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Acoustic Heating in a Weakly Dispersive Fluid Flow

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Summary
Instantaneous acoustic heating in a weakly dispersive fluid flow is the subject of investigation. Splitting of energy balance equation yields in evolution equation for acoustic heating. The specific role of dispersion in this phenomenon is studied and discussed for two examples of weak dispersion mechanisms, relaxation and boundary layer.

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1. Introduction

The dependence of phase speed on frequency is called dispersion. Dispersion and attenuation are connected by the Kramers-Kronig relations [1, 2], therefore, dispersion always accompanies the attenuation of sound. Dispersion in air and seawater is known to alter wave profiles and postpone shock formation. In air, vibrational relaxation is the dominant mechanism at audible frequencies, while chemical reactions is the main cause of seawater dispersion below 500 kHz.

It may be concluded, that dispersion does play role in the secondary phenomena originating from propagation of finite amplitude sound in thermoviscous fluids, the acoustic heating and streaming. Section 2 is devoted to deriving of instantaneous dynamic equation of acoustic heating in a weakly dispersive fluid flow. Analysis is based on the technique similar to that worked out by the author relatively to instantaneous acoustic heating and streaming in non-dispersive flows [3, 4].

The basic idea consists in determination of eigenvectors (modes) of the unbounded flow of infinite small amplitude and respective linear projectors, uniquely extracting the corresponding mode from the overall vector of wave perturbations at any moment. As a result of projecting, the equations governing the weakly nonlinear dynamics of every mode follow from the equations of continuum mechanics [3, 4, 5]. Projecting in fact is a linear combining of equations regarding the links of hydrodynamic variables specific for every mode. The number of final dynamic equations equals to the number of hydrodynamic equations. Each dynamic equation contains the first order derivative with respect to time. In the three dimensional flow, there are five final equations, two for acoustic branches, two for the vortex branches everyone, and one for the entropy mode [3, 6].

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The problem of acoustic heating presupposes the dominative sound though of small amplitude ($M \ll 1$, were M is the Mach number, it is a ratio of magnitude of particles velocity to the sound speed in a fluid), and produced by it entropy comparatively small. Over the time domain, where it holds true, the instantaneous enlargement of static temperature (or, equivalently, isobaric decrease in density) in the leading order is described by the diffusion equation with an acoustic nonlinear source. Nonlinearity and absorption (and corresponding dispersion) are the origins of acoustic heating. The sound itself must satisfy the dynamic equation following from the initial conservational equations.

Traditional theory of acoustic heating rigorously applies to periodic sound and gives answer how to estimate increase in background temperature averaged over period of sound [7, 8]. Procedure of projecting is instantaneous and therefore gives possibility not only to trace temporal development of heating in details, but to include in consideration different aperiodic acoustic sources. The results and discussion of instantaneous acoustic heating induced by pulses in a thermoviscous medium with standard absorption without account of dispersion may be found in paper [4] and referred there papers.

Section 2 below considers the acoustic heating in a weak dispersive fluid. Though only fluids satisfying the equation of state for ideal gases are under investigation, the theory may be easily extrapolated into other media. Analytical examples of heating associating with two examples of weak dispersion mechanisms, relaxation and boundary layer, are presented in section 3.

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2. Acoustic instantaneous heating in a weakly dispersive fluid flow

2.1. Dynamic equations in a fluid with general attenuation and dispersion properties

Let consider the unbounded, homogeneous in composition fluid with uniform unperturbed density and pressure, constant shear viscosity and heat conduction. We proceeded from the equations for the planar fluid flow in the differential form [9, 10]. Three equations, among thermodynamic state, are required: momentum conservation, energy balance, and mass conservation:

$$\begin{aligned}
&\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right)+\frac{\partial p}{\partial x}=-\frac{\partial}{\partial x}\widehat{A}v\\
&\rho\left(\frac{\partial e}{\partial t}+v\frac{\partial e}{\partial x}\right)+p\frac{\partial v}{\partial x}=\chi\frac{\partial^2 T}{\partial x^2}-\frac{\partial v}{\partial x}\widehat{A}v\\
&\frac{\partial \rho}{\partial t}+\frac{\partial(\rho v)}{\partial x}=0,
\end{aligned} \tag{1}$$

where ρ, p, v, e, T denote density, pressure, velocity of fluid particles, internal energy per unit mass and temperature, relatively, x, t are spacial co-ordinate and time, and χ marks thermal conductivity. The linear operator $\widehat{A}$ describes the weak attenuation and dispersion properties of a fluid. Analytic expressions for $\widehat{A}$ are known for several cases, for example, for attenuation and dispersion due to propagation in relaxing medium or due to boundary-layer effects in ducts. These examples are considered in the present study. So that, $\widehat{A}$ may be thought as a sum of specific operators as follows:

$$\widehat{A}=\widehat{A}_1+\widehat{A}_2+\widehat{A}_3, \tag{2}$$

where $\widehat{A}_1$ describes the standard viscous attenuation, $\widehat{A}_2$ is responsible for relaxation effects, and $\widehat{A}_3$ represents the Stoke's boundary-layer effect,

