable rectangular nanoplates based on the surface stress elasticity theory. Compos. Part B: Eng. **95** (2016) 301–316.

[13] R. Ansari, R. Gholami, M. Faghih Shoejaei, V. Mohammadi, S. Sahmani: Surface stress effect on the pull-in instability of circular nanoplates. Acta Astron. **118** (2016) 72–89.

[14] R. Ansari, R. Gholami, A. Norouzzadeh, M. A. Darabi: Wave Characteristics of Nanotubes Conveying Fluid Based on the Non-classical Timoshenko Beam Model Incorporating Surface Energies. Arab. J. Sci. Eng. **41** (11) (2016) 4359–4369.

[15] R. Ansari, A. Norouzzadeh, R. Gholami, M Faghih Shojaei, M. A. Darabi: Geometrically nonlinear free vibration and instability of fluid-conveying nanoscale pipes including surface stress effects. Microfluid. Nanofluid. **20** (2016) 1–14.

[16] R. Ansari, R. Gholami, A. Norouzzadeh, M. A. Darabi: Surface stress effect on the vibration and instability of nanoscale pipes conveying fluid based on a size-dependent Timoshenko beam model. Acta Mech. Sin. **31** (2015) 708–219.

[17] R. Ansari, R. Gholami, M. F. Shojaei, V. Mohammadi, S. Sahmani: Surface stress effect on the pull-in instability of circular nanoplates. Acta Astron. **102** (102) (2014) 140–150.

[18] H. Rouhi, R. Ansari, M. Darvizeh: Size-dependent free vibration analysis of nanoshells based on the surface stress elasticity. Appl. Math. Model. **40** (2016) 3128–3140.

[19] H. Rouhi, R. Ansari, M. Darvizeh: Analytical treatment of the nonlinear free vibration of cylindrical nanoshells based on a first-order shear deformable continuum model including surface influences. Acta Mech. **227** (2016) 1767–1781.

[20] H. Rouhi, R.Ansari, M. Darvizeh: Nonlinear free vibration analysis of cylindrical nanoshells based on the Ru model accounting for surface stress effect. Int. J. Mech. Sci. **113** (2016) 1–9.

[21] X.-F. Li, G.-J. Tang, Z.-B. Shen, K. Y. Lee: Size-dependent resonance frequencies of longitudinal vibration of a nonlocal Love nanobar with a tip nanoparticle. Math. Mech. Solids. **22** (2017) 1529–1542.

[22] X.-F. Li, Z.-B. Shen, K. Y. Lee: Axial wave propagation and vibration of nonlocal nanorods with radial deformation and inertia. ZAMM **97** (2017) 602–616.

[23] M. R. Falvo, G. J. Clary, n. Taylor, R. M., V. Chi, J. Brooks, F. P., S. Washburn, R. Superfine: Bending and buckling of carbon nanotubes under large strain. Nature **389** (1997) 582–584.

[24] K. F. Wang, B. L. Wang, T. Kitamura: A review on the application of modified continuum models in modeling and simulation of nanostructures. Acta Mech. Sin. **32** (2016) 83–100.

[25] J. He, C. M. Lilley: Surface effect on the elastic behavior of static bending nanowires. Nano Lett. **8** (2008) 1798–1802.

[26] Y. Yao, S. Chen: Surface effect on resonant properties of nanowires predicted by an elastic theory for nanomaterials. J. Appl. Phys. **118** (2015) 044303.

[27] R. F. Fung, S. C. Huang: Dynamic modeling and vibration analysis of the atomic force microscope. Vib. Acous. **123** (2001) 502–509.

[28] A. Sadeghi: The flexural vibration of V-shaped atomic force microscope cantilevers by using the Timoshenko beam theory. ZAMM **92** (2012) 782–800.

[29] E. Rezaei, J. A. Turner: Free vibrations of u-shaped atomic force microscope probes. J. Appl. Phys. **115** (2014) 174302.

[30] M. E. Gurtin, A. I. Murdoch: Continuum theory of elastic-material surfaces. Arch. Ration. Mech. Anal. **57** (1975) 291–323.

[31] H. Ibach: The role of surface stress in reconstruction, epitaxial growth and stabilization of mesoscopic structures. Surf. Sci. Rep. **29** (1997) 195–263.

[32] G.-F. Wang, X.-Q. Feng: Effects of surface elasticity and residual surface tension on the natural frequency of microbeams. Appl. Phys. Lett. **90** (2007) 231904.

[33] X. F. Li, J. Zou, S. N. Jiang, K. Y. Lee: Resonant frequency and flutter instability of a nanocantilever with the surface effects. Compos. Struct. **153** (2016) 645–653.

