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A Formulation to Evaluate the Pressure Radiated from Thin Body Subjected to the Broadband Random Excitations Based on FAQP Method

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Summary

A thin-body frequency averaged quadratic pressure (TFAQP) formulation is derived for the numerical solution of the acoustic radiation from thin bodies subjected to broadband random excitations. The singular kernel of $1/r^2$ and hypersingular kernel of $1/r^3$, respectively, are solved by the explicit evaluation method with constant elements. Numerical example is given to verify the formulation for predicting the pressure radiated from thin bodies under broadband random excitations.

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1. Introduction

The numerical modeling of radiation of the sound from thin bodies, such as transducers, fins and wings, has many important applications in acoustics [1]. It has been well known that nearly singular behavior may occur for predicting acoustic radiation from thin bodies with the conventional boundary element method (conventional BEM) using the Helmholtz integral equation. Although, theoretically, using the multi-domain BEM formulation is the simplest way to circumvent the nearly singular difficulty, much pre-processing and larger system of equations are required for the relative large imaginary interface surface [2, 3, 4]. A more elegant way to solve the thin-body problem is to use the thin-body integral formulation in spite of involving evaluation of a hypersingular integral in the order of $1/r^3$ [5]. However, the prediction of noise radiated from the thin bodies presents difficulties for four main reasons as the same with conventional BEM: firstly, due to the huge size of matrices to handle at medium and high frequency, sufficient elements are dispersed to predict the pressure radiated from thin bodies for accuracy. Secondly, for the real-life thin bodies with a very sensitive vibro-acoustic behavior, the radiating pressure strongly varies at a given frequency for different objects of the identical industrial production due to small dispersion of geometry, material and manufacturing process. Thirdly, frequency averaging manipulation of each pure tone prediction at medium and high frequencies makes conventional BEM troublesome in the process of predicting noise radiated from thin bodies subjected to broad band excitation. Fourthly, it is impossible to describe the vibrational behavior of large and complex machines with the field-measured vibration greatly changing, especially at high frequencies, even if the same measurements are repeated [6, 7, 8, 9].

This paper proposes a method of robust prediction of the radiating frequency averaged quadratic pressure from thin bodies subjected to broadband random excitations.

In order to get robust predictions at medium and high frequencies, a general integral equation approach to predict the Frequency Averaged Quadratic Pressure (FAQP) radiated from vibrating bodies was proposed by Guyader in [7]. As mentioned in [7], the frequency averaging does not suppress the characteristic wavenumber problem appearing in the classical Helmholtz integral equation and regularization methods have to be used in a similar way to avoid the instability of the calculation. A combined energy boundary integral equation formulation (CEBIEF) was proposed by Gao and Li to overcome the non-uniqueness problem of the integral formulation of FAQP [10]. Based on the explicit Rayleigh integral equation, the direct prediction of FAQP was developed to predict the radiated noise from a baffled plate [9].

The goal of this paper is to derive a thin-body frequency averaged quadratic pressure (TFAQP) formulation for the prediction of the radiated noise from thin bodies submitted to a broadband random excitation, basing on the FAQP method. For the thin-body problem, the thin-body integral formulation involves the hypersingularity in the order of $1/r^3$. In the TFAQP formulation, the explicit evaluation

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method [11] to evaluate the hypersingular integral is employed.

2. Theoretical Formulation

2.1. Thin-body integral formulation [12]

Assume a thin body surrounded by an infinite acoustic medium of mean density ρ, and speed of sound c, as shown in Figure 1. In the thin-body formulation, the mid surface of the thin body is modeled and denoted by S_N, and the outer surface of the thin body is denoted by S. The thin body radiates the pressure with vibration in the infinite acoustic medium. For derivation purposes, an imaginary surface S_i is constructed to divide the acoustic domain into an exterior subdomain V^* and an interior subdomain V^-. Note that this imaginary surface will not be discretized in real numerical implementation. The imaginary surface is simply used here to derive the thin body boundary integral equation. The unit normal vector can be pointing into either side of the thin body. As shown in Figure 1, the side that is facing in the same direction as the normal vector $\boldsymbol{n}$ is called the "positive side", and the opposite side is called the "negative side". Consequently, the pressure on the positive side of the surface S_N is denoted by p^+, and that on the negative side is denoted by p^-.

