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Labyrinthine Acoustic Metamaterials with Space-Coiling Channels for Low-Frequency Sound Control

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Summary

We numerically analyze the performance of labyrinthine acoustic metamaterials with internal channels folded along a Wunderlich space-filling curve to control low-frequency sound in air. In contrast to previous studies, we perform direct modeling of wave propagation through folded channels without introducing effective theory assumptions. We reveal that metastructures with channels that allow wave propagation in the opposite direction to incident waves, have different dynamics as compared to those for straight slits of equivalent length. These differences are attributed to tortuosity effects and result in 100% wave reflection at band gap frequencies. This total reflection phenomenon is found to be insensitive to thermo-viscous dissipation in air. For labyrinthine channels generated by recursive iteration levels, one can achieve broadband total sound reflection by using a metamaterial monolayer, and efficiently control the amount of absorbed wave energy by tuning the channel width. Thus, the work contributes to a better understanding of labyrinthine metamaterials with potential applications for reflection and filtering of low-frequency airborne sound.

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1. Introduction

Acoustic metamaterials are composites with an engineered structure providing remarkable functionalities, e.g. acoustic cloaking, transformation acoustics, and subwavelength-resolution imaging [1, 2]. Apart from unusual effective properties, metamaterials offer various possibilities to control propagation of sound or elastic waves at deep sub-wavelength scales [3, 4, 5]. This can be achieved by incorporating heavy resonators [3], Helmholtz resonators [6, 7], tensioned membranes [8, 9], and sub-wavelength perforations or slits [10, 11, 12, 13] in a material structure. A class of acoustic metamaterials with internal slits is also known as “labyrinthine”. These have recently attracted considerable attention due to their abilities to exhibit an exceptionally high refractive index and to efficiently reflect sound waves, while preserving light weight and compact dimensions [12, 13, 14].

Labyrinthine metamaterials enable to slow down the effective speed of acoustic waves due to path elonga-

tion by means of folded narrow channels [13, 15]. Their high efficiency in manipulating low-frequency sound has been experimentally demonstrated for various channel geometries. For example, Xie *et al.* have shown the existence of a negative effective refractive index at broadband frequencies for labyrinthine metastructures with zig-zag-type channels [16]. For the same configuration, Liang *et al.* have demonstrated extraordinary dispersion, including negative refraction and conical dispersion for low-frequency airborne sound [15]. Frenzel *et al.* have used the zig-zag channels to achieve broadband sound attenuation by means of three-dimensional labyrinthine metastructures [17, 18]. The issue of poor impedance matching for labyrinthine metamaterials has been addressed by exploiting tapered and spiral channels [19] and hierarchically structured walls [20]. Cheng *et al.* have proven almost perfect reflection of low-frequency sound by sparsely arranged unit cells with circular-shaped channels that support generation of strong artificial subwavelength Mie resonances [12]. In our previous work, we have proposed a simple modification to the latter design (by adding a square frame) to achieve a wider bandwidth tunability

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[14]. Moleron *et al.* have emphasized the importance of thermo-viscous effects on the performance of labyrinthine structures with sub-wavelength slits [21].

Most of the mentioned studies analyze labyrinthine metamaterials with curved channels by replacing a real material structure with a simplified one. In a simplified configuration, wave propagation in folded channels is described by the dynamics of straight slits of an effective length, which equals to the shortest path taken by a wave within the structure [13, 15, 17, 20, 21]. This approach provides reliable results for channels, in which wave propagation direction does not deviate much from that of incident waves. Hence, it appears that the channel tortuosity plays no role. Possible effects of the path tortuosity, e.g. when a wave is allowed to propagate in the opposite direction relative to the incident wave field, remain to be investigated. A limited number of papers have analyzed labyrinthine metamaterials of this type.

In [19], Xie *et al.* investigated metastructures with spiral channels to introduce tunability of effective structural parameters, such as refractive index and impedance. Song *et al.* considered hierarchically organized walls to achieve a broadband wave absorption [20]. These works are mainly focused on the experimental validation of the mentioned features, and lack a theoretical analysis of wave behavior in a tortuous channel.

The goal of this work is to numerically investigate dispersion and propagation properties of airborne sound in labyrinthine metamaterials with channels that allow a change in the direction of wave propagation, and to compare their performance with that of the corresponding straight slits. For this purpose, we design sub-wavelength paths in metamaterial unit cells along a hierarchically-organized curve. In particular, we consider a space-filling curve with self-similar organization, and a simple algorithm to derive length elongation. We perform a complete theoretical analysis of the wave dispersion in the designed metamaterials complemented by the study of acoustic transmission, reflection, and absorption for a single slab in the absence or presence of thermo-viscous losses. Results show that the proposed metamaterials have a great potential as efficient reflectors for low-frequency airborne sound. Moreover, to facilitate their practical exploitation, we propose to assemble reconfigurable structures from thin panels of constant thickness (sheets), providing an inexpensive alternative to an additive manufacturing approach.

2. Space-filling curves

As mentioned above, the wave path in the designed labyrinthine metamaterials can be elongated along space-filling curves [22]. These were first described by Peano [23] (later named after him), and since then many other curves were proposed [24]. An attractive property of these curves is that they go through every point of a bounding domain provided an unlimited number of iterations is assumed. After initially being studied as a curiosity, nowa-

days space-filling curves are widely applied, e.g. for indexing of multi-dimensional data [25], transactions and disk scheduling in advanced databases [26], building routing systems [27], etc.

Among various curves, we have chosen the Wunderlich two-dimensional curve filling a square [22]. It can be constructed as follows. At the 1st iteration level, one draws an “S”-shaped curve starting at the bottom-left corner of a bounding square and ending at the top-right corner. At the n th ($n \geq 2$) iteration level, 3 copies of the $(n - 1)$ th-level curve are arranged along each side of a square with every copy being rotated by 90° relative to the previous one. The curves are joined into an S-shaped route starting from the up-direction for the left column, then down for the middle column, and finally again up for the right column. At every n th iteration level, the length of the Wunderlich curve is $(3^n - 1/3^n)$, while that of e.g. Hilbert’s curves is $(2^n - 1/2^n)$ [22]. Such fast length elongation enables more compact channel folding in a labyrinth (and thus, increases the tortuosity effect), justifying the choice of the Wunderlich curve for this study.

3. Models and methods

Figure 1 presents labyrinths of a square form with internal channels shaped along the Wunderlich curves of the three iteration levels. These are used to construct “unit cell 1” (UC1), “unit cell 2” (UC2), and “unit cell 3” (UC3), respectively. The structural material is aluminum with mass density $\rho_{Al} = 2700 \text{ kg/m}^3$ and speed of sound $c_{Al} = 5042 \text{ m/s}$. The thickness of bounding walls is fixed for all the unit cells and equals $d = 0.5 \text{ mm}$.

The channel width is w , and the size of a square domain occupied by a single labyrinth is $a = 3^N(w + d) + d$, where N is the iteration level. We preserve an interconnecting cavity of width w between adjacent labyrinths. Thus, the metamaterial unit cell size is $a_{uc} = a + w$ (see Figure 1a for notations).

We analyze plane waves propagating in the plane of a unit cell cross-section. The metamaterial geometry is assumed to be constant in an out-of-plane direction without a possibility to excite a momentum in this direction. Hence, the pressure field is always constant in the out-of-plane direction, and the wave dynamics is two-dimensional (2D). The validity of this assumption is confirmed by a good agreement by using the results of three-dimensional (3D) simulations given further in the Section 4.

