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Sound Insulation Characteristics of Functionally Graded Panels

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Summary

An exact treatment based on the two-dimensional elasticity theory, is employed to investigate sound transmission through an arbitrarily thick isotropic functionally graded infinite plate subjected to an oblique plane acoustic incident wave. The mechanical properties of the graded plate are assumed to vary smoothly and continuously with the change of volume concentrations of the constituting materials across the thickness of the plate according to a power law distribution. The original inhomogeneous structure is approximated by a laminate model, for which the solution is expected to gradually approach the exact one as the number of layers increases. The T -matrix solution technique, which involves a system global transfer matrix formed as the product of the individual transfer matrices by applying continuity of the displacement and stress components at the interfaces of neighboring layers along with the pertinent boundary conditions at the upper and lower interfaces of the plate with the surrounding acoustic fluid (air) are employed to solve for the unknown plane wave reflection and transmission coefficients. The analytical results are illustrated with numerical examples in which three frequently used metal-ceramic FGM panels with distinct compositional gradient profiles are subjected to plane incident waves at selected angles of incidence. Furthermore, a Gaussian weighting function for describing the directional distribution of incident energy is adopted for predicting the sound transmission loss along with the weighted sound reduction index, R_w (as specified in ISO 717-1), of the FGM panels subjected to a perfectly diffuse sound field. Primary attention is focused on the effects of material compositional gradient profiles on the sound insulation characteristics of the panels. Limiting cases are considered and good agreements with the solutions available in the literature are obtained.

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1. Introduction

Advances in technology have accelerated the use of laminated lightweight panels in various spheres of engineering, ranging from the construction of buildings to the automobiles, machines and structures, shipbuilding and aerospace industries, where there are considerable interest in knowing their capacity for acoustic insulation. Research teams have been working for many years on the development of analytical and numerical techniques for determining how these elements perform acoustically. Numerous investigators have studied various aspects of this problem in great details. The most relevant recent contributions are only cited here. Jackins and Gaunaurd [1] formulated the resonance scattering theory (RST) for the general case of acoustic reflection from (and transmission through) a stack of arbitrarily many elastic plates. Chonan and Kugo [2, 3] used two-dimensional elasticity theory to study the sound transmission characteristics of two- and three-layered infinite plates excited by a plane acoustic wave. Sastry and

Munjal [4] employed a transfer matrix approach to evaluate the response of a multi-layer infinite plate to a given two-dimensional pressure excitation on one of its faces or alternatively to evaluate the acoustic pressure distribution excited by the normal velocity components of the radiating surfaces. Cai *et al.* [5] presented an exact matrix formulation for computing reflection and transmission of sound waves by anisotropic laminates submerged in fluids. Kang *et al.* [6] used a Gaussian distribution function to propose a new prediction method for sound transmission loss of multilayered panels of infinite extent. Just recently, Lin *et al.* [7] provided a mathematical model for predicting sound transmission loss characteristics of an especially orthotropic laminate composite panel, where the material properties in thickness direction are different from those in in-plane directions but it is isotropic in the laminating plane. Also, Tadeu *et al.* [8] used Green's functions to assess the airborne sound and impact insulation provided by homogeneous and multilayer infinite partitions.

The study of functionally graded materials (FGMs) has recently attracted a lot of attention. FGMs are advanced composites, microscopically engineered to have a smooth spatial variation of material properties in order to im-

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prove overall performance. This is achieved by fabricating the composite material to have a gradual spatial variation in the constituent materials' relative volume fractions and microstructure, thus tailoring its material composition based on functional performance requirements. FGMs offer great promise in applications where the operating conditions are severe, including spacecraft heat shields (thermal barrier structures), heat exchanger tubes, biomedical implants, flywheels, fusion reactors, blades, storage tanks, pressure vessels, and general wear and corrosion resistant coatings or for joining dissimilar materials in aerospace, automobile and defense industries [9]. A great deal of research work has recently investigated dynamic characteristics of inhomogeneous panels or plates [10, 11]. In particular, several researchers have focused on wave propagation in FGM plates [12]. Liu and Tani [13] used a strip element method to investigate surface waves in inhomogeneous plates. They found that Lamb and SH waves propagate in a form of surface waves on the softer surface of the plates. Miura *et al.* [14] proposed an analytical method to analyze elastic (SH-) wave reflection from a gradient inhomogeneous layer. Han *et al.* [15] employed Fourier transformation and modal analysis to analyze transient waves in FGM plates excited by impact loads. Han and Liu [16] used a quadratic layer element to investigate SH waves in an FGM plate where the material property variation is assumed as quadratic function in the thickness direction. Chakraborty and Gopalakrishnan [17] used a spectral element (SE) model to analyze wave propagation in anisotropic inhomogeneous plates. Just recently, Chen *et al.* [18] studied the dispersion behavior of waves in an elastic plate with material properties varying along the thickness direction over a wide range of wave numbers. They approximated the inhomogeneous plate by a laminate plate model consisting of a sufficiently large number of homogeneous layers.

The above review indicates that while there exists a notable body of literature investigating sound transmission (dynamic) characteristics of multilayered or laminated (functionally graded) plates, rigorous analytic or numerical solutions for airborne sound insulation characteristics of a functionally graded panel seem to be absent. The primary purpose of the current work is to fill this gap. The proposed model is of noble interest basically due to its inherent value as a canonical problem in structural acoustics. It is of practical value for noise control engineers involved in dynamic analysis and design of FGM composite panels or plates which are of widespread use in a variety of engineering applications such as thermal-barrier structures, wear- and corrosion-resistant coatings, armor plating, and heat engine components [9]. It can lead to the development of reliable analytical and/or experimental tools for the design of FGM plates or panels with optimal sound insulation characteristics. It can also aid in nondestructive characterization and evaluation of FGM panel structures [19, 20]. The presented exact solution can serve as the benchmark for comparison to solutions obtained by strictly numerical or asymptotic approaches.

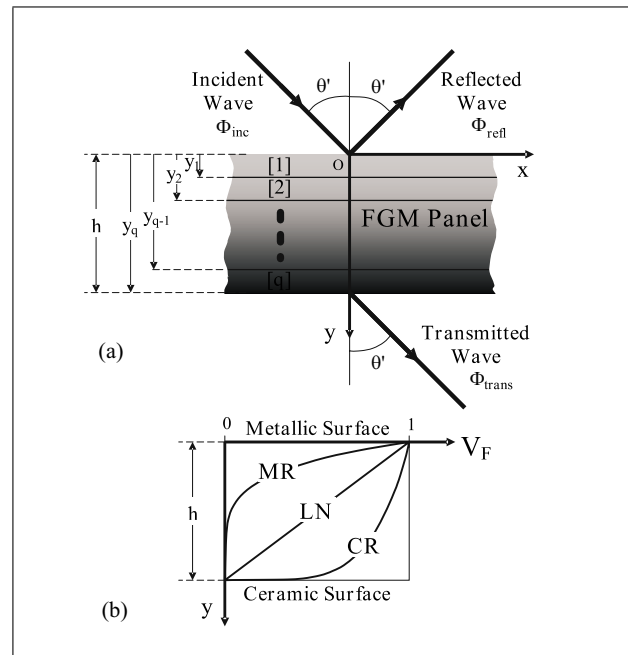


Figure 1. a. Problem geometry. b. Variations in the volume fraction of metallic constituent along the y -axis of the FGM plate.