The two separate Helmholtz integral equations can be written as Equation(1) and Equation (2), and one for the exterior problem and the other for the interior problem.

$$\begin{aligned}\int_{S_N+S_i}&\Big(p^+(Q)\frac{\partial\psi}{\partial n}(Q,L)\\&\quad+\mathrm{i}\rho\omega\psi(Q,L)V_n^+(Q)\Big)\,\mathrm{d}S(Q)\\&=\begin{cases}\frac{1}{2}p^+(L) & L\in S_N,\\ p(L) & L\in V^+,\\ 0 & L\in V^-,\end{cases}\end{aligned}\tag{1}$$

$$\begin{aligned}-\int_{S_N+S_i}&\Big(p^-(Q)\frac{\partial\psi}{\partial n}(Q,L)\\&\quad+\mathrm{i}\rho\omega\psi(Q,L)V_n^-(Q)\Big)\,\mathrm{d}S(Q)\\&=\begin{cases}\frac{1}{2}p^-(L) & L\in S_N,\\ 0 & L\in V^+,\\ p(L) & L\in V^-,\end{cases}\end{aligned}\tag{2}$$

where the function

$$\psi(Q,L)=\mathrm{e}^{-\mathrm{i}kR}/4\pi R\tag{3}$$

is the free-space Green's function in which $R=|Q-L|$, Q is any point on S_N, and L may be on S_N or in domain $V^+\cup V^+$. $\boldsymbol{n}$ is the normal direction of the boundary point Q. $\mathrm{i}=\sqrt{-1}$. $k=\omega/c$ is the wavenumber, ω is the angular frequency, c is the sound speed in the acoustic medium and ρ is the density of the acoustic medium. $p^+(Q)$ and $p^-(Q)$ denote the pressure on the positive and negative side of the surface S_N, respectively. $p(L)$ is the pressure at field point L. $V^+(Q)$ and $V^-(Q)$ denote the normal velocities on the positive and negative side of the surface S_N, respectively.

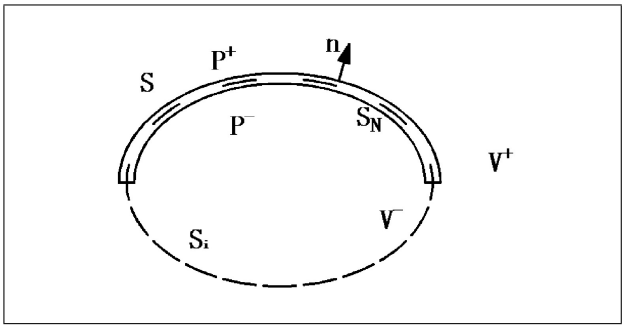

Figure 1. A thin body radiating pressure.

Because the contributions from the imaginary surface simply cancel out due to continuity of variables, Equation (4) can be derived by adding Equation (1) to Equation (2) with the assumption that the two sides of the thin body have the same normal velocity

$$\begin{aligned}&\int_{S_N}\big(p^+(Q)-p^-(Q)\big)\frac{\partial\psi}{\partial n}(Q,L)\,\mathrm{d}S(Q)\\&=\begin{cases}\frac{1}{2}\big(p^+(L)+p^-(L)\big) & L\in S_N,\\ p(L) & L\in V^+\cup V^-.\end{cases}\end{aligned}\tag{4}$$

And by first differentiating in the normal direction of a boundary point $\overline{L}$ on S_N, and then taking the limit as L approaches $\overline{L}$, a hypersingular integral equation can be written as

$$\begin{aligned}\int_{S_N}&\big(p^+(Q)-p^-(Q)\big)\frac{\partial\psi^2}{\partial n\partial n^L}(Q,L)\,\mathrm{d}S(Q)\\&=-\mathrm{i}\rho\omega V_n(L),\end{aligned}\tag{5}$$

where L is on S_N and n^L is the normal direction of the boundary point L.