First, we analyze sound wave dispersion in the labyrinthine metamaterials infinitely extending in both in-plane directions. By neglecting any losses in air, small-amplitude variations of harmonic pressure $p(\mathbf{x}, t) = p(\mathbf{x})e^{i\omega t}$ (with angular frequency $\omega = 2\pi f$, where f indicates the frequency in Hz) are governed by the homogeneous Helmholtz equation,

$$\nabla \cdot \left(-\frac{1}{\rho_0} \nabla p \right) - \frac{\omega^2 p}{\rho_0 c_0} = 0, \quad (1)$$

with air density $\rho_0 = 1.225 \text{ kg/m}^3$ and speed of sound $c_0 = 343 \text{ m/s}$ at a temperature of $T = 20^\circ \text{C}$. Since the

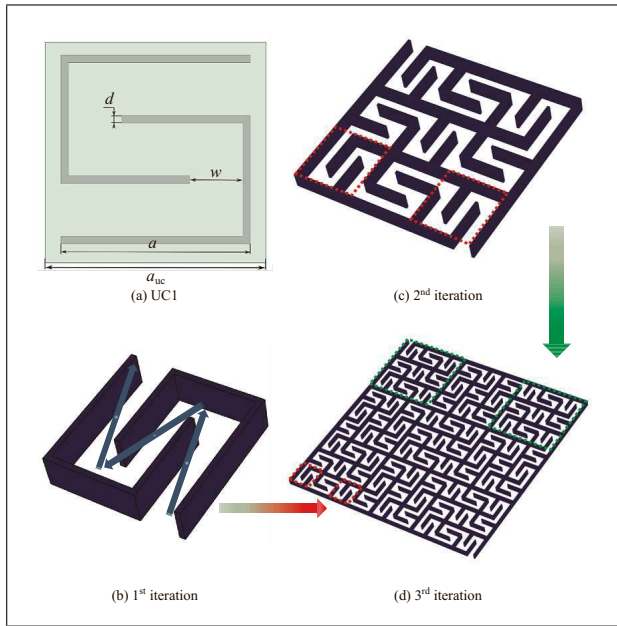


Figure 1. (Colour online) (a) Unit cell at the 1st iteration level (UC1) and the corresponding dimensions. (b-d) Labyrinths with air channels shaped according to the Wunderlich space-filling curve. The first three iteration levels are indicated as UC1, UC2, and UC3. Solid walls are shown in dark blue. The shortest path taken by a wave within UC1 is indicated by blue arrows in (b).

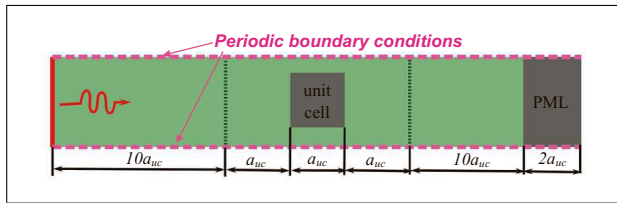


Figure 2. (Colour online) Schematic of the frequency domain model. Green area corresponds to an air domain, green dashed lines indicate locations, at which reflection and transmission coefficients are evaluated. The plane wave radiation condition is applied along the bold red line.

characteristic acoustic impedance of aluminum is around 4 orders of magnitude larger than that of air, we assume zero displacements for the structural walls and apply sound-hard boundary conditions at air-structure interfaces. The pressure distribution at opposite unit cell boundaries is constrained by the Floquet-Bloch periodic conditions,

$$p(\mathbf{x} + \mathbf{a}) = p(\mathbf{x})e^{i\mathbf{k} \cdot \mathbf{a}}, \quad (2)$$

with $\mathbf{q} = (a_{uc}, a_{uc}, 0)$ and wave vector $\mathbf{k} = (k_x, k_y, 0)$. More details about the dispersion analysis can be found in [14].

Next, we evaluate homogeneous wave propagation through a metamaterial monolayer. A sketch of the model is presented in Figure 2. Plane wave radiation occurs at the left domain boundary at a distance of $10a_{uc}$ from the slab. At the right boundary, a perfectly matched layer of width $2a_{uc}$ is added to eliminate unwanted wave reflection. At the bottom and top boundaries, the Floquet-Bloch

periodic boundary conditions (2) enable to infinitely extend the air domain in the vertical direction. The reflection $R = |p_r/p_i|^2$, transmission $T = |p_t/p_i|^2$, and absorption $A = 1 - R - T$ coefficients are evaluated by averaging incident p_i , reflected p_r , and transmitted p_t pressure fields along the lines located at a distance a_{uc} from the metastructure.

In order to understand how the tortuosity of a labyrinthine channel influences sound wave characteristics, we compare the evaluated T and A values for the metastructures with those for straight slits of width w and length $L = L_{eff}$ or $L = a_{uc}$, which are distributed at distances a along the vertical direction. In the case of $L = a_{uc}$, the slits are located between silod blocks of the same size as labyrinthine structures, while the latter do not contain any internal channels. The effective channel length L_{eff} is approximately equal to the shortest wave path from the input to the output through a labyrinthine channel (as shown e.g. by light-blue lines in Figure 1b).

If the channel width is small compared to the wavelength of a propagating wave, thermal and viscous boundary layers near walls cause loss effects (*lossy air*). The thickness of these layers decreases with increasing frequency. The thickness of the thermal boundary layer δ_{th} is evaluated as

$$\delta_{th} = \sqrt{\frac{k}{\pi f \rho_0 C_p}}, \quad (3)$$

where $k = 25.8 \text{ mW/(m K)}$ is the thermal conductivity, and $C_p = 1.005 \text{ kJ/(m}^3 \text{ K)}$ is the heat capacity at constant pressure. The thickness of the viscous boundary layer δ_{vis} is

$$\delta_{vis} = \sqrt{\frac{\mu}{\pi f \rho_0}}, \quad (4)$$

with dynamic viscosity $\mu = 1.814 \cdot 10^{-5} \text{ Pa s}$. The graphical representation of Equations (3)–(4) is given in Figure 3. At 20°C and 1 atm, the viscous and thermal boundary layers are of thickness 0.22 mm and 0.26 mm at 100 Hz, respectively.

As the designed labyrinthine channels are relatively easy to model, we directly include thermal conduction and viscous attenuation into the computational model. Thus, the linearized system consists of a linearized Navier-Stokes equation, the continuity equation, and the energy equation given in [28]. This system is solved for acoustic pressure variations p , the fluid velocity variations $\mathbf{u}$, and temperature variations T . The variations describe small harmonic oscillations around a steady state. The mentioned equations are implemented in the Thermoacoustic interface of Comsol Multiphysics [29].

The dispersion and transmission studies are implemented as eigenvalue and frequency-domain finite-element simulations. The described acoustic domains are discretized with the maximum element size of $\lambda_{min}/12$, where $\lambda_{min} = c_0/f_{max}$, and f_{max} is the maximum considered frequency. Such a mesh resolves the smallest wavelength of the study with 12 elements. To properly capture

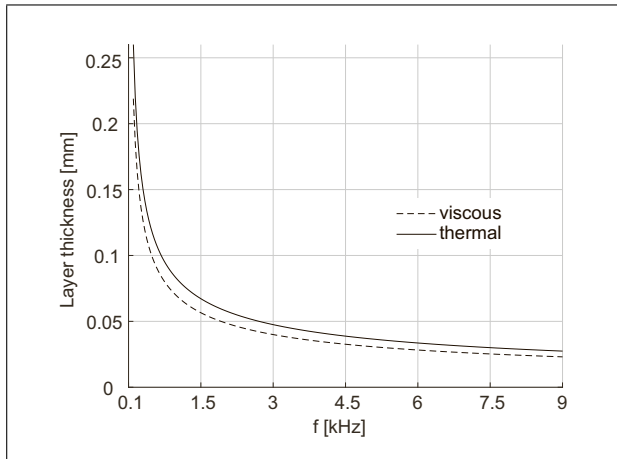


Figure 3. Thickness of viscous and thermal boundary layers according to relations (3) and (4).

the wave field variations within the viscous and thermal boundary layers, we implemented a frequency-varying mesh with 3–5 boundary layers along the thickness of the viscous layer.