2. Formulation

2.1. Acoustic field equations

The problem geometry is depicted in Figure 1a, where (x, y, z) is the Cartesian coordinate system with origin at O . The plate is infinite in the x, z -plane, and the y -axis is normal to the plate boundary. It is submerged in an unbounded acoustic fluid medium, and is insonified from above by a time-harmonic plane acoustic wave with an angle θ with respect to the normal of the plate. Following the standard methods of theoretical acoustics, the field equations for an inviscid and ideal compressible medium that can not support shear stresses may be expressed in terms of a scalar velocity potential as [21]

$$\mathbf{v} = -\nabla\varphi, \quad p = \rho_0\dot{\varphi}, \quad (1)$$

where ρ_0 is the fluid density, $\mathbf{v}$ is the fluid particle velocity vector, p is the acoustic pressure, ∇ is the gradient operator, and the (scalar) velocity potential satisfies the classical wave equation [21]

$$\nabla^2\varphi = \frac{1}{c_0^2} \frac{\partial^2\varphi}{\partial t^2}, \quad (2)$$

in which c_0 is the speed of sound, and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian. The solution of above equation may be assumed in the separable form [3]

$$\varphi(x, y, t) = \Phi(y)e^{i(\bar{k}x - \omega t)}, \quad (3)$$

where $\bar{k} = k \sin \theta$ is the wave number in x -direction, $k = \omega/c_0$ is the acoustic wave number, ω is the circular frequency, and $\Phi(y)$ is an undetermined function. Assuming that the incident wave is time-harmonic, while keeping

in mind the radiation condition, and making use of (3) in (2), the solutions of resulting Helmholtz equation in the upper and lower fluid mediums may respectively written as [3]

$$\begin{aligned}\Phi^{(1)}(y) &= \Phi_{\text{inc}} e^{-iyk \cos \theta} + \Phi_{\text{refl}} e^{iyk \cos \theta}, \\ \Phi^{(2)}(y) &= \Phi_{\text{trans}} e^{-iyk \cos \theta},\end{aligned}\quad (4)$$

where Φ_{inc} , Φ_{refl} and Φ_{trans} are the amplitudes of incidence, reflected and transmitted waves, respectively (see Figure 1a), and the superscript (1) refers to the field variable in the upper fluid medium, while the superscript (2) signifies those in the lower fluid medium. Here we note that in region 1, both incoming and outgoing waves exist while in region 2, only outgoing waves are possible. Also, using the field equations (1), the normal fluid velocity and acoustic pressure in the upper and lower fluid mediums are respectively written as

$$\begin{aligned}\mathbf{v}_y^{(1)}(x, y, t) &= -\frac{\partial \Phi^{(1)}}{\partial y} \\ &= ik \cos \theta \left(\Phi_{\text{inc}} e^{-iyk \cos \theta} - \Phi_{\text{refl}} e^{iyk \cos \theta} \right) e^{i(\bar{k}x - \omega t)}, \\ \mathbf{v}_y^{(2)}(x, y, t) &= -\frac{\partial \Phi^{(2)}}{\partial y} \\ &= ik \cos \theta \Phi_{\text{trans}} e^{-iyk \cos \theta} e^{i(\bar{k}x - \omega t)},\end{aligned}\quad (5)$$

and

$$\begin{aligned}p^{(1)}(x, y, t) &= -i\omega \rho_0 \Phi^{(1)} \\ &= -i\omega \rho_0 \left(\Phi_{\text{inc}} e^{-iyk \cos \theta} + \Phi_{\text{refl}} e^{iyk \cos \theta} \right) e^{i(\bar{k}x - \omega t)}, \\ p^{(2)}(x, y, t) &= -i\omega \rho_0 \Phi^{(2)} \\ &= -i\omega \rho_0 \Phi_{\text{trans}} e^{-iyk \cos \theta} e^{i(\bar{k}x - \omega t)}.\end{aligned}\quad (6)$$

2.2. The FGM plate

For an isotropic material with λ and μ being Lamé constants, the equations of motion in the absence of body forces are expressed as [22]

$$\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla (\nabla \cdot \mathbf{u}) = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}, \quad (7)$$

where $\mathbf{u}$ is the displacement vector, ρ and is the solid material density. Also, the constitutive relations are [22]

$$\sigma_{ij} = \lambda (\nabla \cdot \mathbf{u}) \delta_{ij} + \mu (u_{i,j} + u_{j,i}), \quad (8)$$

where σ_{ij} is the component of stress tensor, and δ_{ij} is the Kronecker delta function. The Helmholtz theorem, allows us to divide the displacement field into an irrotational part and a rotational part as [22]

$$\mathbf{u} = \nabla \phi + \nabla \times \boldsymbol{\psi}, \quad (9)$$

where ϕ is the scalar potential, and $\boldsymbol{\psi}$ is the vectorial potential. Thus, considering the plane strain problem with the

displacement component in the y -direction, i.e., assuming $\boldsymbol{\psi} = (0, 0, \psi)$, the x - and y -displacement components are written as

$$u_x = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}, \quad u_y = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}, \quad (10)$$

where the elastic wave potentials satisfy the Helmholtz equations [22]

$$\nabla^2 \phi = \frac{1}{c_p^2} \frac{\partial^2 \phi}{\partial t^2}, \quad \nabla^2 \psi = \frac{1}{c_s^2} \frac{\partial^2 \psi}{\partial t^2}, \quad (11)$$

in which $c_p = \sqrt{(\lambda + 2\mu)/\rho}$ and $c_s = \sqrt{\mu/\rho}$ are the velocities of P- and S- waves, respectively. Also, the relevant stress components are [22]

$$\begin{aligned}\sigma_{yy} &= (\lambda + 2\mu) \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) - 2\mu \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \psi}{\partial x \partial y} \right), \\ \sigma_{xy} &= \mu \left(2 \frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2} \right).\end{aligned}\quad (12)$$

Next, assuming the wave solutions in the form [18]

$$\phi = \phi_0(y) e^{i(\bar{k}x - \omega t)}, \quad \psi = \psi_0(y) e^{i(\bar{k}x - \omega t)}, \quad (13)$$

where $\phi_0(y)$ and $\psi_0(y)$ are undetermined functions. Substituting equation (13) into equation (11) gives

$$\frac{d^2 \phi_0}{dy^2} + K_p^2 \phi_0 = 0, \quad \frac{d^2 \psi_0}{dy^2} + K_s^2 \psi_0 = 0, \quad (14)$$

where $K_p = \sqrt{(\omega/c_p)^2 - \bar{k}^2}$ and $K_s = \sqrt{(\omega/c_s)^2 - \bar{k}^2}$ are the wave numbers along the y -direction for the P- and S- waves, respectively [22]. The solution to equation (14) is well known [18]

$$\begin{aligned}\phi_0(y) &= A_1 e^{iK_p y} + A_2 e^{-iK_p y}, \\ \psi_0(y) &= B_1 e^{iK_s y} + B_2 e^{-iK_s y}.\end{aligned}\quad (15)$$

Making use of equations (13) and (15) in equations (10) and (12), the displacements and stresses can then be respectively written as

$$\begin{aligned}u_x(x, y, t) &= \left[\bar{k} \left(A_1 e^{iK_p y} + A_2 e^{-iK_p y} \right) \right. \\ &\quad \left. + K_s \left(B_1 e^{iK_s y} - B_2 e^{-iK_s y} \right) \right] i e^{i(\bar{k}x - \omega t)}, \\ u_y(x, y, t) &= \left[K_p \left(A_1 e^{iK_p y} - A_2 e^{-iK_p y} \right) \right. \\ &\quad \left. - \bar{k} \left(B_1 e^{iK_s y} + B_2 e^{-iK_s y} \right) \right] i e^{i(\bar{k}x - \omega t)},\end{aligned}\quad (16)$$

and

$$\begin{aligned}\sigma_{yy}(x, y, t) &= \left[2\mu \bar{k} K_s \left(B_1 e^{iK_s y} - B_2 e^{-iK_s y} \right) \right. \\ &\quad \left. - (\lambda \bar{k}^2 + (\lambda + 2\mu) K_p^2) \right. \\ &\quad \left. \cdot \left(A_1 e^{iK_p y} + A_2 e^{-iK_p y} \right) \right] e^{i(\bar{k}x - \omega t)}, \\ \sigma_{xy}(x, y, t) &= \left[(\bar{k}^2 - K_s^2) \left(B_1 e^{iK_s y} + B_2 e^{-iK_s y} \right) \right. \\ &\quad \left. - 2\bar{k} K_p \left(A_2 e^{-iK_p y} - A_1 e^{iK_p y} \right) \right] \mu e^{i(\bar{k}x - \omega t)}.\end{aligned}\quad (17)$$