2.2. Thin-body frequency averaged quadratic pressure formulation (TFAQP)

For the point P in the space $V^+\cup V^-$, Equation (4) can be written as

$$\begin{aligned}\int_{S_N}&\big(p^+(Q)-p^-(Q)\big)\frac{\partial\psi}{\partial n}(Q,L)\,\mathrm{d}S(Q)\\&=p(P),\quad P\in V^+\cup V^-.\end{aligned}\tag{6}$$

And the complex conjugate form of Equation(4) can be written as

$$\begin{aligned}\int_{S_N}&\big(p^{*+}(Q)-p^{*-}(Q)\big)\frac{\partial\psi^*}{\partial n}(Q,L)\,\mathrm{d}S(Q)\\&=p^*(P),\quad P\in V^+\cup V^-.\end{aligned}\tag{7}$$

Multiplying Equation (6) by Equation (7), one can get

$$\begin{aligned}\big|p(P)\big|^2=\int_S\int_S&\big(p^+(Q)-p^-(Q)\big)\big(p^{+*}(Q')-p^{-*}(Q')\big)\\&\cdot\frac{\partial\psi}{\partial n}(Q,P)\frac{\partial\psi^*}{\partial n}(Q',P)\,\mathrm{d}S(Q)\,\mathrm{d}S(Q').\end{aligned}\tag{8}$$

Expanding Equation (8), it follows the Equation (9) and Equation (10) for calculating the frequency averaged quadratic pressure radiated from thin bodies at point P in the acoustic medium.

$$\left|p(P)\right|^2 = \int_S \int_S \lambda \frac{\partial \psi}{\partial n}(Q, P) \frac{\partial \psi^*}{\partial n}(Q', P)\, \mathrm{d}S(Q)\, \mathrm{d}S(Q'), \tag{9}$$

$$\lambda = p^+(Q)p^{+*}(Q') + p^-(Q)p^{-*}(Q') - p^+(Q)p^{-*}(Q') - p^-(Q)p^{+*}(Q'), \tag{10}$$

where the point Q and Q' are located on the surface and the superscript "*" denotes the complex conjugate, and the collocation point P is outside S_N (i.e., in the acoustic domain).

As mentioned in [7], in general, statistical independence is expected for two signals having a lot of oscillations in a frequency band (statistical behavior) but also having different periodicity of frequency oscillations (independent phenomenon). In vibro-acoustics this behavior is in general true at medium and high frequency for two signals describing uncoupled phenomena. Hence, the basic assumption in [7], the statistical independence of boundary pressure and velocity and of the Green's function, means that the frequency average of a product is equal of the product of frequency averages. Equation (9) in a frequency band leads to

$$\left\langle \left|p(P)\right|^2 \right\rangle = \int_S \int_S \left\langle \lambda \right\rangle \left\langle \frac{\partial \psi}{\partial n}(Q, P) \frac{\partial \psi^*}{\partial n}(Q', P) \right\rangle \cdot \mathrm{d}S(Q)\, \mathrm{d}S(Q'), \tag{11}$$

$$\left\langle \lambda \right\rangle = S_{p^+p^+}(Q, Q') + S_{p^-p^-}(Q, Q') - S_{p^+p^-}(Q, Q') - S_{p^-p^+}(Q, Q'), \tag{12}$$

where the expressions of the sources are given by Equations (13)–(16), respectively.

$$S_{p^+p^+}(Q, Q') = \left\langle p^+(Q)p^{+*}(Q') \right\rangle, \tag{13}$$

$$S_{p^-p^-}(Q, Q') = \left\langle p^-(Q)p^{-*}(Q') \right\rangle, \tag{14}$$

$$S_{p^+p^-}(Q, Q') = \left\langle p^+(Q)p^{-*}(Q') \right\rangle, \tag{15}$$

$$S_{p^-p^+}(Q, Q') = \left\langle p^-(Q)p^{+*}(Q') \right\rangle, \tag{16}$$

where the points Q and Q' are located on the surface and the superscript '*' denotes the complex conjugate.

$$\left\langle \ \right\rangle = \frac{1}{2\Delta k} \int_{\Omega-\Delta k}^{\Omega+\Delta k} \mathrm{d}k. \tag{17}$$

Here,

$$\Delta k = \frac{2\pi\Delta}{c}, \tag{18}$$

where Δ is half of frequency bandwidth. The frequency average is made on a wavenumber bandwidth $2\Delta k$ centered at Ω.

The frequency averaged sound power can be calculated by

$$\left\langle \mathrm{power} \right\rangle = \sum_1^N \frac{1}{2} \Re e\left\{ S_{up}(i, i) \right\} S_i, \tag{19}$$

where S_i represents the area of the ith element and N denotes the total number of elements.