4. Results and discussion

We consider labyrinthine metamaterials of two structural sizes. In the first case, defined as a “fixed channel” case, we consider a constant channel width, $w = \text{const}$, at each iteration step. Thereby, we aim to evaluate effects of tortuosity on sound propagation in elongated paths. For $w = 4$ mm, the metamaterial unit cell sizes are $a_{\text{uc}} = 18$ mm for UC1, 45 mm for UC2, and 126 mm for UC3. For the second case, indicated as “fixed unit cell” case, we assume a fixed unit cell size, $a_{\text{uc}} = \text{const}$, with the channel width becoming smaller at each iteration. In particular, we fix $a_{\text{uc}} = 14$ mm that corresponds to the channel width $w = 3$ mm for UC1 and 0.9 mm for UC2. For UC3, the internal channel disappears for the specified wall thickness $d = 0.5$ mm. The channel width in the “fixed unit cell” case is smaller than that in the “fixed channel” case at the same iteration level. Thus, by comparing wave propagation in these two cases, we can evaluate how different amounts of thermo-viscous losses influences the wave dynamics in labyrinthine channels of the same structure.

In both cases, labyrinthine channels are shaped according to the Wunderlich curve. However, the channel length is scaled differently than that of the Wunderlich fractal curve due to deviations in construction approaches. Specifically, when constructing a fractal curve, one assumes that it is a mapping from a low-dimensional space into a 2D domain, the area of which is fixed for all iteration levels [22]. In contrast to this, for our unit cells, we assume a constant wall thickness that implies variations in the channel length relative to that of the Wunderlich curve. Hence, in the “fixed channel” case, when the area of a bounding square increases at each iteration step, the channel length is elongated by a factor of 3^N relative to a .

In the “fixed unit cell” case, the elongation factor equals $3^N a - 1$.

4.1. “Fixed-channel” case

Figure 4 shows evaluated dispersion relations for homogeneous waves in UC1, UC2, and UC3 propagating along ΓX direction in k -space. The horizontal axis indicates normalized wavenumber $k^* = a_{\text{uc}} k / \pi$, and the vertical axes represent frequencies f in kHz and normalized frequencies $f^* = f a_{\text{uc}} / c_0$. Note the different frequency ranges for each unit cell. The frequencies are limited to a sub-wavelength range, namely up to $f a_{\text{uc}} / c_0 = 0.5$. For UC1, we consider modes forming the lowest band gap separated into two parts and extending up to 9 kHz. For the UC2 and UC3, the frequency range includes the first 4 separated band gaps, and thus, are limited to 4 kHz and 500 Hz, respectively.

The dash-dot lines represent phase velocities of the lowest fundamental mode in lossless air within a unit cell (green curve) and in homogeneous air, when a unit cell is removed (red curve). As can be expected, the velocity is reduced when a wave propagates through a labyrinthine channel. The reduction factor is 1.63 (UC1), 2.91 (UC2), and 5.28 (UC3) compared to homogeneous air.

The dispersion relations in Figure 4 are characterized by several frequency band gaps in the sub-wavelength region. Hence, the designed labyrinthine metamaterials can control sound waves at sub-wavelength scales. As N increases, the band gaps are shifted down to lower frequencies. The shifts are directly related to the path elongation. For example, the 1st band gap starting from $f a / c_0 = 0.21$ for UC1, is shifted to about a 3 times lower frequency, $f a / c_0 = 0.069$, for UC2, as the channel length in UC2 is 3 times longer than that in UC1.

The band-gap bounds are formed by flat parts of dispersion bands that correspond to localized modes. The pressure distributions for these modes are given in the 1st and 3rd columns of Table I for the 1st band gap bounds and Table II for the 2nd and 3rd band gap bounds. Red and blue colors represent maximum and minimum values of pressure, while green color indicates near-zero pressure. Strong pressure localization is observed within the labyrinthine channels. It is easy to estimate that regardless of the iteration level, these localized modes correspond to Fabry-Perot resonances in a straight slit of width w and length L_{eff} [21, 13],

$$f_l^{FP} = l c_0 / 2 L_{\text{eff}}, \quad (5)$$

where l is a positive integer. In the “fixed channel” case, L_{eff} equals $2.305 d_{\text{uc}}$ for UC1, $L_{\text{eff}} = 5.667 d_{\text{uc}}$ for UC2, and $L_{\text{eff}} = 16.642 d_{\text{uc}}$ for UC3 with $d_{\text{uc}} = a_{\text{uc}} \sqrt{2}$. Note that odd l values correspond to the lower band-gap bounds, while even l values allow approximating the upper band-gap bounds in Figure 4.

The fact that the band gap bounds are formed by multiple Fabry-Perot resonances explains a similar structure of various dispersion bands in Figure 4, which have close values of phase and group velocities.