Now let us consider a dual-phase functionally graded plate of uniform thickness h with variable material properties along the y -direction (Figure 1b). Adopting a laminate model, the FGM plate is assumed to be composed of layers of homogeneous isotropic materials which are perfectly bonded at their interface. Assuming that the applied incident field causes equal strains in the two phases of the FGM plate (i.e., the overall composite stress is the sum of stresses carried by each phase), the mechanical properties within each layer of uniform thickness $h_j = y_j - y_{j-1}$ ($j = 1, 2, \dots, q$; $y_0 = 0$, $y_q = h$) may quite accurately be estimated using a simple effective medium theory such as the Voigt rule of mixtures [23, 24]. Accordingly, the effective mechanical properties in each layer are expressed as a linear combination of those of individual constituents weighted by the volume fraction of each phase [25, 26], i.e.,

$$\begin{aligned}\lambda^{[j]} &= V_F(\bar{y}_j)\lambda^{[1]} + [1 - V_F(\bar{y}_j)]\lambda^{[q]}, \\ \mu^{[j]} &= V_F(\bar{y}_j)\mu^{[1]} + [1 - V_F(\bar{y}_j)]\mu^{[q]}, \\ \rho^{[j]} &= V_F(\bar{y}_j)\rho^{[1]} + [1 - V_F(\bar{y}_j)]\rho^{[q]},\end{aligned}\quad (18)$$

where $\bar{y}_j = (y_j + y_{j+1})/2$, and $(\lambda^{[j]}, \mu^{[j]}, \rho^{[j]})$, and $V_F(\bar{y}_j)$ are the Lamé constants, mass density, and volume fraction of constituting material in the j -th layer of the multilayered plate, respectively. Furthermore, keeping equations (13) and (15) in view, the relevant potentials corresponding to the compressional and shear waves transmitted into the j -th layer may conveniently be represented in the form

$$\begin{aligned}\phi_j &= \left[A_1^{[j]} e^{iyK_p^{[j]}} + A_2^{[j]} e^{-iyK_p^{[j]}} \right] i e^{i(\bar{k}x - \omega t)}, \\ \psi_j &= \left[B_1^{[j]} e^{iyK_s^{[j]}} + B_2^{[j]} e^{-iyK_s^{[j]}} \right] i e^{i(\bar{k}x - \omega t)},\end{aligned}\quad (19)$$

where $K_p^{[j]} = \sqrt{(\omega/c_p^{[j]})^2 - \bar{k}^2}$, $K_s^{[j]} = \sqrt{(\omega/c_s^{[j]})^2 - \bar{k}^2}$ ($j = 1, 2, \dots, q$) are the compressional and shear wave numbers in each layer, respectively, and $A_{1,2}^{[j]}$ and $B_{1,2}^{[j]}$ are unknown wave transmission constants.

2.3. Method of solution (T-matrix approach)

The specific steps taken in constructing the model for making use of the transfer matrix solution technique are described as follows [27]. After constructing the formal solution for the relevant field variables (i.e., stresses and displacements) associated with each layer in the multilayered (functionally graded) plate in terms of reflection and transmission coefficients, these solutions are specialized to the upper and lower interfaces of the layer. Subsequently, the local transfer matrix for each layer is constructed by elimination of the common transmission coefficients. Such a matrix relation can be used, in conjunction with satisfying appropriate interface conditions across neighboring layers, to directly relate the field variables of the lower face of one layer to the upper face of its outer neighbor. When this procedure is carried out consecutively for all layers in the bundle, a global transfer matrix (i.e., the product of the individual transfer matrices) results, which connects

the boundary variables at the upper and lower surfaces of the multi-layered plate. In this way, the noble advantage of using the global transfer matrix saves us from tedious implementation of the boundary conditions at all $q+1$ interfaces (i.e., only the boundary conditions on the first and the last interface need to be imposed). Consequently, substitution of the scalar potentials into the constitutive relations, leads to the matrix form of the formal solutions for the field variables (stress and displacement components) associated with the j -th layer [27], i.e.,

$$\mathbf{W}^{[j]} = \mathbf{Q}^{[j]} \mathbf{X}^{[j]}, \quad (20)$$

where

$$\begin{aligned}\mathbf{W}^{[j]} &= [\sigma_{yy}^{[j]}, \sigma_{xy}^{[j]}, u_x^{[j]}, u_y^{[j]}]^T, \\ \mathbf{X}^{[j]} &= [A_1^{[j]}, A_2^{[j]}, B_1^{[j]}, B_2^{[j]}]^T,\end{aligned}\quad (21)$$

in which the elements of the coefficient matrix $\mathbf{Q}^{[j]}$ are given in the Appendix, and the displacement and stress components in the j -th layer are respectively expressed as

$$\begin{aligned}u_x^{[j]} &= \frac{\partial \phi_j}{\partial x} + \frac{\partial \psi_j}{\partial y}, \quad u_y^{[j]} = \frac{\partial \phi_j}{\partial y} - \frac{\partial \psi_j}{\partial x}, \\ \sigma_{yy}^{[j]} &= (\lambda^{[j]} + 2\mu^{[j]}) \left(\frac{\partial^2 \phi_j}{\partial x^2} + \frac{\partial^2 \phi_j}{\partial y^2} \right) \\ &\quad - 2\mu^{[j]} \left(\frac{\partial^2 \phi_j}{\partial x^2} + \frac{\partial^2 \psi_j}{\partial x \partial y} \right), \\ \sigma_{xy}^{[j]} &= \mu^{[j]} \left(2 \frac{\partial^2 \phi_j}{\partial x \partial y} + \frac{\partial^2 \psi_j}{\partial y^2} - \frac{\partial^2 \psi_j}{\partial x^2} \right).\end{aligned}\quad (22)$$

The matrix relation (20) can advantageously be specialized for the lower and upper interfaces of the j -th layer as

$$\mathbf{W}^{[j+]} = \mathbf{Q}^{[j+]} \mathbf{X}^{[j]}, \quad \mathbf{W}^{[j-]} = \mathbf{Q}^{[j-]} \mathbf{X}^{[j]}, \quad (23)$$

where the superscripts $[j-]$ and $[j+]$ denote the quantities evaluated at $y = y_{j-1}$ and $y = y_j$, respectively. The above equations can be supplemented with the continuity conditions between each interface layer, i.e., $\mathbf{W}^{[j+]} = \mathbf{W}^{[(j+1)-]}$ at $y = y_j$. Next, by eliminating the common amplitude vector $\mathbf{X}^{[j]}$ in equation (23), the field variable vector $\mathbf{W}^{[j+]}$ may be related to $\mathbf{W}^{[j-]}$ by $\mathbf{W}^{[j+]} = \mathbf{M}^{[j]} \mathbf{W}^{[j-]}$, where $\mathbf{M}^{[j]} = \mathbf{Q}^{[j+]}[\mathbf{Q}^{[j-]}]^{-1}$, is the local transfer matrix for j -th layer, which relates the field variables at its upper surface to those at the lower surface. Subsequently, by invoking the continuity conditions between all interface layers, the field variables at the upper boundary of the total system (i.e., at $y = y_0 = 0$) is related to those at the lower boundary (i.e., at $y = y_q = h$) by

$$\mathbf{W}^{[q+]} = \mathbf{T} \mathbf{W}^{[1-]}, \quad (24)$$

where $\mathbf{T} = \mathbf{M}^{[q]} \mathbf{M}^{[q-1]} \dots \mathbf{M}^{[1]}$ is the so called (4×4) global transfer matrix [28].