2.3. Integral equations for determination of boundary sources

With the assumption that the two sides of the thin body have the same normal velocity and multiply Equation (4) and Equation (5) by $V_n^*(L')$, the products write, respectively

$$\int_{S_N} \left(p^+(Q) - p^-(Q)\right) V_n^*(L') \frac{\partial \psi}{\partial n}(Q, L)\, \mathrm{d}S(Q) = \frac{1}{2}\left(p^+(L) + p^-(L)\right) V_n^*(L'), \tag{20}$$

$$\int_S \left(p^+(Q) - p^-(Q)\right) V_n^*(L') \frac{\partial \psi^2}{\partial n \partial n^L}(Q, L)\, \mathrm{d}S(Q) = -\mathrm{i}\rho k c V_n^*(L'), \tag{21}$$

where L and L' are located at the surface S_N.

With the frequency averaging (Equation 17) of Equation (20) and Equation (21) and the assumption [7], the statistical independence of boundary pressure and velocity and of the Green's functions, Equation (22) and Equation (23) are derived, respectively,

$$\int_{S_N} \left(S_{p^+u}(Q, L') - S_{p^-u}(Q, L')\right) \left\langle \frac{\partial \psi}{\partial n}(Q, L) \right\rangle \mathrm{d}S(Q) = \frac{1}{2}\left(S_{p^+u}(Q, L') + S_{p^-u}(Q, L')\right), \tag{22}$$

$$\int_{S_N} \left(S_{p^+u}(Q, L') - S_{p^-u}(Q, L')\right) \left\langle \frac{\partial \psi^2}{\partial n \partial n^L}(Q, L) \right\rangle \mathrm{d}S(Q) = \left\langle -\mathrm{i}\rho k c \right\rangle S_{uu}(L, L'). \tag{23}$$

Multiplying Equation (5) by $p^{+*}(L')$ and $p^{-*}(L')$, one can get the products, respectively,

$$\int_{S_N} \left(p^+(Q) - p^-(Q)\right) p^{+*}(L') \frac{\partial \psi^2}{\partial n \partial n^L}(Q, L)\, \mathrm{d}S(Q) = -\mathrm{i}\rho k c V_n(L) p^{+*}(L'), \tag{24}$$

$$\int_{S_N} \left(p^+(Q) - p^-(Q)\right) p^{-*}(L') \frac{\partial \psi^2}{\partial n \partial n^L}(Q, L)\, \mathrm{d}S(Q) = -\mathrm{i}\rho k c V_n(L) p^{-*}(L'). \tag{25}$$

With the frequency averaging (Equation(17)) of Equation (24) and Equation (25) and the assumption [7], the statistical independence of boundary pressure and velocity and of the Green's functions, Equation (26) and Equation (27) are derived, respectively

$$\int_{S_N} \left(S_{p^+p^+}(Q, L') - S_{p^-p^+}(Q, L')\right) \left\langle \frac{\partial \psi^2}{\partial n \partial n^L}(Q, L) \right\rangle \mathrm{d}S(Q) = \left\langle -\mathrm{i}\rho k c \right\rangle S_{up^+}(L, L'), \tag{26}$$

$$\int_S \left(S_{p^+p^-}(Q, L') - S_{p^-p^+}(Q, L')\right) \left\langle \frac{\partial \psi^2}{\partial n \partial n^L}(Q, L) \right\rangle \mathrm{d}S(Q) = \left\langle -\mathrm{i}\rho k c \right\rangle S_{up^-}(L, L'). \tag{27}$$

The resolution of the linear system (23) gives the frequency averaged cross intensity jump (FACIJ) source, $S_{p^+u}(Q, L') - S_{p^-u}(Q, L')$, from the frequency averaged cross velocity (FACV) source, $S_{uu}(L, L')$, that is supposed to be known. For the actual situations, the normal velocities of the thin-body strucure can be measured or calculated. The frequency averaged cross velocity sources, $S_{uu}(L, L')$, can be got by calculating average value of cross velocity in the bandwidth between normal velocities at point L on the element S_L and at point L' on the element $S_{L'}$. Then one can substitute this FACIJ source into Equation (22) to get the frequency averaged cross intensity sum (FACIS) source, $S_{p^+u}(Q, L') + S_{p^-u}(Q, L')$ across the thin surface. Once the FACIJ source and FACIS source are known, the frequency averaged cross intensity (FACI) source, $S_{p^+u}(Q, L')$ and $S_{p^-u}(Q, L')$, on either side of the thin body can be easily obtained. Because $S_{p^+u}(Q, L')$ is the conjugate transpose of $S_{up^+}(L', Q)$, FACI source can be used to get the frequency averaged cross pressure jump source, $S_{p^+p^+}(Q, L') - S_{p^-p^+}(Q, L')$ and $S_{p^+p^-}(Q, L') - S_{p^-p^-}(Q, L')$, on either side of the thin body by Equation (26) and Equation (27), respectively.