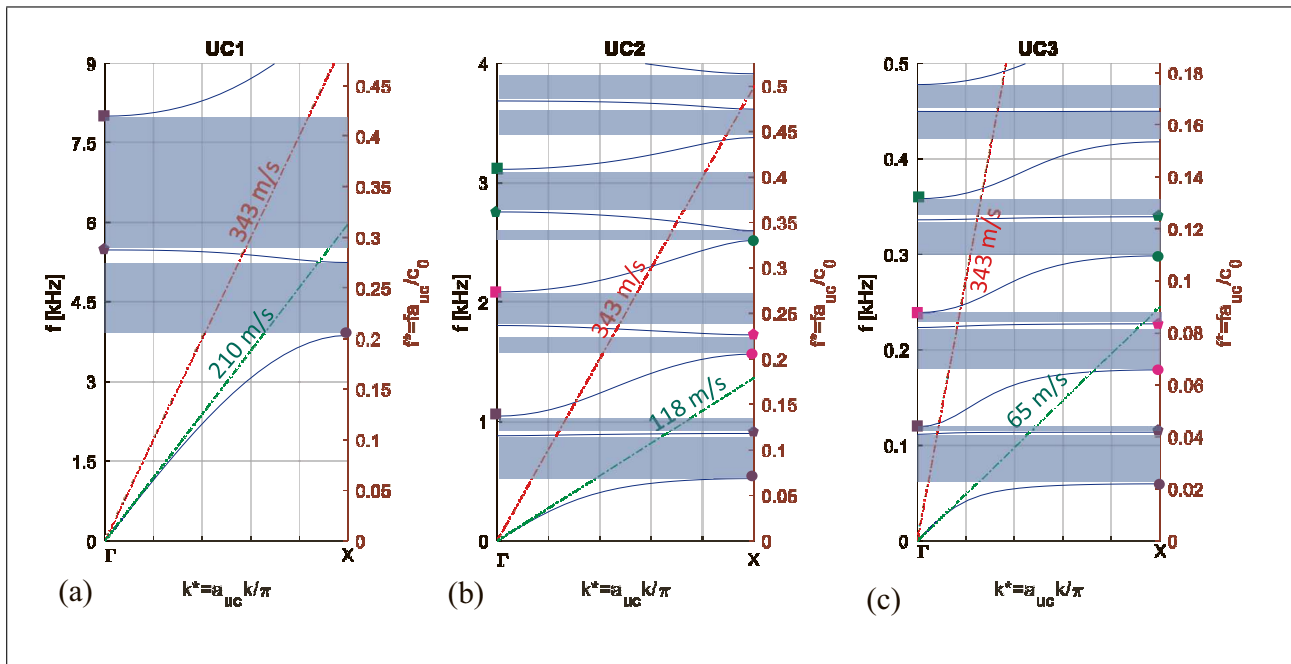


Figure 4. (Colour online) “Fixed channel” case: Dispersion relations for the labyrinthine unit cells of the 3 iteration levels with a fixed channel width, $w=4$ mm. Band gaps are shown by shaded rectangles. The slope of the green and red dash-dot lines indicates phase velocities of the fundamental mode within a unit cell and in homogenous air (when a unit cell is removed). Bold points designate frequencies with the pressure distributions given in Tables I and II.

The pressure distributions given in Tables I–II also resemble those of artificial monopole, dipole and multipole resonances described in [12]. For example, the patterns at the lower bound of the 1st band gap (the 1st column in Table I) are similar to a monopole, in which the pressure is concentrated in the central part of a channel, equally radiating along two propagation directions [12, 14]. Thus, the monopole and multipole resonances in folded channels originate from the tortuosity effect of the Fabry-Perot resonances.

Since an effective dynamic bulk modulus (not evaluated in this study) is typically negative above the monopole resonance, one can expect a high wave reflectance at these frequencies [12]. This behavior has been experimentally observed in [12] for circular-shaped folded channels.

Apart from the Fabry-Perot resonances, wave dispersion in the designed labyrinthine metamaterials is also characterized by the presence of bands within the band gap frequencies. These bands are found within every band gap of the analyzed unit cells (see curves separating band gaps in Figure 4). Pressure distributions for these modes (the 2nd column in Tables I–II) resemble those for the dipole and its higher harmonics (compare to 3rd column of Tables I–II), but the pressure is not localized inside a channel. Hence, these modes are not localized, rather they are propagating waves with very small (and often negative) group velocity. They may be analogous to slow modes inside phononic band gaps for elastic waves [30, 31]. The mechanism of the slow mode excitation in acoustics and their dynamics will be investigated in future work. Here, we consider these modes as included in a single band gap (shown as separated into two parts), since we did not de-

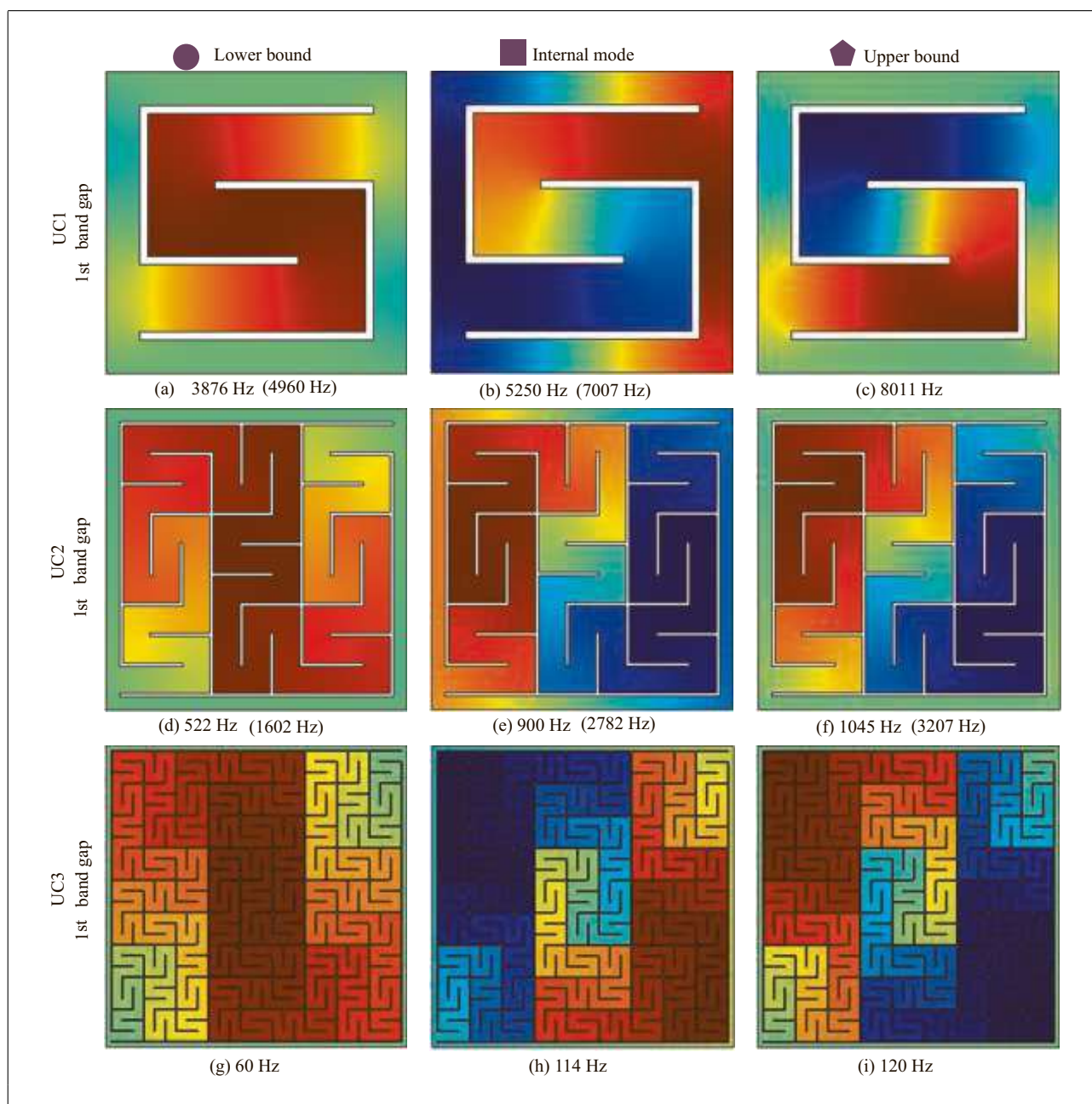
tect their presence in the frequency-domain simulations (for lossless and lossy air), even for a very fine frequency step (see Figures 5–6).

Frequency-domain simulation results are given in Figures 5–6 in terms of transmission and absorption coefficients for lossless and lossy air. (The reflection coefficient can be directly derived from these data, and thus is not shown.) We analyze waves propagating through a monolayer composed of the labyrinthine unit cells UC1–UC3 (Figures 5a, 6a, 6c) and periodic straight slits of length L_{eff} (Figure 5b, 6b, 6d) or a_{uc} (Figure 5c). Note that at very low frequencies, the transmission and absorption coefficients for lossy air appear to be mesh-dependent, and are not shown here due to their limited reliability..

When losses in air are neglected, incoming waves are either transmitted or reflected for all the considered geometries, and thus, the absorption coefficient is zero (not shown in the graphs). Total transmission is achieved at frequencies of the Fabry-Perot resonances given by Equation (5). As can be seen, this effect is independent of the channel tortuosity and occurs in folded labyrinthine channels of any iteration level at almost the same frequencies as for the equivalent straight slits. For the slit of length a_{uc} , the fundamental Fabry-Perot resonance appears to be at higher frequencies than the analyzed frequency range. Thus, straight slits of length a_{uc} are not considered further.