$$\begin{aligned}
\widehat{A}_1 v &= \frac{4}{3}\frac{\eta}{c_0}\frac{\partial v}{\partial t},\\
\widehat{A}_2 v &= mc_0\rho_0\int_{-\infty}^{t}\frac{\partial v(x,t')}{\partial t'}\exp\left(-(t-t')/t_R\right)\mathrm{d}t',\\
\widehat{A}_3 v &= -\frac{2c_0\rho_0}{R}\sqrt{\frac{\eta}{\rho_0\pi}}\int_{-\infty}^{t}\frac{v(x,t')}{\sqrt{t-t'}}\mathrm{d}t'.
\end{aligned} \tag{3}$$

In the formulas before, η denotes shear viscosity, ρ_0 is the static density of a fluid, and R is the tube radius. The integro-differential operator $\widehat{A}_2$ is responsible for a single relaxation process (including bulk viscosity) with characteristic time of relaxation t_R and dispersion $m = c_\infty^2/c_0^2 - 1$, where c_0 and c_∞ are the small-signal sound speeds at zero frequency and infinite frequency, respectively [1, 7, 11, 12, 13]. Operators (3) should in addition contain complementary nonlinear terms which are not affect further analysis because a matter of interest are quadratic nonlinear effects. It is convenient to regard $\widehat{A}$ as a time operator, although it might in general be a space operator. The fact that $\widehat{A}v$ must be a small term implies that space operations in $\widehat{A}$ may be replaced by the time operators, for example, $\partial v/\partial t = -c_0\partial v/\partial x$ for a rightwards acoustic wave, and so on.

The system (1) should be completed by the equations of state. In the present investigation, only ideal gases will be considered. Internal energy and temperature of an ideal gas are functions of ρ, p as follows ($\gamma = C_p/C_v$ is the ratio of the specific heats at constant pressure, C_p and constant volume, C_v):

$$e = C_v T = \frac{p}{(\gamma-1)\rho}. \tag{4}$$

Using the dimensionless quantities

$$p' = \frac{p-p_0}{c_0^2\cdot\rho_0}, \rho' = \frac{\rho-\rho_0}{\rho_0}, v' = \frac{v}{c_0}, \tag{5}$$

and operators

$$\partial/\partial x' = \lambda\partial/\partial x, \partial/\partial t' = \frac{\lambda}{c_0}\partial/\partial t, \widehat{A}' = \frac{\widehat{A}}{c_0\rho_0}, \tag{6}$$

where λ is a characteristic scale of a flow, one can rewrite (1) as the system (7),

$$\begin{aligned}
&\frac{\partial v}{\partial t}+\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\widehat{A}v=-v\frac{\partial v}{\partial x}+\rho\frac{\partial p}{\partial x}+\rho\frac{\partial}{\partial x}\widehat{A}v,\\
&\frac{\partial p}{\partial t}+\frac{\partial v}{\partial x}-\frac{\delta}{\gamma-1}\frac{\partial^2(\gamma p-\rho)}{\partial x^2}=\\
&-v\frac{\partial p}{\partial x}-\gamma p\frac{\partial v}{\partial x}-(\gamma-1)\frac{\partial v}{\partial x}\widehat{A}v+\frac{\delta}{\gamma-1}\frac{\partial^2(\rho^2-\gamma\rho p)}{\partial x^2}\\
&\frac{\partial \rho}{\partial t}+\frac{\partial v}{\partial x}=-v\frac{\partial \rho}{\partial x}-\rho\frac{\partial v}{\partial x}.
\end{aligned} \tag{7}$$

Here and everywhere below in the text, primes by dimensionless quantities are dropped. Among already mentioned quantities,

$$\delta = \chi\left(1/C_v - 1/C_p\right)/\left(\rho_0 c_0\lambda\right) = \chi\left(\gamma-1\right)/\left(\rho_0 c_0\lambda\right)C_p$$

is the dimensionless thermal attenuation. The third equation from the set (7) is exact, the first and second ones are derived with the accuracy up to the second-order nonlinear terms of the order M^2, including those ones standing by the thermal coefficient $O(\delta M^2)$ and responsible for weak dispersion.

While transforming the system of equations (1) into the system (7), the dimensionless density is expanded in the series in the vicinity of zero perturbation, so that $(1+\rho)^{-1} \approx 1-\rho+\rho^2$. We use this expansion also in the expressions for the internal energy and temperature in accordance to equation (4). Indeed, the second equation from the set (7), is a combination of the second and the third equations from the system (1) necessary for excluding the partial derivative of density with respect to time. It appears after expanding the internal energy into the series.