3. The numerical implementation

For the computational ease, the surface is discretized into N constant boundary elements. The application of the collocation method to Equation (23) and Equation (22) lead to the linear system, respectively

$$\sum_{Q=1}^{N} A(Q, L)\big(S_{p^+u}(Q, L') - S_{p^-u}(Q, L')\big) = \sum_{Q=1}^{N} B(Q, L) S_{uu}(Q, L'), \tag{28}$$

$$\sum_{Q=1}^{N} C(Q, L)\big(S_{p^+u}(Q, L') - S_{p^-u}(Q, L')\big) = \sum_{Q=1}^{N} D(Q, L)\big(S_{p^+u}(Q, L') - S_{p^-u}(Q, L')\big), \tag{29}$$

where $A(Q, L)$ and $B(Q, L)$ are calculated by Equation (30) and Equation (31),

$$A(Q, L) = \int_{S_Q} \left\langle \frac{\partial \psi^2}{\partial n \partial n^L}(Q, L) \right\rangle \mathrm{d}S, \tag{30}$$

$$B(Q, L) = \left\langle -\mathrm{i}\rho k c \right\rangle, \tag{31}$$

where $\left\langle \partial^2 G(Q, L)/\partial n \partial n^L \right\rangle$ and $\left\langle -\mathrm{i}\rho k c \right\rangle$ are given in Equation (32) and Equation (36), respectively.

$$\left\langle \frac{\partial^2 \psi(Q, P)}{\partial n \partial n^L} \right\rangle = -\frac{\mathrm{i}\rho c}{8\pi \Delta k R^4}\left(\lambda_1 + \lambda_2 + \lambda_3\right) \cdot \exp\left(-\mathrm{i}kR\right)\Big|_{\Omega-\Delta k}^{\Omega+\Delta k}, \tag{32}$$

$$\lambda_1 = -8\cos(R, n^L)\cos(R, n) + 2\cos(n, n^L), \tag{33}$$

$$\lambda_2 = \mathrm{i}kR\big(-5\cos(R, n^L)\cos(R, n) + \cos(n, n^L)\big), \tag{34}$$

$$\lambda_3 = k^2 R^2 \cos(R, n^L)\cos(R, n), \tag{35}$$

$$\left\langle -\mathrm{i}\rho k c \right\rangle = -\mathrm{i}\rho k c. \tag{36}$$

Here, R is the distance between point Q and collocation point L.

The $C(Q, L)$ and $D(Q, L)$ are calculated by Equation (37) and Equation (38)

$$C(Q, L) = \int_{S_Q} \left\langle \frac{\partial \psi}{\partial n}(Q, L) \right\rangle \mathrm{d}S, \tag{37}$$

$$D(Q, L) = \frac{1}{2}, \tag{38}$$

where $\left\langle \partial\psi/\partial n(Q, L) \right\rangle$ is given in Equation (39),

$$\left\langle \frac{\partial \psi}{\partial n}(Q, L) \right\rangle = \frac{\mathrm{i}}{8\pi \Delta k R^3}\frac{\partial R}{\partial n}\left[\mathrm{e}^{-\mathrm{i}kR}\left(2 + \mathrm{i}hR\right)\right]\Big|_{\Omega-\Delta k}^{\Omega+\Delta k}, \tag{39}$$

where R is the distance between point Q and collocation point L. Δk is the half of wavenumber bandwidth.

The application of the collocation method to Equation (26) and Equation (27) lead to the linear system

$$\sum_{Q=1}^{N} A(Q, L)\big(S_{p^+p^+}(Q, L') - S_{p^-p^+}(Q, L')\big) = \sum_{Q=1}^{N} B(Q, L) S_{up^+}(L, L'), \tag{40}$$

$$\sum_{Q=1}^{N} A(Q, L)\big(S_{p^+p^-}(Q, L') - S_{p^-p^-}(Q, L')\big) = \sum_{Q=1}^{N} B(Q, L) S_{up^-}(L, L'), \tag{41}$$

where $A(Q, L)$ and $B(Q, L)$ can also be calculated by Equation (30) and Equation (31).

