Bibliographical reference

Measuring Dispersion Curves for Bending Waves in Beams: A Comparison of Spatial Fourier Transform and Inhomogeneous Wave Correlation, Bart Van Damme and Armin Zemp, Acta Acustica **vol. 104** (Number 2), 2018, pp. 228-234

DOI

https://doi.org/10.3813/AAA.919164

ACTA ACUSTICA UNITED WITH ACUSTICA
Vol. 104 (2018) 228 – 234

DOI 10.3813/AAA.919164

Measuring Dispersion Curves for Bending Waves in Beams: A Comparison of Spatial Fourier Transform and Inhomogeneous Wave Correlation

Bart Van Damme, Armin Zemp
Empa, Laboratory for Acoustics/Noise Control, Ueberlandstrasse 129, 8600 Dübendorf, Switzerland.
bart.vandamme@empa.ch

Summary
In this paper, two methods to acquire the dispersion of bending waves in beams are compared: the spatial Fourier transform and inhomogeneous wave correlation. Two examples of beams with non-standard dynamic properties are given: layered beams made out of wood (cross laminated timber) and beams with a periodically varying thickness profile. Dynamic testing, such as modal analysis or wave speed measurements, can be used to determine the elastic properties of materials. A direct measurement of the dispersion facilitates an easy assessment of the stiffness parameters over a wide frequency range for layered beams made out of orthotropic layers. The efficiency of structural metamaterials, elastic structures engineered to exhibit band gaps in which waves are quickly attenuated, can also be addressed through the dispersion relation.

PACS no. 43.40.At, 43.40.Cw, 43.20.Jr, 43.40.Tm

1. Introduction

Nondestructive evaluation of material parameters can be done in a number of ways. Static testing, such as stress-strain tensile tests [1, 2] or three-point bending tests [3, 4] are widely used to determine stiffness properties such as Young's modulus, Poisson ratio, and bending stiffness. These methods can only be performed on specified, often small, geometries (tensile tests), or address the stiffness locally between the support points (bending tests). This is problematic for materials with large local elastic variability such as composites and wood [1, 2, 5]. Dynamic testing employs properties of waves and vibrations to indirectly calculate the underlying material parameters. For large structures, modal analysis is a popular method [3, 6, 7, 8, 9]: After measuring the eigenfrequencies, the results are compared to a model in which the unknown stiffness values are chosen in such a way that the difference between model and measurements is minimized. This model can be analytical for simple geometries, but often requires numerical techniques such as finite elements. Since the eigenfrequencies depend on the specimen's boundary conditions, these have to be meticulously described in the model. Measuring the wave speed is another popular approach, mostly performed using ultrasound transducers [10]. The wave velocity can also be obtained for lower frequencies, e.g. using time-of-flight measurements [11] or phase measurements [12]. Once the velocity of longitudinal, shear, surface and bending waves is known, it can be related to the stiffness. It is in general not straightforward to separate the different wave types, or account for geometrical dispersion effects. Additionally, early reflections in small specimens interact with the source signal, and thus complicate the wave speed measurement.

Alternatively, several methods are available to determine the dispersion relation of waves in a large frequency band, such as air-coupled ultrasound [13], wave form imaging [14] or the waveguide finite element method [15]. In this paper, we compare two of these methods for bending wave propagation in complex beams: spatial fast Fourier transformations [16, 17] and inhomogeneous wave correlation [18, 19]. Both schemes require the knowledge of the complex velocity (or displacement) response to a broadband excitation, measured in many points along a significant part of the beam under consideration. Two very diverse materials are investigated in the following sections: cross laminated timber (Figure 1a-b) and beams with a periodically varying thickness profile, an example of phononic crystals (Figure 1c). In the former, the dispersion depends on the orthotropic properties of each layer in the composite structure. The latter is an example of phononic crystals, a class of materials showing high wave attenuations in certain frequency bands, so-called band gaps.

Received 12 June 2017,
accepted 8 August 2017.