[34] H. Wang, X. F. Li, G. J. Tang, Z. B. Shen: Effect of surface stress on stress intensity factors of a nanoscale crack via double cantilever beam model. J. Nanosci. Nanotechnol. **13** (2013) 477–482.

[35] Z.-L. Hu, K. Y. Lee, X.-F. Li: Crack in an elastic thin-film with surface effect. Int. J. Eng. Sci. **123** (2018) 158-173.

[36] P. A. T. Olsson, H. S. Park: On the importance of surface elastic contributions to the flexural rigidity of nanowires. J. Mech. Phys. Solids **60** (2012) 2064–2083.

[37] X.-F. Li, H. Zhang, K. Y. Lee: Dependence of Young's modulus of nanowires on surface effect. Int. J. Mech. Sci. **81** (2014) 120–125.

[38] K. Kiani: Axial buckling analysis of a slender current-carrying nanowire acted upon by a magnetic field using the surface energy approach. J. Phys. D: Appl. Phys. **48** (2015) 245302.

[39] J.-X. Wu, X.-F. Li, A.-Y. Tang, K. Y. Lee: Free and forced transverse vibration of nanowires with surface effects. J. Vib. Contr. **23** (2017) 2064–2077.

[40] J. A. Turner, J. S. Wiehn: Sensitivity of flexural and torsional vibration modes of atomic force microscope cantilevers to surface stiffness variations. Nanotechnology **12** (2001) 322–330.

[41] W. J. Chang: Sensitivity of vibration modes of atomic force microscope cantilevers in continuous surface contact. Nanotechnology **13** (2002) 510–514.

[42] A. W. Rodriguez, F. Capasso, S. G. Johnson: The Casimir effect in microstructured geometries. Nat. Photonics **5** (2011) 211–221.

[43] S. K. Lamoreaux: The Casimir force: background, experiments, and applications. Rep. Prog. Phys. **68** (2005) 201–236.

[44] A. Farrokhabadi, N. Abadian, R. Rach, M. Abadyan: Theoretical modeling of the Casimir force-induced instability in freestanding nanowires with circular cross-section. Physica E **63** (2014) 67–80.

[45] A. A. Chumak, P. W. Milonni, G. P. Berman: Effects of electrostatic fields and casimir force on cantilever vibrations. Phys. Rev. B **70** 085407.

[46] A. Koochi, A. S. Kazemi, Y. T. Beni, A. Yekrangi, M. Abadyan: Theoretical study of the effect of Casimir attraction on the pull-in behavior of beam-type NEMS using modified Adomian method. Physica E **43** (2010) 625–632.

[47] Y. T. Beni, M. Abadyan, A. Koochi: Effect of the casimir attraction on the torsion/bending coupled instability of electrostatic nano-actuators. Physica Scripta **84** (2011) 065801.

[48] N. Kacem, S. Baguet, S. Hentz, R. Dufour: Computational and quasi-analytical models for non-linear vibrations of resonant MEMS and NEMS sensors. Int. J. Non Linear Mech. **46** (2011) 532–542.

[49] B. Y. Tadi, A. Koochi, M. Abadyan: Theoretical study of the effect of Casimir force, elastic boundary conditions and size dependency on the pull-in instability of beam-type NEMS. Physics E **43** (2011) 979–988.

[50] M. E. Gurtin, X. Markenscoff, R. N. Thurston: Effect of surface stress on natural rrequency of thin crystals. Appl. Phys. Lett. **29** (1976) 529–530.

[51] G. F. Wang, X. Q. Feng: Surface effects on buckling of nanowires under uniaxial compression. Appl. Phys. Lett. **94** (2009) 149.

[52] R. E. Miller, V. B. Shenoy: Size-dependent elastic properties of nanosized structural elements. Nanotechnology **11** (2000) 139–147.

[53] V. B. Shenoy: Atomistic calculations of elastic properties of metallic fcc crystal surfaces. Phys. Rev. B **71** (2005) 4104.

[54] G. Bressi, G. Carugno, R. Onofrio, G. Ruoso: Measurement of the casimir force between parallel metallic surfaces. Phys. Rev. Lett. **88** 041804.

[55] X. F. Li, A. Y. Tang, L. Y. Xi: Vibration of a Rayleigh cantilever beam with axial force and tip mass. J. Constr. Steel Res. **80** (2013) 15–22.

[56] Y. Feng, Y. Liu, B. Wang: Finite element analysis of resonant properties of silicon nanowires with consideration of surface effects. Acta Mech. **217** (2011) 149-155.