It should be mentioned that the hypersingular integrals of $1/r^3$ arises in the process of evaluating the Equation (23) and Equation (26) and Equation (27). We use the explicit evaluation method [11] to solve this difficulty.

Once the boundary unknowns are solved, the frequency averaged quadratic pressure radiated from the thin bodies at any field point in the acoustic domain can be calculated by Equation (42)

$$\left\langle \left| p(P) \right|^2 \right\rangle = \sum_{Q=1}^{N}\sum_{Q'=1}^{N} E(Q, Q')\left\langle \lambda(Q, Q') \right\rangle, \tag{42}$$

where $E(Q, Q')$ can be calculated by

$$E(Q, Q') = \begin{cases} \dfrac{\mathrm{i}\frac{\partial R}{\partial n}\frac{\partial R'}{\partial n}}{32\pi^2 \Delta k R^2 R'^2 (R - R')} \cdot \Big[\mathrm{e}^{-\mathrm{i}k(R-R')}\Big(2 - \dfrac{2RR'}{(R-R')^2} + \mathrm{i}\dfrac{R^2 + R'^2 - 4RR'}{(R-R')}k + RR'k^2\Big)\Big]\Big|_{\Omega-\Delta k}^{\Omega+\Delta k}, & R \neq R', \\ \dfrac{\frac{\partial R}{\partial n}\frac{\partial R'}{\partial n}}{32\pi^2 \Delta k R^2 R'^2}\Big(k + \dfrac{RR'}{3}k^3\Big)\Big|_{\Omega-\Delta k}^{\Omega+\Delta k}, & R = R', \end{cases} \tag{43}$$

where R is the distance between point Q and point P. R' is the distance between point Q' and point P.

Table I. The Cartesian coordinate of excitation point and field points.

	Excitation point	Field point A	Field point B	Field point C	Field point D
Cartesian coordinate (m)	(0,0,0)	(0,0,2)	(0.5,0.5,1)	(1.5,1.5,1.5)	(2,2,20)

4. Validation of the approach

In order to demonstrate the validity of TFAQP method, a numerical example is designed to perform for a thin square plate with the length of the side $l = 2$ m and the thickness $t = 0.04$ m in an unbounded medium. The medium density $\rho = 1.2\,\mathrm{kg/m^3}$ and the sound speed $c = 340$ m/s. The thin plate with clamped supported boundary condition is subjected to broadband excitations and radiates the pressure.

As shown in the Figure 2, the Cartesian coordinate system is established at the geometric center O of thin plane. In the numerical example, a broadband random excitation F with the expectation of excitations $\mu = 100\,\mathrm{N}$ is applied at the geometric center O of thin plane divided into 400 constant elements. The frequency range of the broadband random excitation is from 1 Hz to 565 Hz and the frequency step Δf is equal to 1 Hz. The randomness of the excitation F increases with the increase of frequency. The variance of excitations is proportional to the frequency. Correspondingly, the maximum variance of excitations $\sigma = 30\,\mathrm{N^2}$ at the maximum frequency $f = 565$ Hz. Suppose the thin plate is excited at the center of the thin plate by ten sets of the broadband random excitations.

The coordinates of excitation point and field points are listed in Table I. The response of thin plate excited by the broadband random excitations can be determined with the finite element method, and then the frequency averaged cross velocity sources, $S_{uu}(L, L')$, can be got by calculating average value of cross normal velocity between two points on surface in the bandwidth.

For the percentage bandwidth analysis, ψ and ε are of the $1/n$-octave band determined by the following equations using the geometric center frequency of the percentage band, ψ_c, as

$$\psi = \frac{\psi_c}{2}\left(2^{\frac{1}{2n}} + 2^{-\frac{1}{2n}}\right), \tag{44}$$

$$\varepsilon = \frac{\psi_c}{2}\left(2^{\frac{1}{2n}} - 2^{-\frac{1}{2n}}\right), \tag{45}$$

where ψ denotes the arithmetic center frequency of the percentage band and ε denotes the half-bandwidth in linear scale. Considering the acoustic radiation from thin-body structures at a certain geometric center frequency, ψ_c, the frequency averaged cross velocity sources should be determined by averaging the cross velocities between two points on surface in a range of frequency from $\psi - \varepsilon$ to $\psi + \varepsilon$. Correspondingly, the frequency averaged quadratic pressure determined by Equation(11) represents the average sound pressure radiated from thin-body structures in the corresponding frequency range from $\psi - \varepsilon$ to $\psi + \varepsilon$.