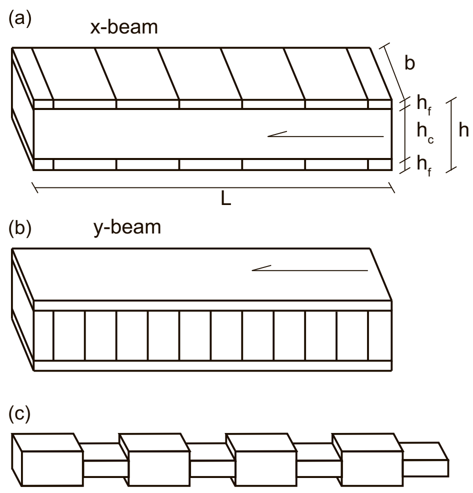

Figure 1. Schematic representation of the beam geometries. CLT beams cut in two principles directions x (a) and y (b) of a plate. The fiber direction is denoted by the half arrow. Beam with periodically varying thickness (c).

2. Analysis methods

2.1. Spatial Fourier transform

The two dimensional Fourier transform (2D-FFT) is a popular nondestructive method to collect the frequency-wave number relation in the ultrasound domain. The method requires a time response signal $s(x, t)$ to a broadband source, measured along a line in equally spaced points. The spacing has to be small enough to meet the Nyquist criterion, at least half of the smallest wavelength of interest. On the other hand, the span of the measurement has to be large enough in order to achieve a fine resolution in the wave number domain. This last claim leads to practical problems for frequencies in the acoustic range, since large samples are needed to span several wavelengths. Zero padding can be applied in order to artificially increase the number of measurement points, especially if the sample has clamped boundary conditions. In case of a freely vibrating test object, zero padding leads to an abrupt change in amplitude, thereby introducing wave number harmonics.

In this paper, the temporal Fourier transformed data $\tilde{s}(x, \omega)$ were gathered directly from the measurement system. The additional spatial Fourier transform is performed in Matlab on the raw, zero-padded data. No additional window or signal processing was applied. Since the considered beams achieve high vibration amplitudes at the eigenfrequencies, the amplitude of the 2D-FFT fluctuates strongly. To avoid these large amplitude variations, the maximum value is normalized to 1 for each frequency.

2.2. Inhomogeneous wave correlation

Using the same measurements as for the 2D-FFT method, the dispersion relation can be found in a different way. For a chosen frequency f_0, the spatial response $\tilde{s}(f_0, x)$ is compared with an inhomogeneous running wave $o(t, x) = \exp[\mathrm{i}(2\pi f t - kx)]$, which yields $\tilde{o}(x, k_r, k_i) = \exp[\mathrm{i}k_r x - k_i x]$ after Fourier transformation in the time domain. The unknown wave number $k = k_r + \mathrm{i}k_i$ can be found as the location of the maximum of the normalized correlation function

$$\Im(k_r, k_i) = \frac{\left| \int \tilde{s}(f_0, x) \cdot \tilde{o}^*(x, k_r, k_i)\,\mathrm{d}x \right|}{\sqrt{\int \left|\tilde{s}(f_0, x)\right|^2 \mathrm{d}x \cdot \int \left|\tilde{o}(x, k_r, k_i)\right|^2 \mathrm{d}x}}, \quad (1)$$

where * stands for the complex conjugate. For measurements in discrete points x_i, the integrals are replaced by summations. The inhomogeneous wave correlation (IWC) function $\Im$ typically has a well-defined maximum, denoting the point where the measured signal correlates best with the damped wave $o(t, x)$. This allows for an accurate estimation of the complex wave number.

If multiple inhomogeneous waves with distinguished wave numbers are present for one single frequency, they will show up as secondary maxima in the IWC function. However, in this work, the right and left travelling flexural waves overwhelm all other possible wave modes, such as longitudinal and torsional waves, or higher order Lamb waves. This is due to the perpendicular excitation on the one hand, and the measurement of the out-of-plane velocity on the other. When limiting the search for the maximum of $\Im$ for positive values of k_r only, the dispersion of the right travelling wave with the highest amplitude can be discerned. A recent study based on the Laplace transform of similar measurements shows how multiple wave modes can be distinguished [20]. It should be noticed that this analysis method, and the k-space method described in [18] are very similar in terms of their mathematical description.

For this paper, the IWC method was implemented in Matlab. The point of best correlation was found using the fmincon function for finding minima within given boundaries.