The unknown coefficients Φ_{refl} and Φ_{transl} , in addition to the elements of the field variable vector, $\mathbf{W}^{[1-]}$ can be

determined from the appropriate boundary conditions imposed at the upper and the lower boundaries of the multilayered plate. Thus, assuming continuity of normal fluid and solid velocities, normal stress and fluid pressure, and vanishing of tangential stress at $y = 0, h$ implies that [28]

$$\begin{aligned} -i\omega u_y(x, y, t) \Big|_{y=0,h} &= v_y^{(1,2)}(x, y, t) \Big|_{y=0,h}, \\ \sigma_{yy}(x, y, t) \Big|_{y=0,h} &= -p^{(1,2)}(x, y, t) \Big|_{y=0,h}, \\ \sigma_{xy}(x, y, t) \Big|_{y=0,h} &= 0. \end{aligned} \quad (25)$$

Finally, making use of equations (5), (6), and (25) in equation (24), we obtain

$$\begin{aligned} \omega T_{1,3} u_x^{[1-]} + (k \cos \theta T_{1,4} - i\rho_0 \omega^2 T_{1,1}) \Phi_{\text{refl}} \\ + i\rho_0 \omega^2 e^{-ihk \cos \theta} \Phi_{\text{trans}} &= (i\rho_0 \omega^2 T_{1,1} + k \cos \theta T_{1,4}) \Phi_{\text{inc}}, \\ \omega T_{2,3} u_x^{[1-]} + (k \cos \theta T_{2,4} - i\rho_0 \omega^2 T_{2,1}) \Phi_{\text{refl}} \\ &= (i\rho_0 \omega^2 T_{2,1} + k \cos \theta T_{2,4}) \Phi_{\text{inc}}, \\ \omega T_{3,3} u_x^{[1-]} + (k \cos \theta T_{3,4} - i\rho_0 \omega^2 T_{3,1}) \Phi_{\text{refl}} \\ - \omega u_x^{[h+]} &= (i\rho_0 \omega^2 T_{3,1} + k \cos \theta T_{3,4}) \Phi_{\text{inc}}, \\ \omega T_{4,3} u_x^{[1-]} + (k \cos \theta T_{4,4} - i\rho_0 \omega^2 T_{4,1}) \Phi_{\text{refl}} \\ + k \cos \theta e^{-ihk \cos \theta} \Phi_{\text{trans}} &= (i\rho_0 \omega^2 T_{4,1} + k \cos \theta T_{4,4}) \Phi_{\text{inc}}. \end{aligned} \quad (26)$$

The above system of equations can readily be solved for the unknown coefficients Φ_{trans} , $u_x^{[1-]}$, Φ_{refl} and $u_x^{[h+]}$ once Φ_{inc} , θ and ω are known.

2.4. Transmission loss

The most relevant field quantity for acoustics and noise control engineers is the sound transmission loss (TL) through the plate. It is defined as [29]

$$\text{TL (dB)} = 10 \log_{10}(1/\tau), \quad (27)$$

where the transmission coefficient, $\tau(\omega, \theta) = |\Phi_{\text{trans}}|^2 / |\Phi_{\text{inc}}|^2$, is the ratio of the transmitted to incident sound intensities. Also, the frequency that causes a resonance effect in the plate and makes most of the incident energy passes through it (i.e., $|\Phi_{\text{trans}}| \approx |\Phi_{\text{inc}}|$) is referred to as the coincidence frequency f_c . At this frequency the wavelength of the sound waves projected on the panel equals the wavelength of guided waves traveling along the panel, leading to increased movement of the panel, leading to low sound insulation. The approximate value of f_c , according to the thin plate theory, is given by [29]:

$$f_c = \frac{c_0^2}{2\pi \sin^2 \theta} \sqrt{\frac{\rho h}{D}}, \quad (28)$$

where $D = Eh^3/12(1-\nu^2)$ is the plate's bending stiffness, with E and ν being the Young's modulus and the Poisson ratio, respectively.

In case of a diffuse sound field, Kang *et al.* [6] introduced a directional weighting function for incidence energy, making the TL prediction very accurate compared to the field incidence calculations. Accordingly,

the angle-averaged diffuse-field transmission coefficient, which takes into account the angular characteristic of incident energy, may be defined as [6]

$$\tau_d = \frac{\int_0^{\pi/2} G(\theta) \tau(\omega, \theta) \cos \theta \sin \theta d\theta}{\int_0^{\pi/2} G(\theta) \cos \theta \sin \theta d\theta}, \quad (29)$$

where $G(\theta)$ is a directional weighting function for the incident sound energy on the test specimen. Although there are various kinds of functions (e.g., cosine and polynomial functions) approximating the numerical and experimental curves for the directional distribution of incident energy, the use of Gauss function in prediction of sound transmission loss of partitions is highly recommended as it has been the most general function for describing phenomena in nature, and also it is one of the simplest functions that match the calculated and measured results in an actual test environment [6, 30]. The normalized Gaussian weighting function, which assumes zero mean and vertical symmetry of the incident energy, is given by

$$G(\theta) = e^{-\beta \theta^2}, \quad (30)$$

where $1 \leq \beta \leq 2$ is a constant depending on measurement parameters such as frequency, facilities dimensions, geometry, source and microphones positions [30].

3. Numerical results

In order to illustrate the nature and general behavior of the solution, we consider some numerical examples. Realizing the large number of parameters involved here while keeping in view our computing hardware limitations, we confine our attention to a particular model. The surrounding fluid is assumed to be air ($\rho = 1.2 \text{ kg/m}^3$, $c_0 = 344 \text{ m/s}$) at atmospheric pressure and ambient temperature. A dual-phase metal/ceramic functionally graded panel of total thickness $h = h_1 + h_2 \dots + h_q = 0.02 \text{ m}$ is considered. The volume fraction of metal in the FGM panel is varied from 100% on the upper interface to zero on the lower interface. Three distinct material gradient profiles in the FGM panel are considered. The volume fraction of metal in $0 \leq y \leq h$ for these situations are assumed according to a power law variation as follows [31]

$$V_F(y) = \left(1 - \frac{y}{h}\right)^\gamma, \quad \text{where} \quad (31)$$

$$\begin{cases} \gamma = 0.2 : & \text{Metal Rich (MR),} \\ \gamma = 1.0 : & \text{Linear (LN),} \\ \gamma = 5.0 : & \text{Ceramic Rich (CR).} \end{cases}$$

The corresponding variations in the volume fraction of metal along the y -axis are shown in Figure 1b. It is clear that MR simulates a metal-rich profile, CR a ceramic-rich profile, and LN a linear profile in the FGM plate. Also, due to the changes in the relative proportions of ceramic and metal, elastic properties of the graded panel should also