In Figures 3–7, BEM-1/n octave represents that the sound pressure and sound power radiated from the thin plate under broadband random excitations for ten times

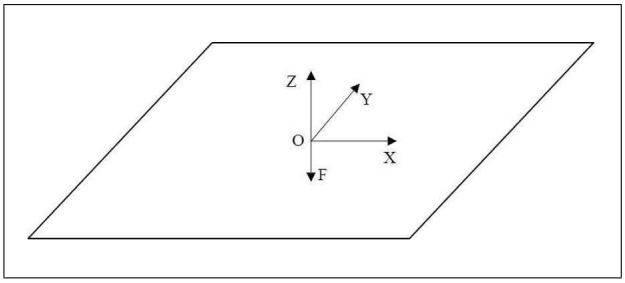

Figure 2. The schematic diagram of thin plate.

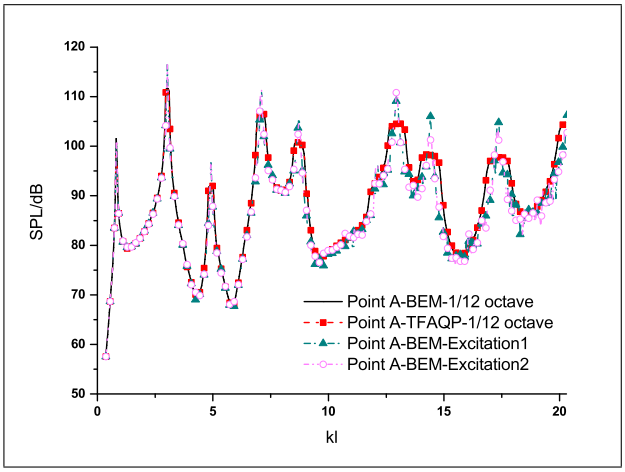

Figure 3. (Colour online) The SPL of averaged pressure with conventional BEM and the TFAQP method within 1/12 octave and the SPL of conventional BEM with two random excitations at field point A.

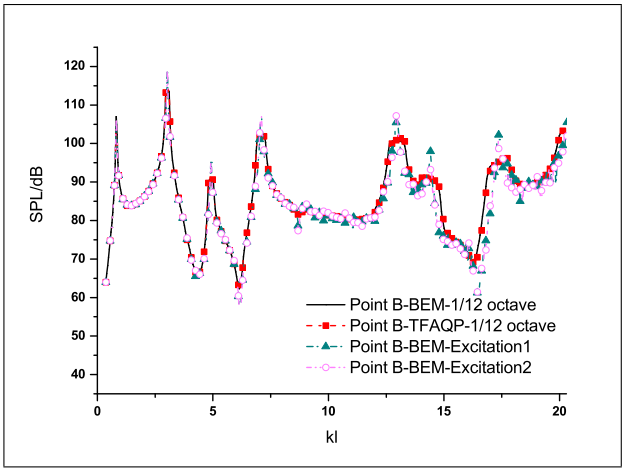

Figure 4. (Colour online) The SPL of averaged pressure with conventional BEM and the TFAQP method within 1/12 octave and the SPL of conventional BEM with two random excitations at field point B.

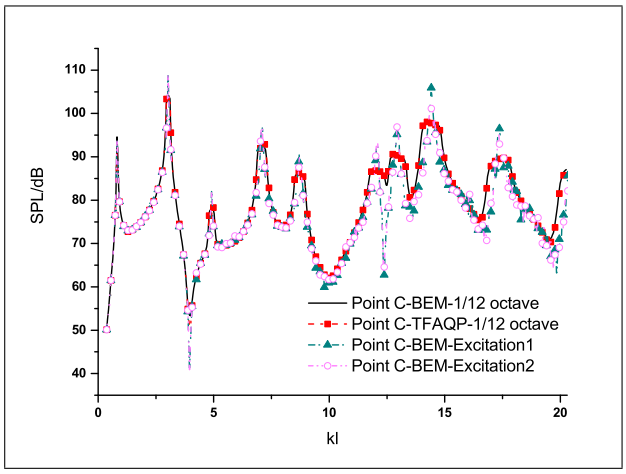

Figure 5. (Colour online) The SPL of averaged pressure with conventional BEM and the TFAQP method within 1/12 octave and the SPL of conventional BEM with two random excitations at field point C.