3. Cross laminated timber beams

3.1. Description of the material

Cross laminated timber (CLT) is a lightweight construction material quickly gaining in popularity [21, 22]. In order to achieve a uniform high static strength, timber beams are ordered in an odd number of layers, each layer having the wood fibers perpendicular to the neighboring ones. The layers are connected using polyurethane glue, whereas the individual beams in a layer might or might not be glued. Although the material has advantageous structural, safety and economical properties [23, 24], its acoustic performance is poor due to the high stiffness/mass ratio [25]. The layered structure introduces direction dependent material parameters, and the authors have shown before that vibrations have to be modeled separately in both directions. We have previously performed a study on beams cut out of a three-layer CLT plate, using modal analysis to find the stiffness parameters relevant for bending vibrations [8]. The results show that samples cut along the fiber direction of the inner layer behave like thick beams (Figure 1a), whereas beams cut in the other main direction are better described by a model for sandwich beams with a soft core (Figure 1b). The inverse problem requires 1) solving a nonlinear equation to find the eigenfrequencies and 2) optimization through a genetic algorithm to ensure convergence to the absolute minimum.

3.2. Dispersion of bending waves in thick beams and sandwich beams

Wood is an orthotropic material with stiffness values that can vary of several orders of magnitude. The Young's modulus along the fiber direction is typically 10-100 times higher than the other stiffness and shear moduli. A low shear modulus implies that through-the-thickness shear force variations are important, even if the beams are thin compared to the wavelength. This can be taken into account when using Timoshenko's model for vibrating beams, which holds for modeling waves in a large frequency range. According to this model, the dispersion relation is given by

$$EIk^4 - \rho A\omega^2 - \left(\rho I + \frac{EI\rho}{KG}\right)k^2\omega^2 + \frac{\rho^2 I}{KG}\omega^4 = 0. \quad (2)$$

The equation contains the Young's modulus E and the shear modulus G, the beam's cross section A and second moment of area $I = bh^3/12$. The constant $K = 5/6$ captures the shear force variation.

Waves in a three layer sandwich beam have been investigated by Nilsson and Nilsson [26], under the assumption of soft core materials. This is applicable to CLT beams in the y-direction, whose core Young's modulus E_c is extremely low due to the loose layering of the lamellas. To incorporate the boundary conditions between the layers, finally a sixth-order differential equation is retrieved. From this, the dispersion relation for plane running waves is found to be

$$\begin{aligned} &2D_1D_2k^6 - 2D_2I_\rho k^4\omega^2 \\ &-\left(D_1\mu + 2D_2 + I_\rho G_c h_c\right)k^2\omega^2 \\ &+G_c h_c\left(D_1k^4 - \mu\omega^2\right) + I_\rho\mu\omega^4 = 0. \end{aligned} \quad (3)$$

The geometric and material parameters introduced here are defined as

$$\begin{aligned} D_1 &= E_c h_c^3/12 + E_f\left(h_c^2 h_f/2 + h_c h_f^2 + 2h_f^3/3\right), \\ D_2 &= E_f h_f^3/12, \\ I_\rho &= \rho_c h_c^3/12 + \rho_f\left(h_c^2 h_f/2 + h_c h_f^2 + 2h_f^3/3\right), \\ \mu &= 2h_f\rho_f + h_c\rho_c, \end{aligned}$$

in which the subscripts c and f refer to core and face layers, respectively, as shown in Figure 1a. Material damping was introduced in both models by adding an imaginary part to the Young's and shear moduli: $E_d = E(1 - i\eta_E)$ and $G_d = G(1 - i\eta_G)$.

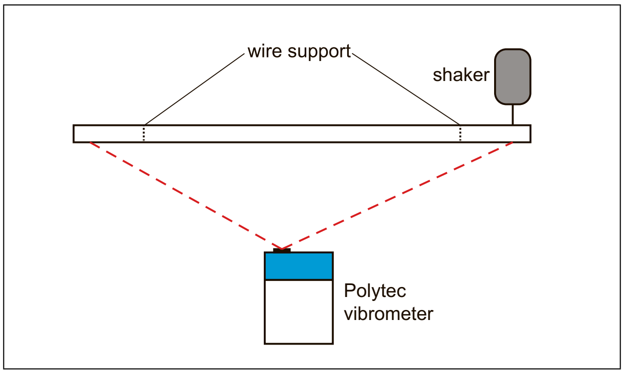

Figure 2. Experimental setup for the CLT beam measurements.