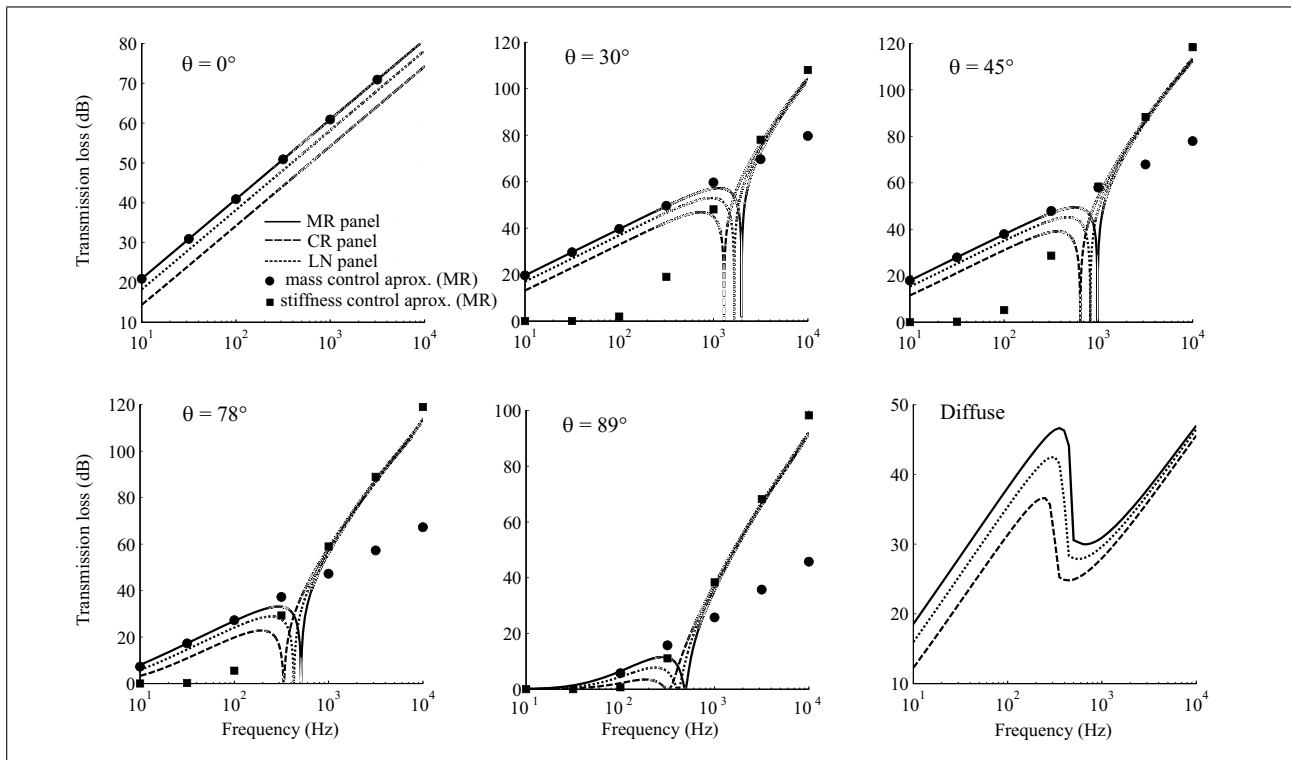


Figure 2. Variation of sound transmission loss with incident wave frequency for a SUS304/Si₃N₄ FGM panel for selected material gradient profiles and incident field conditions.

vary across its thickness. The effective structural properties for an equivalent single layer panel may simply be approximated by making use of the calculated total volume fraction of the metallic constituent,

$$\bar{V}_F = \frac{1}{h} \int_0^h V_F(y) dy,$$

in (18) as follows [31]:

$$\xi_{\text{eqv}} = \bar{V}_F \xi_{\text{metal}} + (1 - \bar{V}_F) \xi_{\text{ceramic}}, \quad (32)$$

where ξ may be any of the panel material parameters ρ , λ , μ , E , ν . Consequently, using equation (28), the coincidence frequency of the equivalent panel may be written as

$$f_c^{\text{eqv}} = \frac{c_0^2}{2\pi \sin^2 \theta} \sqrt{\frac{\rho_{\text{eqv}} h}{D_{\text{eqv}}}}, \quad (33)$$

where equation (32) should be used to calculate the equivalent material properties of the panel.

A Mathematica code was constructed for computing the global transfer matrix, T treating the linear system (26), and to calculate the unknown transmission and reflection coefficients, Φ_{refl} and Φ_{trans} , as functions of the frequency for a unit amplitude plane wave ($\Phi_{\text{inc}} = 1$) incident at an angle θ (Figure 1). For a perfectly diffuse incident sound field, $\beta = 2$ is selected for the value of the exponential parameter [6], and the integral equation (28) is numerically evaluated using the command “GaussianQuadrature” from

Table I. Material properties of the constituents.

Material	ρ (kg/m ³)	λ (GPa)	μ (GPa)
SUS 304	8166	179	102.7
Si ₃ N ₄	2370	119.9	129.9
Ti-6Al-4V	4429	60.1	40.7
Al ₂ O ₃	3750	150.3	138.7
Al	2700	50.4	25.9
TiC	4920	133.3	200
Steel	7900	121.1	80.8
Brass	8100	51	34
Foamed Concrete	600	0.2	0.7
Glass	2500	20	25.4

“NumericalMath” package available in Mathematica. The computations were performed on a network of Pentium IV personal computers with a maximum number of $q_{\text{max}} = 100$ layers, a maximum number of 10 Gauss points in each interval of integration, where over 50 intervals are selected within the range $0 \leq \theta \leq \pi/2$. The convergence was systematically checked in a simple trial and error manner, by increasing the number of layers, the number of Gauss points and also the number of intervals within the selected range, while looking for steadiness or stability in the numerical value of the solutions [32].

Figures 2 through 4 show the variation of sound transmission loss (TL) with incident wave frequency for three frequently used metal / ceramic FGM panels: Stainless Steel- Silicon Nitride (SUS304/Si₃N₄), Titanium-Aluminum Oxide (Ti-6Al-4V/Al₂O₃), and Aluminum-Titanium

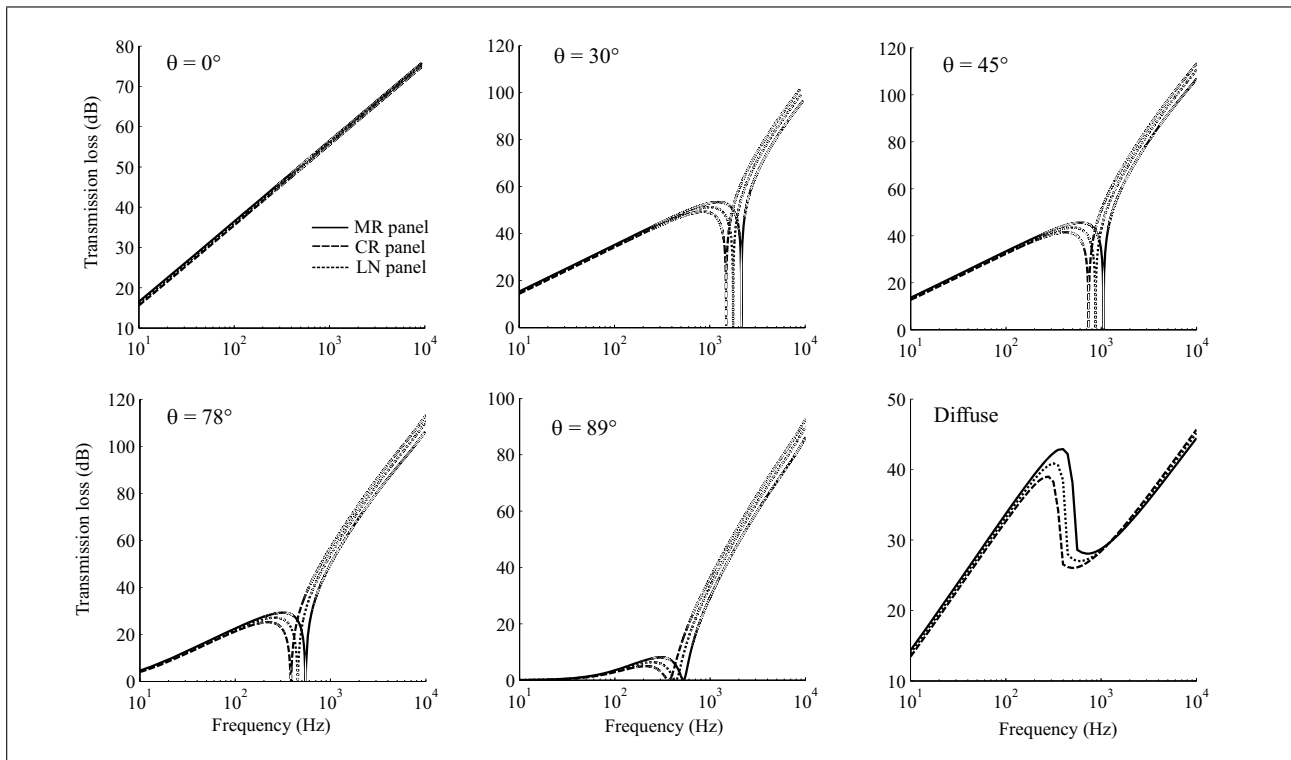


Figure 3. Variation of sound transmission loss with incident wave frequency for a Ti-6Al-4V/Al₂O₃ FGM panel for selected material gradient profiles and incident field conditions.