When thermo-viscous losses are included, the transmission peaks decrease in magnitude and are shifted to lower frequencies compared to the lossless case. The latter occurs due to the reduction of the sound propagation velocity in dissipative air and is confirmed by experimental measurements in [21].

Table I. (Colour online) “Fixed channel” case (“Fixed unit cell” case): Pressure distributions around the 1st band gap for the labyrinthine metamaterial unit cells of the 3 iteration levels. Red and blue colors represent maximum and minimum pressure, while green color indicates (almost) zero pressure. The frequencies in brackets are referred to the “fixed unit cell” case.

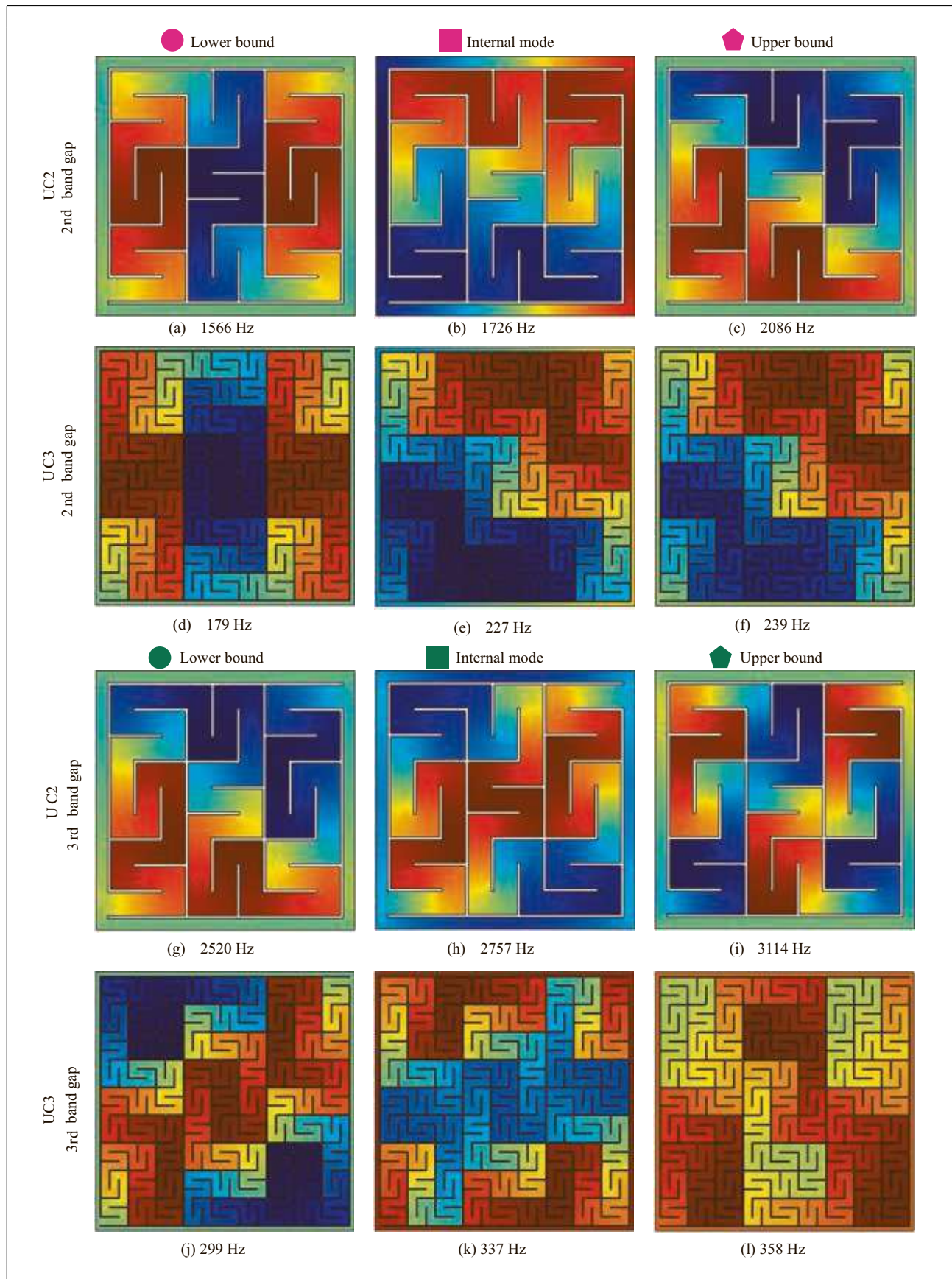


The striking difference in wave propagation through the unfolded (straight) and labyrinthine channels occurs between the frequencies of Fabry-Perot resonances. In the case of straight slits, the main part of the incoming waves is reflected, while about 15–20% of the wave energy is transmitted through a slit. For the labyrinthine channels, the same behavior is observed in the propagating frequency range, while within the band gaps total wave reflection occurs with zero transmission coefficient. As mentioned above, the fundamental Fabry-Perot resonance at the lower band gap bound corresponds to the monopole, and thus, the observed total reflectance can be justified by a negative values of effective bulk modulus within the band

gap. While experimental data for circular-folded channels indicate about 84% insertion loss, in good agreement with calculated transmission for equivalent straight slits at frequencies between the Fabry-Perot resonances (see e.g. in Figure 6b), our labyrinthine structures demonstrate total zero transmission even if thermo-viscous losses are taken into account. We attribute this to the fact that the designed wave path redirects a propagating wave in the opposite direction relative to incident waves and thus, to the channel tortuosity, since all the other structural parameters are the same as for a straight slit.

Therefore, the tortuosity of the labyrinthine channels significantly modifies the wave dynamics at band-gap fre-

Table II. (Colour online) “Fixed channel” case: Pressure distributions around the 2nd and 3rd band gaps for the labyrinthine metamaterial unit cells of the 2nd and 3rd iteration levels. Red and blue colors represent maximum and minimum pressure, and green color indicates (almost) zero pressure.