In the case of Timoshenko beams, four complex wave numbers exist for each frequency, in three layer sandwich beam this increases to six. Only two wave numbers however have a real part which is larger than the imaginary part, the other possible solutions belong to evanescent waves. These highly damped waves only play a role close to the excitation source and boundaries of the sample.

3.3. Measurement and results

For the experiments, an x- and y-beam were cut from a three-ply CLT plate provided by Pius Schuler AG (Kronenstrasse 12, 6418 Rothenthurm, Switzerland). The raw material for the lamellas is Norway spruce with a strength class of C24 according to standard DIN EN 338 for wood. The outer layers have a thickness $h_f = 15$ mm and the core is $h_c = 50$ mm thick. Measurements were performed on $L = 3$ m long beams with a width $b = 60$ mm, hung by two light strings at the location of the first bending vibration nodes, 0.67 m from the beam's ends. The beams are excited by a shaker positioned at one end, using a sinusoidal sweep signal with a 100–3000 Hz range. The input force $F(t)$ is measured by a force sensor (PCB 208C02), and the velocity response $v(x,t)$ is measured in 89 points with a 32 mm spacing, almost over the entire length of the beams. A Polytec PSV-400 scanning laser Doppler vibrometer is used to acquire the out-of-plane velocities. A schematic representation of the setup is shown in Figure 2. The 2D-FFT and IWC analysis is performed on the temporal transfer functions $TF(x,\omega) = \mathcal{F}[v(x,t)]/\mathcal{F}[F(t)]$, which is collected directly as the H1 transfer function from the Polytec measurement software. For the IWC analysis, the first and last 5 points are omitted to avoid the presence of any evanescent wave content due to the source or free boundaries.

The results of both analyses are shown in Figure 3 and Figure 4. The measurement was zero padded to 256 points to perform the 2D-FFT, leading to a discrete wave number step of 0.76 m^{-1}, and a maximum wave number according to the Nyquist criterion of 98.2 m^{-1}. The resolution is sufficient to track the dispersion. Only at very low frequencies, where the wave number increases steeply, discrete steps in the dispersion curve are visible. At higher frequencies, the 2D-FFT method is increasingly more sensitive to noise, as can be seen in Figure 4a. The IWC method

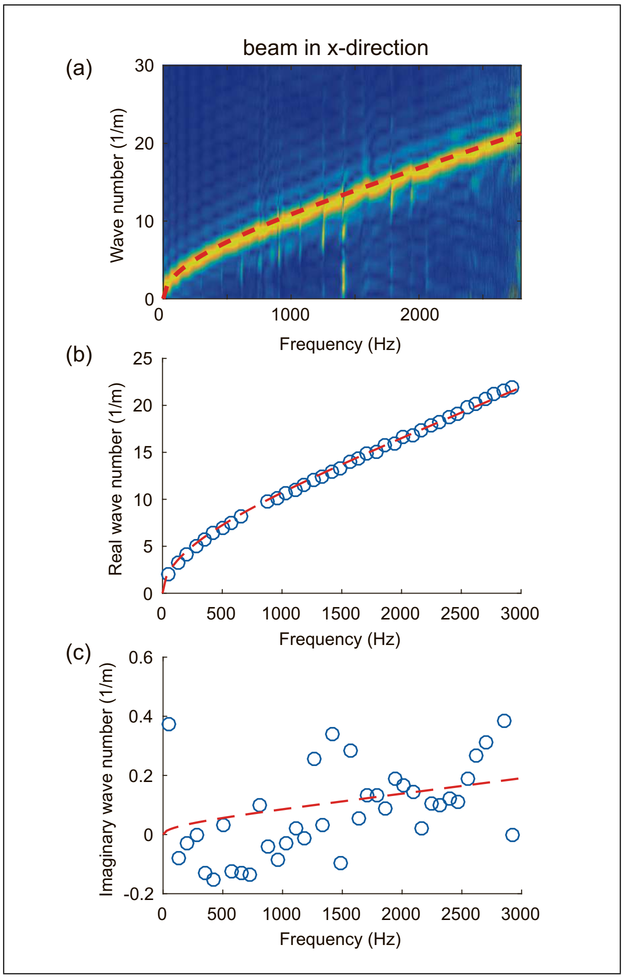

Figure 3. Measured dispersion relation of a CLT beam cut in the x-direction using spatial Fourier transform (a) and inhomogeneous wave correlation (b-c). The analytically predicted dispersion curve is shown as a red dashed line.