Carbide (Al/TiC), for selected material gradient profiles (i.e., MR, LN, CR), and angles of incidence ($\theta = 0, 30, 45, 78, 89^\circ$). The material properties for the constituent materials are listed in Table I [33, 34]. Here it should be noted that, as our analysis is carried out based on the elasticity theory, the sound transmission loss at $\theta = \pi/2$ cannot be computed because the theory has a singularity for this angle of incidence [3]. Also shown are the TL spectra calculated for the FGM panels in a perfectly diffuse sound field (i.e., via equation 29). The most important observations are as follows. It is easy to check that at low frequencies (i.e., at frequencies below the coincidence dip or in the mass controlled frequency region) the TL curves conform with the “Law of Theoretic Mass” according to the classical expression [29]:

$$TL \text{ (dB)} \approx 20 \log_{10} \frac{\omega h \rho_{eqv} \cos \theta}{2 \rho_0 c_0}$$

(e.g., note the good agreements between the circular markers and the solid lines for the metal-rich panel in the low frequency range in Figure 2). The only coincidence effect is found to occur in the mid-frequency range, the value of which strongly depends on the angle of incidence, and less sensitively on the material compositional profile in the FGM panels (note that the coincidence phenomenon is not present in the normal incidence case, since the incident field strikes the plate along the normal direction, and it does not generate waves traveling along the panels; also note the widest coincidence dip occurs for $\theta = 89^\circ$). In particular, the coincidence frequency f_c shifts downwards with an increase of θ and takes a minimum value

(i.e., the so-called critical frequency) near $\theta \approx \pi/2$ [29]. At frequencies above f_c , the stiffness dominates panel’s response and it is easy to check that TL curves reasonably agree with the well known high frequency approximation for the transmission loss based on thin plate theory [29]:

$$TL \text{ (dB)} \approx 20 \log_{10} \frac{D_{eqv} k^4 \sin^4 \theta \cos \theta}{2 \rho_0 c_0 \omega}$$

(e.g., note the good agreements between the square markers and the solid lines for the metal-rich panel in the high frequency range in Figure 2). Furthermore, it is clear that the above mentioned theoretical approximations do not predict the dips of insulation which occur particularly when the system vibrates in the vicinity of eigenmodes, such as those caused by the coincidence effect. Nevertheless, the above noted asymptotic agreements suggest that the acoustic insulation can reasonably be predicted using an average elastic constant and an average density.

Further examination of figures indicates that it is possible to control the sound transmission amplification caused by the coincidence effect by changing the ceramic/metal content of the FGM layer. In particular, we note that increasing the ceramic (metal) content causes a shift in the coincidence dip to lower (higher) frequencies. Also, it is well known that the human ear has a maximum sensitivity in the frequency range around 4 kHz, and it is desirable that the coincidence frequency be located at some separation from this range [4]. Consequently, using a ceramic rich FGM panel can lead to a better acoustic insulation performance in the coincidence and high frequency regions.

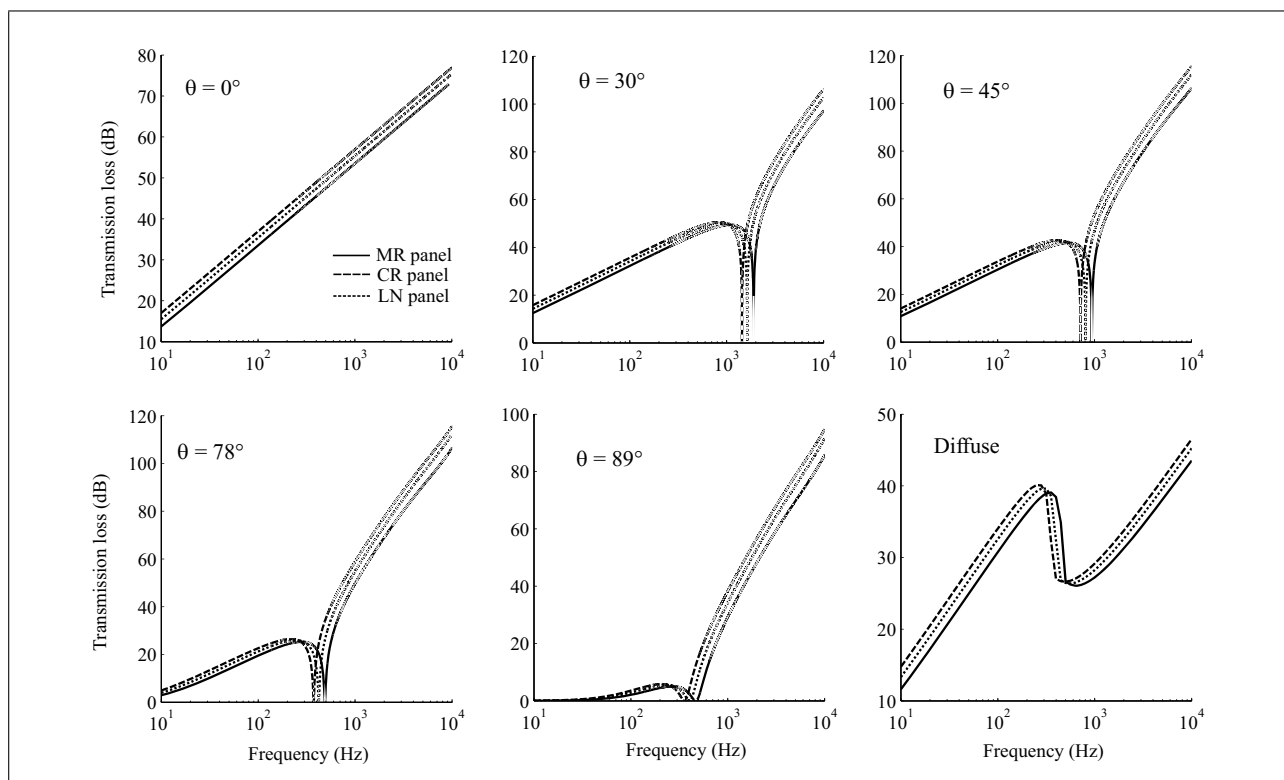


Figure 4. Variation of sound transmission loss with incident wave frequency for a Al/TiC FGM panel for selected material gradient profiles and incident field conditions.

Furthermore, Figures 2 and 3 show the superior performance of the metal rich SUS304/Si₃N₄ panel in the low frequency (mass controlled) range, especially in the diffuse incident sound field situation. This is clearly linked to the much heavier mass of the metallic part in comparison with the ceramic part for this panel. Nevertheless, the situation is reversed for the Al/TiC-panel, as the mass of the ceramic constituent is larger than that for the metallic part (note the slightly higher TL for the ceramic rich panel in the mass controlled region), especially for the diffuse incident sound field. In the high frequency (stiffness controlled) range, on the other hand, while sound transmission is relatively insensitive to the material gradient profiles in the SUS304/Si₃N₄-plate, a noticeable improvement in the TL characteristics is observed for the ceramic rich profile in the Ti-6Al-4V/Al₂O₃ and Al/TiC panels. This may be related to the fact that the difference in the stiffness of the metallic and ceramic constituents for the SUS304/Si₃N₄ plate is relatively smaller than those for Ti-6Al-4V/Al₂O₃ and Al/TiC panels. Lastly, it is observed that the best acoustic insulation performance in the mass controlled (stiffness controlled) region is achieved with the metal rich profile in SUS304/Si₃N₄ plate (the ceramic rich profile in Al/TiC plate), which contains a combination of light and heavy (soft and hard) constituent materials.