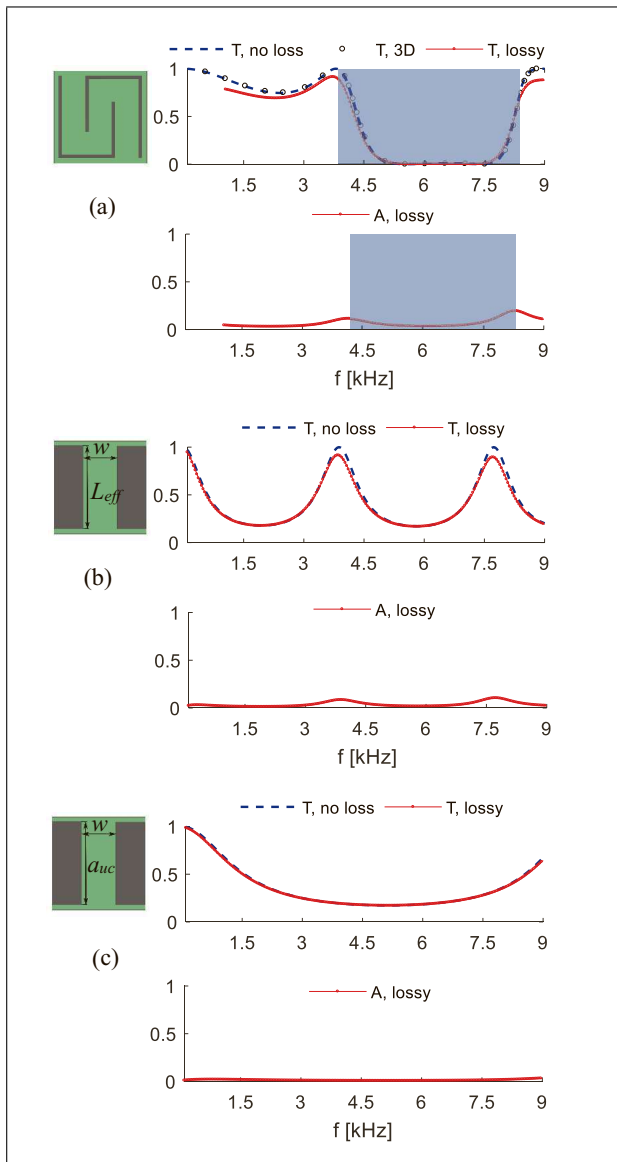


Figure 5. “Fixed channel” case: Transmission (T) and absorption (A) coefficients for acoustic waves in lossless (dashed line) and lossy (solid line) air through (a) a labyrinthine metamaterial UC1; (b) an equivalent straight slit of width $w = 4$ mm and length $L_{eff} = 45.6$ mm; (c) a straight slit of width $w = 4$ mm and length $a_{uc} = 18$ mm. Shaded regions indicate band-gap frequencies shown in Figure 4a. Circular markers in (a) indicate transmission coefficient values in lossless air for the corresponding 3D model of height $4a_{uc}$.

quencies, and these effects are not be captured by considering equivalent straight slits.

While the total transmission at the Fabry-Perot resonances is eliminated by the loss mechanisms in sub-wavelength straight channels [21], the revealed total reflection at band-gap frequencies in the labyrinthine channels is not affected by dissipation. At higher iteration levels, the band gaps are shifted to lower frequencies and decrease in size (compare Figures 5a, 6a, and 6c). At the same time the amount of transmitted energy at the frequencies of propagating modes also decreases, in contrast to the

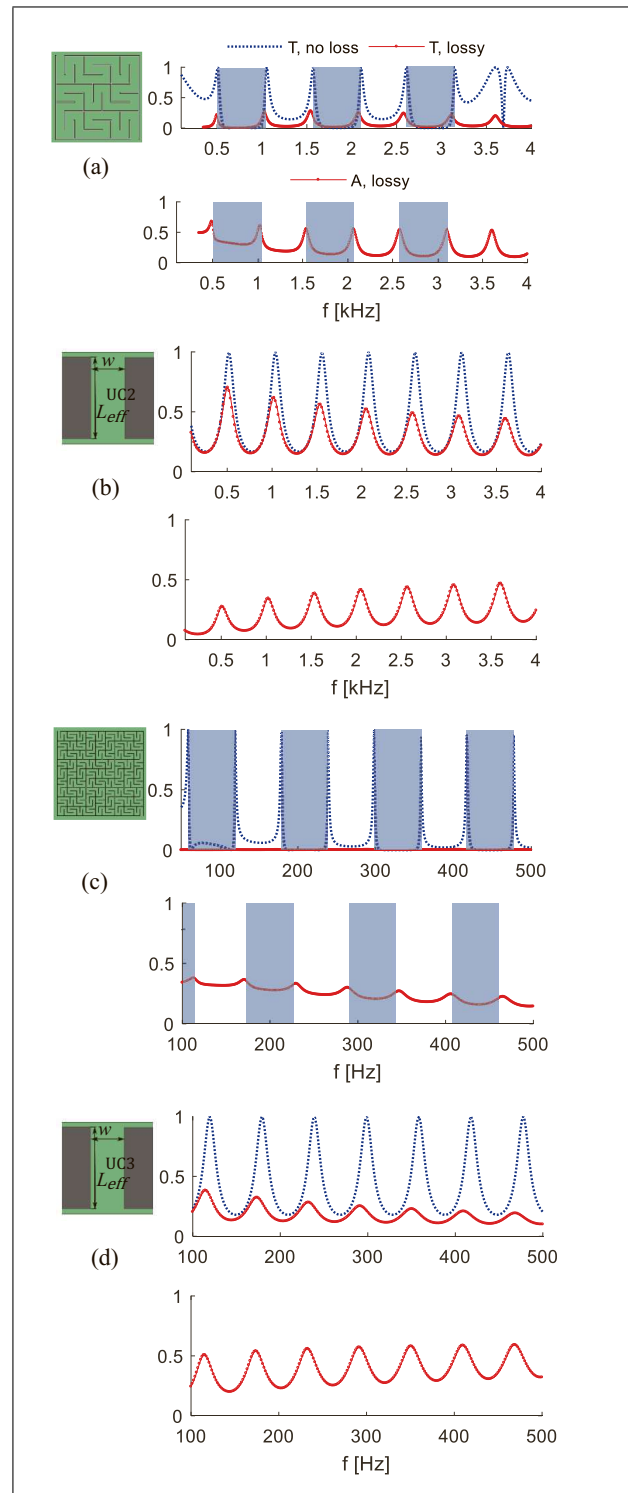


Figure 6. “Fixed channel” case: Transmission (T) and absorption (A) coefficients for acoustic waves in lossless (dotted line) and lossy (solid line) air through (a) a labyrinthine metamaterial UC2; (b) a straight slit of width $w = 4$ mm and length $L_{eff} = 328.5$ mm; (c) a labyrinthine unit cell UC3 and (d) a straight slit of width $w = 4$ mm and length $L_{eff} = 2.871$ m. Shaded regions indicate frequency band gaps shown in Figure 4.

case of equivalent straight slits (compare e.g. Figures 6c and 6d). Thus, the incorporation of third and higher iteration levels for a “fixed channel” unit cell is beneficial for

low-frequency sound control and allows to achieve total sound reflection at broadband frequencies.

To summarize, we can derive two key conclusions. First, wave propagation in the labyrinthine metamaterials with hierarchically-structured channels differs from that through straight slits of an equivalent effective length. The physical mechanism causing this difference is related to the channel tortuosity, which allows wave propagation in the opposite direction relative to an incident pressure field. When one derives effective characteristics for metastructures with complex-shaped internal channels, the mentioned tortuosity effect must be taken into account in order to correctly predict the wave dynamics. Second, the designed labyrinthine metamaterials can be used as compact and broadband low-frequency sound reflectors, since 100% wave reflection can be achieved by using only a single unit cell.

Circular markers in Figure 5a represent the transmission coefficient for a corresponding 3D domain obtained by extruding the 2D model (Figure 2) in the out-of-plane direction by a height of $4u_{uc}$. Excellent agreement between the 3D and 2D results justifies the validity of the introduced assumption on the two-dimensional nature of the analyzed problem.

Finally, we note that the designed metamaterials can be compared with tortuous open-porous materials. The porosity level, evaluated as the ratio of the air-domain area inside a unit cell to the total area of a unit cell, is about 90% for UC1, 88 % for UC2, and 89 % for UC3, which is rather low as compared to the almost 100% porosity of typical foams [32]. The main difference between porous foams and the designed labyrinthine metamaterials is the physical mechanism of wave control. Porous materials attenuate waves due to inherent thermo-viscous losses with the absorption coefficient close to 1 for broad frequency ranges. In contrast to this, the proposed metastructures mainly reflect incident waves with absorption approaching 0.5 at single frequencies of Fabry-Perot resonances (see Figures 6a,c). In the next section, we estimate the metamaterial performance for an increased level of thermo-viscous losses due to a smaller channel width.