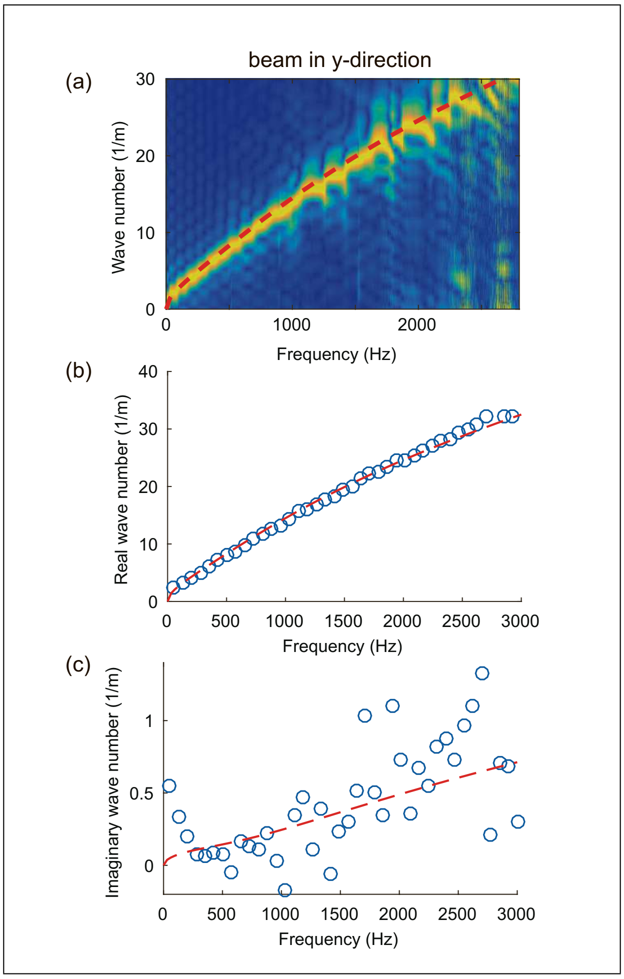

Figure 4. Measured dispersion relation of a CLT beam cut in the y-direction using spatial Fourier transform (a) and inhomogeneous wave correlation (b-c). The analytically predicted dispersion curve is shown as a red dashed line.

doesn't suffer from these problems when retrieving the real part of the wave number. Both in the low and high frequency range, the results are reliable.

A nonlinear least-square fit was performed on the IWC results to retrieve the elastic constants of the CLT beams. The analytical dispersion curves were calculated using the models in [8] and the best fit is shown in Figure 3 and Figure 4 as a red dotted line. The values for the elastic properties are given in Table I. There is a rather large difference in the found values for the Young's moduli. However, modal analysis was only performed for eigenfrequencies up to 1800 Hz, and it is possible that the results would deviate for measurements up to 3000 Hz as done for the dispersion curve. The shear modulus is typically hard to retrieve with great accuracy using modal analysis, since it only affects the highest modes, and the according frequencies are difficult to define accurately due to higher damping. However, the gathered values are in the same range for both methods, and fitting the dispersion values leads to a narrow error band of around 5%. This shows that the wave based approach of 2D-FFT and IWC can be advantageous.

Table I. Elastic constants of CLT beams gathered by dispersion curve fitting and modal analysis according to [8].

x-beam	Dispersion	Modal
E	9.9 ± 0.4 GPa	9.6 GPa
G	0.538 ± 0.013 GPa	0.60 GPa
y-beam	Dispersion	Modal
E_f	8.7 ± 1.3 GPa	11.0 GPa
G_c	0.150 ± 0.014 GPa	0.12 GPa

The imaginary part of the wave number, however, is very noisy due to the extremely low values. In order to reach the same order of magnitude, the imaginary part of the material parameters was set to 2% of the real part. For low-loss materials, modal analysis is a more reliable choice. Application of the linear prediction method on the complex transfer function in a single point of the beam has shown to give accurate attenuation values for each mode [8].