In order to facilitate comparison of different partitions, the acoustic insulation performance may advantageously be represented by a single figure of merit “averaged” across frequency. This is the weighted sound reduction index, R_w , which is developed by International Standards

Organization (ISO 717-1) for rating of airborne sound insulation in buildings and building elements [35]. It is based on curve fitting the measured or calculated transmission loss spectrum with respect to a standardized reference curve (contour) that is defined in one-third octaves in the frequency range 100 Hz–3150 Hz. The reference curve is made from three straight lines with a slope of 9 dB per octave from 100 to 400 Hz, 3 dB per octave from 400 to 1250 Hz, and 0 dB per octave from 1250 to 3150 Hz. When the reference curve is vertically shifted to meet certain criteria [35], the R_w (dB) rating is given by the value of the contour at 500 Hz. Figure 5 illustrates the determination of weighted sound reduction index from the sound transmission loss spectra of the FGM panels in a perfectly diffuse sound field. The resultant R_w (dB) values are summarized in Table III. The most important observations are as follows. The weighted sound reduction index for the metal rich SUS304/Si₃N₄ panel is distinctively higher than those for the ceramic rich and linear profile panels. This may clearly be linked to the higher overall transmission loss for the metal rich panel in both coincidence and high frequency regions (note that the sound transmission behavior at low frequencies do not come into play as all TL curves in the mass controlled region fall above the ISO reference curve). Also, the R_w index for the metal rich Ti-6Al-4V/Al₂O₃ panel is moderately higher than those for the ceramic rich and linear profile panels. Noting the nearly insignificant effect of material gradient profile on the transmission loss spectra of metal rich Ti-6Al-4V/Al₂O₃ panel in the high frequency region, one

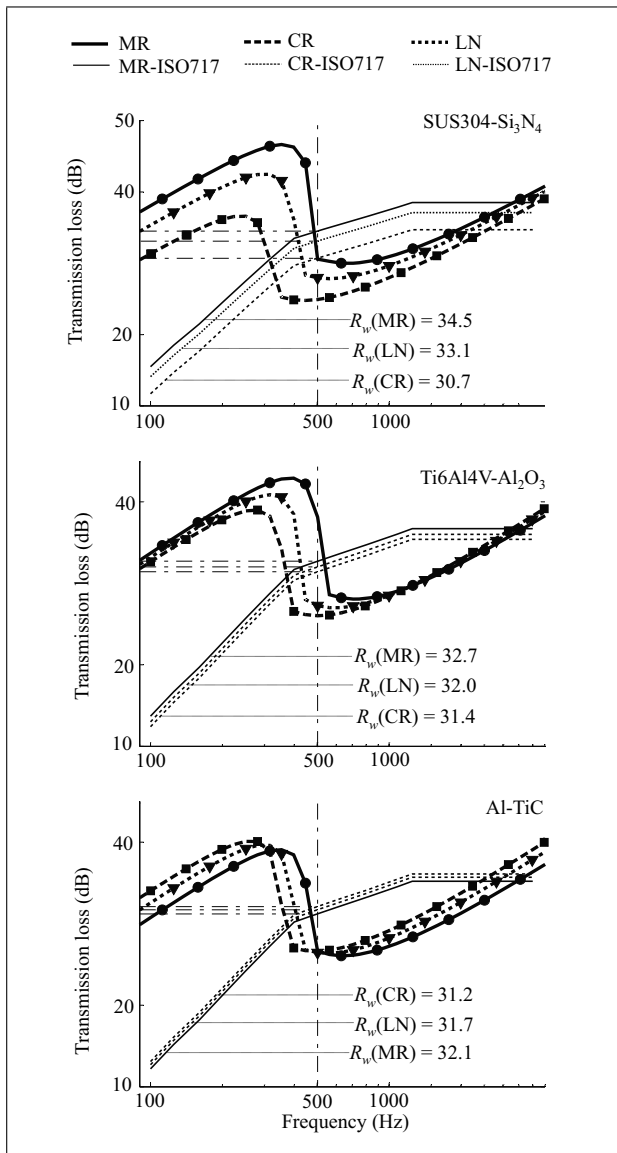


Figure 5. Determination of weighted sound reduction index from the sound transmission loss spectra of the FGM panels in a perfectly diffuse sound field.

may conclude that the sound insulation in the coincidence region dominates the overall panel's characteristics. On the other hand, the weighted sound reduction index for the ceramic rich Al/TiC panel is slightly higher than those for the linear and metal rich panels. This may directly be related to the relatively higher overall transmission loss for the ceramic rich Al/TiC panel in the high frequency region. In other words, sound insulation in the stiffness controlled region barely dominates that in the coincidence region (note the distinctively higher transmission loss values for the metal rich panel at a given frequency in the coincidence region).

In order to check overall validity of the work, we first used our general Mathematica code to compute the sound transmission loss spectra for a sandwich concrete-steel-concrete plate ($q = 3$) of total thickness $h = 0.07$ m, suspended in air for selected steel core thickness values at

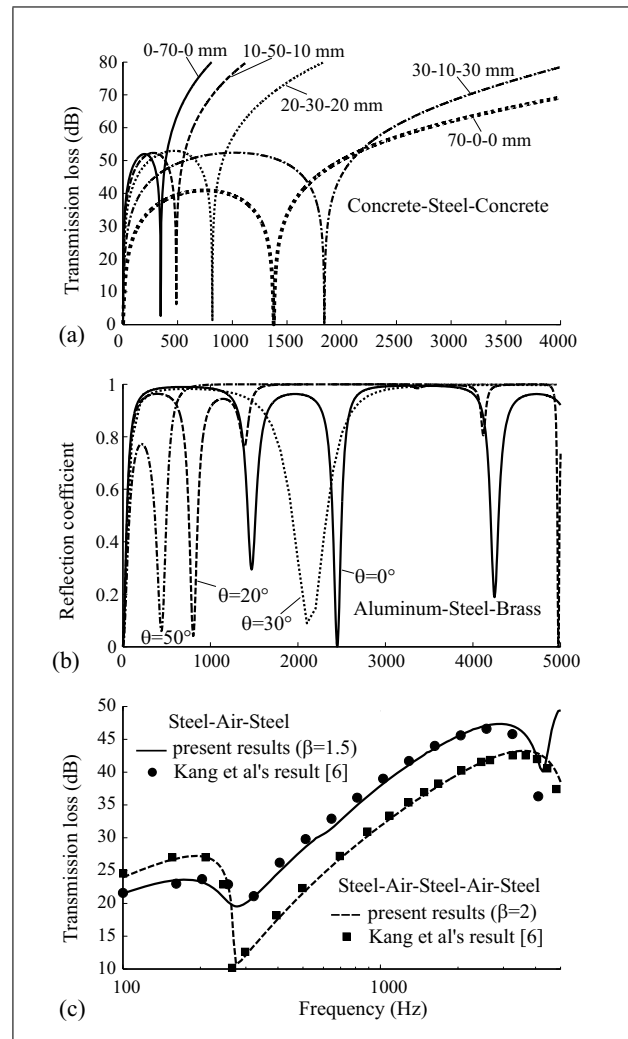


Figure 6. a. Sound transmission loss spectra for a sandwich concrete-steel-concrete plate suspended in air for selected steel core thickness values at $\theta = 45^\circ$ incidence. b. Sound reflection coefficient for a three-layered aluminum-steel-brass plate ($h_1 = 0.001$, $h_2 = 0.0005$, $h_3 = 0.0005$) suspended in air at selected angles of incidence. c. Sound transmission loss spectrum for a steel-air-steel double panel ($h_1 = 0.0022$, $h_2 = 0.006$, $h_3 = 0.0022$ with $\beta = 1.5$) and for a glass-air-glass-air-glass triple panel ($h_1 = h_2 = h_3 = 0.003$, $h_4 = 0.01$, $h_5 = 0.003$ m with $\beta = 2$) in a perfectly diffused incident sound field.