4.2. “Fixed-unit-cell” case

In this case, the unit cell size $a_{uc} = 14$ mm is fixed as for all the iteration levels. Dispersion relations of UC1 and UC2 are shown in Figure 7 for homogeneous waves propagating along the ΓX direction. The dimensional frequency ranges here are the same as those for the corresponding unit cells in the “fixed channel” case (see Figures 4a,b).

The structure of the dispersion relation in Figure 7a is similar to that in Figure 4a, except that the bands are shifted to higher frequencies. This occurs due to a shorter channel length. At first sight, more differences are found by comparing the dispersion relations for UC2 in Figure 4b and Figure 7b. While in Figure 4b there are four band gaps, the relation in Figure 7b is characterized by the presence of a single wide band gap. This occurs because the unit cell area, $A^{(fix_{uc})} = 14^2 \text{ mm}^2$, in the second

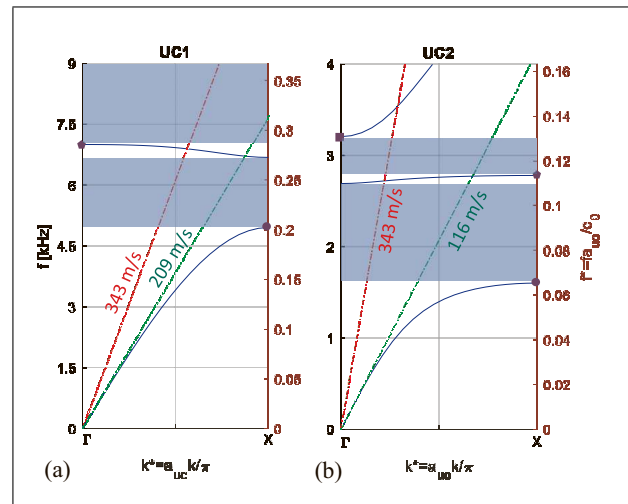


Figure 7. (Colour online) “Fixed unit cell” case: Band structure diagrams for the unit cells UC1 and UC2 of fixed size $a=14$ mm with the channel width of 3 mm and 0.9 mm, respectively. Band gap frequencies are shaded. The slopes of the green and red dash-dot lines indicate the phase velocities of the fundamental pressure wave inside a unit cell and in homogeneous air (when a unit cell is removed).

case, is about 3 times smaller than that for the “fixed channel case”, $A^{(fix_w)} = 41^2 \text{ mm}^2$. As a result, the monopole, dipole and multipole resonances, as well as the related band gaps, are shifted to 3 times higher frequencies. However, in terms of non-dimensional frequencies, the band gap frequencies remain unchanged. The similarity of dispersion relations in Figures 4 and 7 can be expected, since the metamaterial structure is preserved. In contrast to this, one should observe differences in transmission and absorption coefficients between these two cases due to the different amount of thermo-viscous losses in the channels of a various width.

Figure 8 shows the transmission and absorption coefficients for labyrinthine monoslabs of the “fixed unit cell” case and those for straight slits of the effective length $L_{eff} = 34.5$ mm (UC1) and $L_{eff} = 107$ mm (UC2). The key features found in the analysis of the “fixed channel” case are also observed for the “fixed unit cell” case, namely the wave propagation in the labyrinthine channels is not equivalent to that in straight slits due to the occurrence of 100% reflection within band gaps. The total reflection is also independent of losses in air. However, as the channel of UC2 in the “fixed unit cell” case is more than 4 times narrower relative to that in the “fixed channel” case, the influence of thermo-viscous losses becomes more pronounced. This can be seen for larger absorption values at the Fabry-Perot resonances.

Therefore, wave attenuation within labyrinthine channels can be obviously increased by decreasing the channel width. The porosity of the metamaterial then also decreases as a consequence. For UC2, the structural porosity is 64.7% for the “fixed unit cell” case versus 88% for the “fixed channel” case. Therefore, one can consider the wave absorption within the labyrinthine metamaterials as simi-

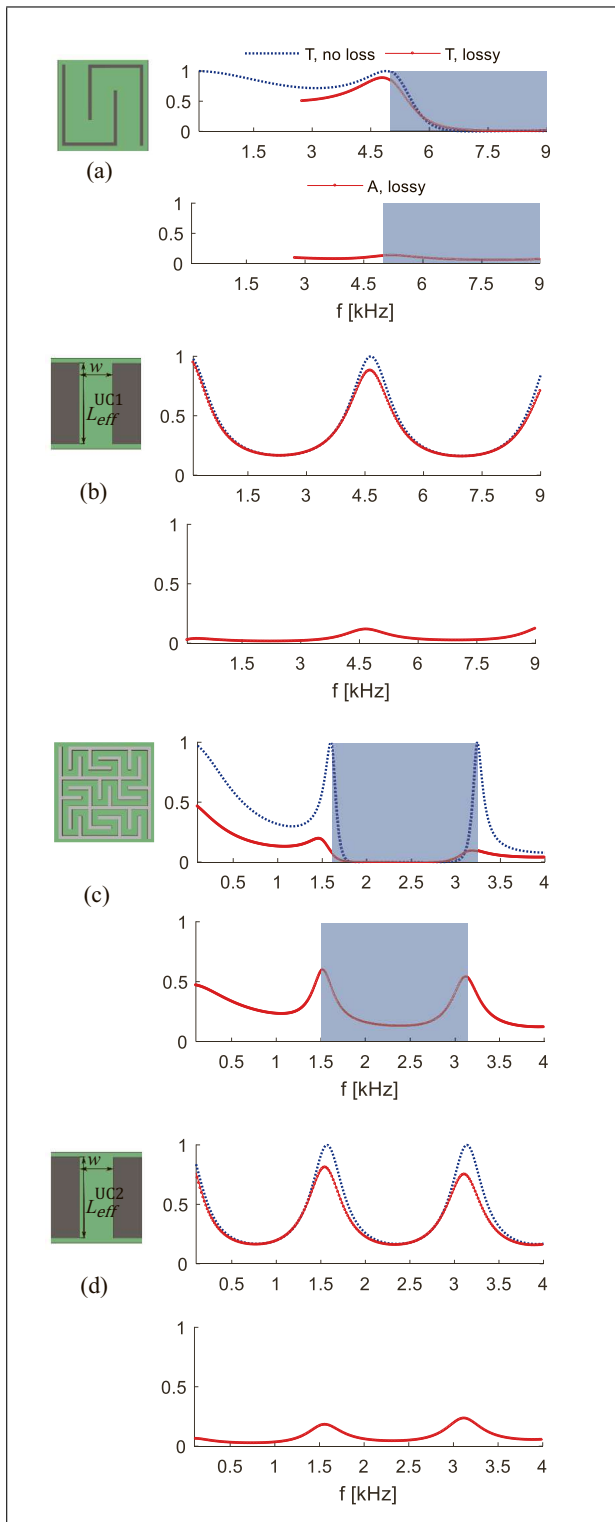


Figure 8. “Fixed unit cell” case: Transmission (T) and absorption (A) coefficients for acoustic waves in lossless (dotted line) and lossy (solid line) air through (a) a labyrinthine unit cell UC1 and (b) a straight slit of width $w = 3$ mm and length $L_{\text{eff}} = 34.6$ mm; (c) a labyrinthine unit cell UC2 and (d) a straight slit of width $w = 0.9$ mm and length $L_{\text{eff}} = 107$ mm. Shaded regions indicate frequency band gaps shown in Figure 7.

lar to that of tortuous foams by decreasing the structural external dimensions and porosity level.