4. Beams with periodically varying thickness

4.1. Dispersion and band gaps

Beams with a periodic variation of the bending stiffness are widely studied as an archetypal example of phononic crystals for bending waves [27, 28, 29]. Due to multiple reflections of the waves at the boundaries, a series of band gaps is created. The geometry and material parameters define the location and width of these forbidden frequency bands. Also the depth of the band gaps, i.e. the efficiency of the spatial damping, varies within each band gap. The dispersion of the structure can be calculated analytically using periodic (Floquet) boundary conditions, yielding both the real and imaginary part of the wave number as a function of frequency [30]. We used the model proposed by Liu and Hussein [27], including shear effects, which provides a general calculation method for dispersion in periodic beams consisting of unit cells with thickness or material variation, or placed on regularly spaced supports. Each unit cell can have an arbitrary number of internal interfaces. The basic idea is to define a transfer matrix for the displacement, rotation angle, bending moment and shear force at each interface within the unit cell. Additionally, the Floquet theorem defines the transfer matrix between the left and right edge of the unit cell, leading to a single equation that relates the frequency and wave number in the periodic beam.

For this work, the method was applied to a 3D-printed polyamid beam ($E = 2.1$ GPa, $G = 1.0$ GPa, $\rho = 880\,\mathrm{kg/m^3}$) with varying thickness: 3 mm and 10 mm in 20 mm long sections. The width of the beam is 10 mm. The beam had 9 periods, resulting in a total length of 0.36 m. The beam was designed to show multiple band gaps with varying width and depth over the entire acoustic frequency range. A first, shallow band gap occurs between 900–1400 Hz, with a maximum value of the imaginare wave number equal to $6\,\mathrm{m^{-1}}$. Two larger band gaps can be found at 4000–8000 Hz and 11000–16000 Hz, with values for the imaginary part of the wave number reaching $30\,\mathrm{m^{-1}}$.

4.2. Measurement and result

Measurements were performed on a 3D-printed beam with properties described above. The beam was light enough to be mounted with one end directly on a shaker, using the same force sensor as described in 3.3. The other end could vibrate freely. The vibration velocity was measured in 87 points with a 3.0 mm spacing. The data were zero padded to 1024 points to reach a k-step of $1.52\,\mathrm{m^{-1}}$.

The 2D-FFT analysis (Figure 5a) is accurate in the first two pass bands up to 4000 Hz, although above 2500 Hz the signal-to-noise ratio diminishes substantially. In the band gaps, especially in the second one, the noise level is too high to discern the main wave number. Remarkably, in the pass band above 8000 Hz, this analysis method shows several branches of the dispersion curve with similar amplitudes. Since the geometry has a period d, the wave number solution shows a π/d-periodicity according to Floquet's theorem. Since the calculation of the dispersion curve is almost always done in the first Brillouin zone, the existence of waves with higher wave numbers at a certain frequency is often overlooked. This experimental method shows the simultaneous presence of the so called Bloch harmonics.

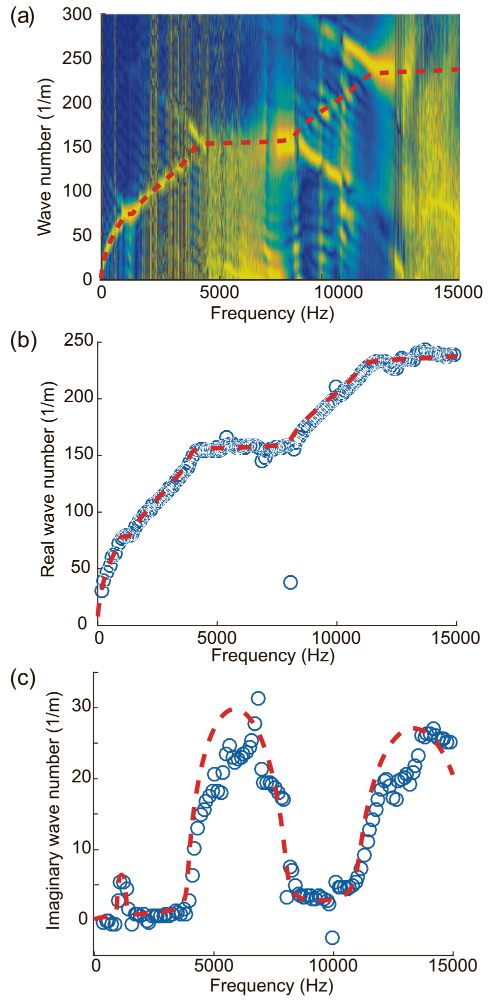

Figure 5. Measured dispersion relation of a polyamid beam with periodically varying thickness using spatial Fourier transform (a) and inhomogeneous wave correlation (b-c). The analytically predicted dispersion curve is shown as a red dashed line.