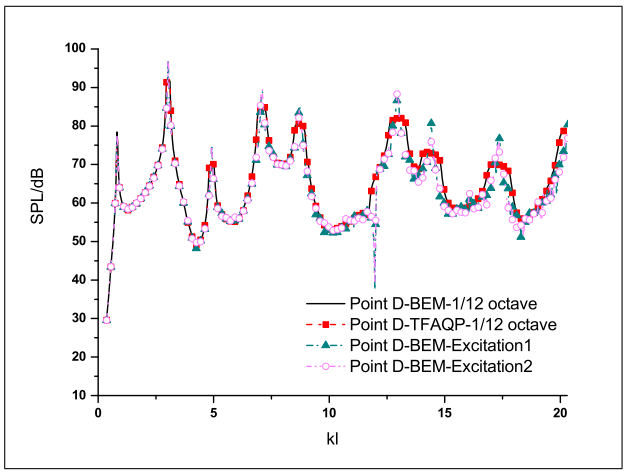

Figure 6. (Colour online) The SPL of averaged pressure with conventional BEM and the TFAQP method within 1/12 octave and the SPL of conventional BEM with two random excitations at field point D.

are calculated with conventional BEM and then the sound pressure and sound power get averaged within 1/n octave. TFAQP-1/n octave represents the frequency averaged quadratic pressure and sound power derived from averaged velocities, which are the response of the thin plate subjected to broadband random excitations. BEM-Excitation1 and BEM-Excitation2 is two of ten random excitations to represent the sound pressure and sound power with randomness. Figures 3–6 show that the difference of sound pressure level (SPL) between two random excitations become large because the randomness of excitations increases with the increase of frequency. The SPL of the TFAQP method gets good agreement with the SPL of averaged pressure with conventional BEM within 1/12 octave. Figure 7 presents the sound power level (PWL) of averaged sound power with conventional BEM and the TFAQP method within 1/12 octave and 1/3 octave and the PWL of conventional BEM under different random excitations.

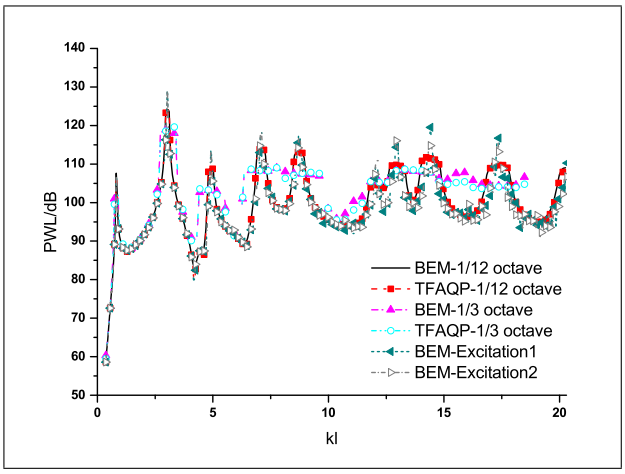

Figure 7. (Colour online) The PWL of averaged pressure with conventional BEM and the TFAQP method within 1/12 octave and 1/3 octave.

Similarly, the difference of PWL between two random excitations become large with the randomness of excitations increasing at high frequency. Comparison of the PWL of the TFAQP method within 1/12 octave and 1/3 octave indicates that the excessively wide bandwidth increases the error at some frequencies, although the PWL of the TFAQP method within 1/3 octave gets good agreement with the PWL of averaged sound power with conventional BEM at most frequencies.

5. Conclusion

In this paper, we proposed a new integral equation, TFAQP, to get robust predictions of the frequency averaged quadratic pressure radiated from the thin bodies subjected to broadband random excitations. The hypersingular integrals of $1/r^3$ involved in the TFAQP method is evaluated with the explicit evaluation method. The method is validated by performing a thin plate subjected to broadband random excitations. In numerical example, the frequency-band averaged quadratic pressure and sound power obtained by the conventional boundary element are taken as a reference to compare the SPL and PWL of the TFAQP method within 1/12 octave and 1/3 octave. The numerical results show that the TFAQP method can be used to evaluate the acoustic pressure radiated from the thin bodies under broadband random excitations.

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