5. Conclusions

In this work, we have theoretically analyzed the characteristics of labyrinthine metamaterials with sub-wavelength channels shaped along a space-filling curve to control airborne sound. We have demonstrated that if a folded channel allows wave propagation in the opposite direction relative to the incident pressure, wave dynamics in the channel is *not equivalent* to that of a straight slit of an effective length. In addition, we have shown that Fabry-Perot resonances of the straight slit correspond to monopole, dipole and multipole resonances in folded channels and govern the generation of band gaps. Within the band gaps, total wave reflection occurs that is not influenced by the presence of dissipation losses in air. Moreover, by increasing the channel tortuosity and further elongating a wave path, one can achieve almost perfect reflection outside the band gaps. Although at higher iteration levels the designed metamaterials resemble a tortuous porous material, they mostly control waves due to interference effects, in contrast to thermo-viscous dissipation mechanism in porous foams. This results in a low wave attenuation within a metastructure for a sufficiently wide channel. The absorption level can be increased by decreasing the channel width and the structural weight.

This is the first time that a space-filling curve has been considered for designing and elongating wave paths in labyrinthine metamaterials. Therefore, further more in-depth analysis is required to study the influence of various geometric factors, e.g. number of angles or turns, as well as the metamaterial performance for inhomogeneous waves in complex-shaped folded channels. The proposed structures show promise as broadband low-frequency sound reflectors that can be assembled inexpensively from thin sheets.

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References

- [1] S. Cummer, J. Christensen, A. Alu: Controlling sound with acoustic metamaterials. *Nat. Rev. Mats.* **16001** (2016).

- [2] R. V. Craster, S. Guenneau: Acoustic metamaterials: Negative refraction, imaging, lensing and cloaking. Springer, Heidelberg, 2012.
- [3] Z. Liu, X. Zhang, Y. Mao, Y. Y. Zhu, Z. Yang, C. Chan, P. Sheng: Locally resonant sonic materials. *Science* **289** (2000) 1734.
- [4] A. Krushynska, V. Kouznetsova, M. Geers: Towards optimal design of locally resonant acoustic metamaterials. *J. Mech. Phys. Solids* **71** (2014) 179–196.
- [5] P. Deymier: Acoustic metamaterials and phononic structures. Springer, Heidelberg, 2013.
- [6] N. Fang, D. Xi, J. Xu, M. Ambati, W. Srituravanich, C. Sun, X. Zhang: Ultrasonic metamaterials with negative modulus. *Nat. Mat.* **5** (2006) 452–456.
- [7] A. Movchan, S. Guenneau: Split-ring resonators and localized modes. *Phys. Rev. B* **70** (2004) 125116.
- [8] G. Ma, M. Yang, S. Xiao, Z. Yang, P. Sheng: Acoustic metasurface with hybrid resonances. *Nat. Mat.* **13** (2014) 873–878.
- [9] Z. Yang, J. Mei, M. Yang, N. H. Chan, P. Sheng: Membrane-type acoustic metamaterial with negative dynamic mass. *Phys. Rev. Lett.* **101** (2008) 204301.
- [10] V. García-Chocano, J. Sánchez-Dehesa: Anomalous sound absorption in lattices of cylindrical perforated shells. *Appl. Phys. Lett.* **106** (2015) 124104.
- [11] A. Bozhko, J. Sanchez-Dehesa, A. Krokhin: Redirection and splitting of sound waves by a periodic chain of thin perforated cylindrical shells. *J. Acoust. Soc. Am.* **141** (2017) 3640.
- [12] Y. Cheng, C. Zhou, B. Yuan, D. Wu, Q. Wei, X. Liu: Ultra-sparse metasurface for high reflection of low-frequency sound based on artificial Mie resonances. *Nat. Mater.* **14** (2015) 1013–1019.
- [13] Z. Liang, J. Li: Extreme acoustic metamaterial by coiling up space. *Phys. Rev. Lett.* **108** (2012) 114301.
- [14] A. Krushynska, F. Bosia, M. Miniaci, N. Pugno: Tunable spider-web inspired hybrid labyrinthine acoustic metamaterials for low-frequency sound control. *arXiv:1701.07622*, 2017.
- [15] Z. Liang, T. Feng, S. Lok, F. Liu, K. Ng, C. Chan, J. Wang, S. Han, S. Lee, J. Li: Space-coiling metamaterials with double negativity and conical dispersion. *Sci. Rep.* **3** (2013) 1614.
- [16] Y. Xie, B.-I. Popa, L. Zigoneanu, S. Cummer: Measurement of a broadband negative index with space-coiling acoustic metamaterials. *Phys. Rev. Lett.* **110** (2013) 175501.
- [17] T. Frenzel, J. Brehm, T. Bueckmann, R. Schittny, M. Kadic, M. Wegener: Three-dimensional labyrinthine acoustic metamaterials. *Appl. Phys. Lett.* **103** (2013) 061907.
- [18] C. Zhang, X. Hu: Three-dimensional single-port labyrinthine acoustic metamaterial: perfect absorption with large bandwidth and tunability. *Phys. Rev. Appl.* **6** (2016) 064025.
- [19] Y. Xie, A. Konneker, B.-I. Popa, S. Cummer: Tapered labyrinthine acoustic metamaterials for broadband impedance matching. *Appl. Phys. Lett.* **103** (2013) 201906.
- [20] G. Song, Q. Cheng, B. Huang, H. Y. Dong, T. Cui: Broadband fractal acoustic metamaterials for low-frequency sound attenuation. *Appl. Phys. Lett.* **109** (2016) 131901.
- [21] M. Moleron, M. Serra-Garcia, C. Daraio: Visco-thermal effects in acoustic metamaterials: from total transmission to total reflection and high absorption. *New J. Phys.* **18** (2016) 033003.
- [22] H. Sagan: Space-filling curves. Springer, 1994.
- [23] G. Peano: Sur une courbe, qui remplit toute une aire plane. *Math. Ann.* **36** (1890) 157–160.
- [24] H. Haverkort, F. van Walderveen: Locality and bounding-box quality of two-dimensional. *Computational Geometry: Theory and Applications* **43** (2010) 131–147.
- [25] J. K. Lawder, P. J. H. King: Using space-filling curves for multi-dimensional indexing. *Advances in Databases. BNCOD 2000, Lecture Notes in Computer Science* 1832, 2000.
- [26] M. Ali, S. Ladhake: Overview of space-filling curves and their applications in scheduling. *International Journal of Advances in Engineering and Technology* **1** (2011) 148–154.
- [27] L. Platzman, J. I. Bartholdi: Spacefilling curves and the planar travelling salesman problem. *Journal of the ACM* **36** (1989) 719–737.
- [28] Acoustics module of comsol multiphysics. User's guide, 2013.
- [29] R. 5. Comsol Multiphysics. www.comsol.com.
- [30] R. Zhu, X. Liu, G. Hu, F. Yuan, G. Huang: Microstructural designs of plate-type elastic metamaterial and their potential applications: a review. *Int. J. Smart and Nano Mater.* **6** (2015) 14–40.
- [31] C.-Y. Sun, J.-C. Hsu, T. Wu: Resonant slow modes in phononic crystal plates with periodic membranes. *Appl. Phys. Lett.* **97** (2010) 031902.
- [32] J. Allard, N. Atalla: Propagation of sound in porous media: modelling sound absorbing materials. Wiley, 2009.