The results of the IWC analysis are astonishingly accurate (Figure 5b-c). The method does not seem to be sensitive to the problems the 2D-FFT technique encounters, such as high noise levels for higher frequencies, and the limited distance over which the measurement was performed. The real part of the wave number fits very well to the analytical model. The discrepancy at high values of the imaginary part (in the second and third band gap) is due to the low amplitude of the measured signals, since the excited vibrations are highly damped. Setting the imaginary part of the Young's modulus to $\eta_E = 3\%$ of the real part even captures the material damping accurately in the pass bands, up to 12000 Hz.

5. Conclusion

This paper compares two methods for the measurement of the dispersion of bending waves in beams. Two applications were discussed: deriving the elastic properties of cross laminated timber, and measuring the spatial damping of waves in the band gaps of a beam with periodic thickness variation.

For long cross laminated timber beams, both methods are equally accurate in the low to mid-frequency range up to 2 kHz. Since the dispersion is more sensitive to the material's shear modulus than the modal frequencies, this elastic constant can be retrieved more accurately than through standard modal analysis. A second advantage is the fact that dispersion is a property independent of the beam's boundary conditions, which is a major concern for modal analysis. A drawback is obviously the need of a larger amount of measurement points, since modal analysis in its most elementary form can be performed on a single transfer function. Moreover, inaccuracies occur in the calculation of the very low imaginary values of the wave number ($< 0.5\,\mathrm{m}^{-1}$) using the IWC method.

In case of small samples or worse signal-to-noise ratios, the IWC method is a better option than 2D-FFT. On short beams with periodically varying cross section, the correlation method performs well over the entire frequency range from 0 to 15 kHz. Moreover, it directly supplies a quantitative value for the spatial damping of traveling waves, which is a key point in the development and assessment of phononic crystals and metamaterials with band gaps.

Although both methods were applied on one-dimensional structures, line measurements can be made in different directions in order to retrieve direction-dependent stiffness parameters, e.g. of orthotropic plates, and to determine the efficiency of band gaps in metamaterial plates.

References

[1] J. M. Hodgkinson: Mechanical testing of advanced fibre composites. Elsevier, 2000.

[2] J. Bodig, B. A. Jayne: Mechanics of wood and wood composites. Krieger Publishing Company, Malabar, Florida, 1993.

[3] R. Steiger, A. Gulzow, D. Gsell: Non-destructive evaluation of elastic material properties of cross-laminated timber (CLT). Conference COST E, number October, 2008, 29–30.

[4] H. Yoshihara, Y. Kubojima, K. Nagaoka, M. Ohta: Measurement of the shear modulus of wood by static bending tests. Journal of Wood Science **44** (1998) 15–20.

[5] Y. H. Chui: Simultaneous evaluation of bending and shear moduli of wood and the influence of knots on these parameters. Wood Science and Technology **25** (1991) 125–134.

[6] A. Bolmsvik: Structural-acoustic vibrations in wooden assemblies: Experimental modal analysis and finite element modelling. Linnaeus University Press, 2013.

[7] S. I. Schubert, D. Gsell, J. Dual, M. Motavalli, P. Niemz: Rolling shear modulus and damping factor of spruce and decayed spruce estimated by modal analysis. Holzforschung **60** (2006) 78–84.

[8] B. Van Damme, S. Schoenwald, A. Zemp:. Modeling the bending vibration of cross-laminated timber beams. European Journal of Wood and Wood Products (2017) published online.

[9] E. Jansson, I. Bork, J. Meyer: Investigations into the acoustical properties of the violin. Acta Acustica united with Acustica **62** (1986) 1–15.

[10] R. Gonçalves, A. J. Trinca, B. P. Pellis: Elastic constants of wood determined by ultrasound using three geometries of specimens. Wood Science and Technology **48** (2014) 269–287.