$\theta = 45^\circ$ incidence (see Table I for constituent material properties). The outcome, as displayed in Figure 6a shows excellent agreement with the numerical results presented in Figure 7 of Chonan and Kugo's [3] work. Subsequently, we calculated the sound reflection coefficient $|\Phi_{\text{refl}}|^2$ for a three-layered aluminum-steel-brass plate ($h_1 = 0.001$, $h_2 = 0.0005$, $h_3 = 0.0005$ m; $q = 3$) submerged in air at selected angles of incidence ($\theta = 0, 20, 30, 50^\circ$; see Table I for constituent material properties). Figure 6b shows very good agreements with the data provided in Figure 10 of Jackins and Gaunaud's paper [1]. Also, to check the accuracy of our numerical results for case of the perfectly diffused incident sound field, we used our general Mathematica code to compute the sound trans-

Table II. Calculated coincidence frequencies (Hz) for the equivalent FGM Panels noted.

	Profile	$\theta = 30^\circ$	$\theta = 45^\circ$	$\theta = 78^\circ$	$\theta = 89^\circ$
SUS304 -Si ₃ N ₄	MR	1949	975	509	487
	LN	1632	816	426	408
	CR	1272	636	332	318
Al-TiC	MR	1812	906	473	453
	LN	1448	724	378	362
	CR	1307	636	342	327
Ti6Al4V -Al ₂ O ₃	MR	2094	1047	547	524
	LN	1641	820	429	410
	CR	1374	687	359	344

Table III. The calculated weighted sound reduction index, R_w (dB), for the FGM panels.

	SUS304-Si ₃ N ₄	Al-TiC	Ti6Al4V-Al ₂ O ₃
MR	34.5	31.2	32.7
LN	33.1	31.7	32.0
CR	30.7	32.1	31.4

mission loss spectrum for a steel-air-steel double panel ($h_1 = 0.0022$, $h_2 = 0.006$, $h_3 = 0.0022$ m; where the physical properties of the air layer are approximated by $\rho^{[2]} = 1.2$ kg/m³, $c_p^{[2]} = 344$ m/s, $\lambda^{[2]} = \rho^{[2]}(c_p^{[2]})^2 \approx 1.5 \cdot 10^5$ Pa, $c_s^{[2]} = \mu^{[2]} \approx 0$; with $\beta = 1.5$) as well as for a glass-air-glass-air-glass triple panel ($h_1 = h_2 = h_3 = 0.003$ m, $h_4 = 0.01$ m, $h_5 = 0.003$ m; where the physical properties of the air layers are again approximated by $c_p^{[2,4]} = 344$ m/s, $\lambda^{[2,4]} = \rho^{[2,4]}(c_p^{[2,4]})^2 \approx 1.5 \cdot 10^5$ Pa, $c_s^{[2,4]} \mu^{[2,4]} \approx 0$; with $\beta = 2$). The outcome, as shown in Figure 6c shows fair agreement with the numerical results presented in Figures 11 and 14 of Kang *et al.*'s [6] work which is restricted by the assumptions of thin plate theory. As a final check, we used equation 33 (which is also restricted by the assumptions of the thin plate theory) to calculate the coincidence frequencies of the equivalent FGM panels. The results, as tabulated in Table II, agree well with the coincidence dips displayed in Figures 2 through 4.

4. Conclusions

Inhomogeneous or multilayered panels are widely used in modern engineering and technological applications. Manufacturers of such panels are confronted with the problem of finding the acoustic transmission characteristics of such structures. The present work is concerned with transmission of sound waves through a thick-walled functionally graded panel of infinite extent. The solution of the problem is derived by means of the very powerful transfer matrix method which effectively reduces the order of the final solving matrix and greatly improves the computational precision. Special attention is paid to the influence of material compositional profiles on the sound insulation ca-

capacity of three frequently used dual-phase metal-ceramic FGM panels. The numerical results reveal that it is possible to control the sound insulation deterioration caused by the coincidence effect by changing the ceramic/metal content of the FGM layers. In particular, we note that increasing the ceramic (metal) content causes a downward (upward) shift in the coincidence dip to lower (higher) frequencies. Furthermore, the superior performance of the metal rich SUS304/Si₃N₄ panel in the low frequency (mass controlled) region is demonstrated, especially in the diffuse incident sound field situation. Nevertheless, a slightly higher TL for the ceramic rich Al/TiC panel is noted in the mass controlled region, as the mass of the ceramic constituent is relatively larger. In the high frequency (stiffness controlled) range, on the other hand, while sound transmission is relatively insensitive to the material gradient profiles for the SUS304/Si₃N₄ plate, a noticeable improvement in the TL is observed for the ceramic rich profile in the Ti-6Al-4V/Al₂O₃ and Al/TiC panels. It is also concluded that the best acoustic performance in the mass controlled (stiffness controlled) region is achieved with the metal rich profile in SUS304/Si₃N₄ plate (the ceramic rich profile in Al/TiC plate), which contains a combination of light and heavy (soft and hard) constituent materials. Lastly, it is demonstrated that while the panel's material gradient profile in the mass controlled region does not wholly affect the numerical value of calculated weighted sound reduction index R_w (as determined according to the ISO-717), it is highly consequential in the coincidence and stiffness controlled regions. Moreover, the effect of gradient profile in the stiffness controlled region is found to dominate the influence of material gradient pattern in the coincidence region. The presented model can aid the acoustics and noise control engineers to design FGM panels with optimum acoustic characteristics through suitable selection of the constituting material gradient profiles.

Appendix

$$\begin{aligned}
 Q_{1,1}[j] &= -\left[\lambda^{[j]}k^2 + (\lambda^{[j]} + 2\mu^{[j]})(k_p^{[j]})^2\right]e^{i(\bar{k}x + K_p^{[j]}y - \omega t)} \\
 Q_{1,2}[j] &= -\left[\lambda^{[j]}k^2 + (\lambda^{[j]} + 2\mu^{[j]})(k_p^{[j]})^2\right]e^{i(\bar{k}x - K_p^{[j]}y - \omega t)} \\
 Q_{1,3}[j] &= 2\mu^{[j]}k\bar{K}_s^{[j]}e^{i(\bar{k}x + K_s^{[j]}y - \omega t)} \\
 Q_{1,4}[j] &= -2\mu^{[j]}k\bar{K}_s^{[j]}e^{i(\bar{k}x - K_s^{[j]}y - \omega t)} \\
 Q_{2,1}[j] &= -2\mu^{[j]}k\bar{K}_p^{[j]}e^{i(\bar{k}x + K_p^{[j]}y - \omega t)} \\
 Q_{2,2}[j] &= 2\mu^{[j]}k\bar{K}_p^{[j]}e^{i(\bar{k}x - K_p^{[j]}y - \omega t)} \\
 Q_{2,3}[j] &= \mu^{[j]}[k^2 - (K_s^{[j]})^2]e^{i(\bar{k}x + K_s^{[j]}y - \omega t)} \\
 Q_{2,4}[j] &= \mu^{[j]}[k^2 - (K_s^{[j]})^2]e^{i(\bar{k}x - K_s^{[j]}y - \omega t)} \\
 Q_{3,1}[j] &= i\bar{k}e^{i(\bar{k}x + K_p^{[j]}y - \omega t)} \\
 Q_{3,2}[j] &= i\bar{k}e^{i(\bar{k}x - K_p^{[j]}y - \omega t)} \\
 Q_{3,3}[j] &= iK_s^{[j]}e^{i(\bar{k}x + K_s^{[j]}y - \omega t)} \\
 Q_{3,4}[j] &= -iK_s^{[j]}e^{i(\bar{k}x - K_s^{[j]}y - \omega t)}
 \end{aligned}$$

$$\begin{aligned} Q_{4,1}[j] &= iK_p^{[j]} e^{i(\bar{k}x + K_p^{[j]}y - \omega t)} \\ Q_{4,2}[j] &= -iK_p^{[j]} e^{i(\bar{k}x - K_p^{[j]}y - \omega t)} \\ Q_{4,3}[j] &= -i\bar{k}e^{i(\bar{k}x + K_s^{[j]}y - \omega t)} \\ Q_{4,4}[j] &= -i\bar{k}e^{i(\bar{k}x - K_s^{[j]}y - \omega t)} \end{aligned}$$

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