[11] A. Santoni, S. Schoenwald, B. Van Damme, P. Fausti: Determination of the elastic and stiffness characteristics of cross-laminated timber plates from flexural wave velocity measurements. Journal of Sound and Vibration **400** (2017) 387–401.

[12] I. Roelens, F. Nuytten, I. Bosmans, G. Vermeir: In situ measurement of the stiffness properties of building components. Applied Acoustics **52** (1997) 289–309.

[13] I. Y. Solodov, R. Stoessel, G. Busse: Material characterization and nde using focused slanted transmission mode of air-coupled ultrasound. Research in Nondestructive Evaluation **15** (2004) 65–85.

[14] I. Solodov, G. Busse: Anisotropy of plate waves in composites: effects and implications. Acta Acustica united with Acustica **97** (2011) 678–685.

[15] R. Pagán, M. Recuero, W. Kropp: Wave content of wooden floors. Acta Acustica united with Acustica **100** (2014) 67–78.

[16] D. M. Profunser, E. Muramoto, O. Matsuda, O. B. Wright, U. Lang: Dynamic visualization of surface acoustic waves on a two-dimensional phononic crystal. Physical Review B - Condensed Matter and Materials Physics **80** (2009) 1–7.

[17] B. Van Damme, K. Van Den Abeele, L. YiFeng, O. Bou Matar: Time reversed acoustics techniques for elastic imaging in reverberant and nonreverberant media: an experimental study of the chaotic cavity transducer concept. Journal of Applied Physics **109** (2011).

[18] J. Berthaut, M. N. Ichchou, L. Jezequel: K-space identification of apparent structural behaviour. Journal of Sound and Vibration **280** (2005) 1125–1131.

[19] L. Van Belle, C. Claeys, E. Deckers, W. Desmet: Measurement of dispersion curves for locally resonant metamaterials with damping. In ISMA, International Conference on Noise and Vibration Engineering, 2016, 2115–2126.

[20] A. Geslain, S. Raetz, M. Hiraiwa, M. Abi Ghanem, S. P. Wallen, A. Khanolkar, N. Boechler, J. Laurent, C. Prada, A. Duclos, *et al.*: Spatial laplace transform for complex wavenumber recovery and its application to the analysis of attenuation in acoustic systems. Journal of Applied Physics **120** (2016) 135107.

[21] J. Vessby, B. Enquist, H. Petersson, T. Alsmarker: Experimental study of cross-laminated timber wall panels. European Journal of Wood and Wood Products **67** (2009) 211–218.

[22] S. Gagnon and C. Pirvu: CLT handbook: cross-laminated timber. FPInnovations, 2011. ISBN 9780864885531.

[23] A. Filiatrault, B. Folz: Performance-based seismic design of wood framed buildings. Journal of Structural Engineering **128** (2002) 39–47.

[24] A. Frangi, M. Fontana, E. Hugi, R. Jübstl: Experimental analysis of cross-laminated timber panels in fire. Fire Safety Journal **44** (2009) 1078–1087.

[25] S. Schoenwald, B. Zeitler, I. Sabourin, F. King: Sound insulation performance of cross laminated timber building systems. Proceedings of Internoise 2013 42 nd Interna-

tional Congress and Exposition on Noise Control Engineering, Innsbruck, Austria, 2013.

[26] E. Nilsson, A. C. Nilsson: Prediction and measurement of some dynamic properties of sandwich structures with honeycomb and foam cores. Journal of Sound and Vibration **251** (2002) 409–430.

[27] L. Liu, M. I. Hussein: Wave Motion in Periodic Flexural Beams and Characterization of the Transition Between Bragg Scattering and Local Resonance. Journal of Applied Mechanics **79** (2012) 011003.

[28] J. Wen, G. Wang, D. Yu, H. Zhao, Y. Liu: Theoretical and experimental investigation of flexural wave propagation in straight beams with periodic structures: Application to a vibration isolation structure. Journal of Applied Physics **97** (2005).

[29] G. Wang, J. Wen, X. Wen: Quasi-one-dimensional phononic crystals studied using the improved lumped-mass method: Application to locally resonant beams with flexural wave band gap. Physical Review B - Condensed Matter and Materials Physics, **71** (2005) 1–5.

[30] D. M. Mead: Wave Propagation in Continuous Periodic Structures: Research Contributions From Southampton, 1964-1995. Journal of Sound and Vibration **190** (1996) 495–524.