The three eigenvectors, or possible types of the linear motion take the form:

$$\psi_{a,1} = \begin{pmatrix} 1 + \frac{1}{2}\left(\widehat{A} - \delta\partial/\partial x\right) \\ 1 - \delta\partial/\partial x \\ 1 \end{pmatrix} \rho_{a,1},$$

$$\psi_{a,2} = \begin{pmatrix} -1 + \frac{1}{2}\left(\widehat{A} - \delta\partial/\partial x\right) \\ 1 + \delta\partial/\partial x \\ 1 \end{pmatrix} \rho_{a,2},$$

$$\psi_e = \begin{pmatrix} -\frac{\delta}{\gamma-1}\partial/\partial x \\ 0 \\ 1 \end{pmatrix} \rho_e, \qquad (8)$$

where $\psi = \left(v(x,t)\ p(x,t)\ \rho(x,t)\right)^T$ is a vector of dimensionless perturbations. These three eigenvectors manifest the existence of three basic types of the plane flow: two first are acoustic, rightwards and leftwards progressive, and the third one is entropy mode. Only one variable is independent for any mode, it determines the two other ones uniquely. The dimensionless infinitely small-signal speed of every mode when dispersion tends to zero is determined by the relations: $c_{a,1} = -c_{a,2} = 1$, $c_e = 0$. It is noticeable that links connecting excess pressure and density p and ρ in both acoustic modes, do not include viscous operator $\widehat{A}$, but only a term proportional to the thermal conductivity.

In the linear flow ($M \to 0$), acoustic rightwards progressive mode and the entropy one are governed correspondingly by the following independent dynamic equations:

$$\frac{\partial\rho_{a,1}}{\partial t} + \frac{\partial\rho_{a,1}}{\partial x} + \frac{1}{2}\left(\widehat{A} - \delta\frac{\partial}{\partial x}\right)\frac{\partial\rho_{a,1}}{\partial x} = 0,$$

$$\frac{\partial\rho_e}{\partial t} - \frac{\delta}{\gamma-1}\frac{\partial^2\rho_e}{\partial x^2} = 0. \qquad (9)$$

In the studies of nonlinear flow, the links for acoustic modes should be corrected by involving of quadratic terms providing approximately isentropic motion accounting for weak losses due to irreversible processes (acoustic mode is strictly isentropic in lossless flows) [7, 14]. Links specific for the Riemann wave may be also determined algorithmically on the base of projecting for a fluid with arbitrary equations of state [3, 5].

Let the overall perturbation consists of only rightwards progressive acoustic mode with an excess density ρ_a. By including the quadratic terms in the eigenvectors (8) the overall perturbations take the form as follows:

$$v(x,t) = \rho_a(x,t) + \frac{\gamma-3}{4}\rho_a^2(x,t) + \frac{1}{2}\left(\widehat{A} - \delta\partial/\partial x\right)\rho_a - \frac{\delta}{\gamma-1}\frac{\partial\rho_e(x,t)}{\partial x},$$

$$p(x,t) = \rho_a(x,t) + \frac{\gamma-1}{2}\rho_a^2(x,t) - \delta\frac{\partial\rho_a(x,t)}{\partial x},$$

$$\rho(x,t) = \rho_a(x,t) + \rho_e(x,t). \qquad (10)$$

It may be easy verified, that considering only acoustic terms in the links (10), yields to the nonlinear equation for sound being inserted into any equation from the system (7):

$$\frac{\partial\rho_a}{\partial t} + \frac{\partial\rho_a}{\partial x} + \frac{\gamma+1}{2}\rho_a\frac{\partial\rho_a}{\partial x} + \frac{1}{2}\left(\widehat{A} - \delta\frac{\partial}{\partial x}\right)\frac{\partial\rho_a}{\partial x} = 0. \quad (11)$$

The overall change of variables (10) which accounts for the entropy quantity ρ_e, yields to a system of coupling nonlinear equations in quantities ρ_a, ρ_e, including quadratic cross terms. At this point, we declare that the entropy mode is a secondary mode with magnitude of ρ_e much less than that of ρ_a. In fact, we use one more small parameter in order to derive dynamic equations keeping the terms of the same order of magnitude. So that, the equation governing the dominative rightwards progressive sound, is (11), which does not account for the interaction with entropy mode at all.

Using change of variables (10) and combining of two last equations from system (7) (the second one minus the third one) allows to exclude the partial derivative $\partial\rho_a/\partial t$. We keep the second-order nonlinear terms and use the governing equation for the sound (11) in calculations of terms including the partial derivative with respect of time,

$$\frac{\partial(p-\rho)}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\gamma-1}{2}\rho_a^2 - \delta\frac{\partial\rho_a}{\partial x} - \rho_e\right)$$

$$\approx (\gamma-1)\rho_a\left(-\frac{\partial\rho_a}{\partial x} - \frac{1}{2}\left(\widehat{A} - \delta\frac{\partial}{\partial x}\right)\frac{\partial\rho_a}{\partial x}\right)$$

$$+ \delta\frac{\partial^2\rho_a}{\partial x^2} + \delta\frac{\gamma+1}{4}\frac{\partial^2\rho_a^2}{\partial x^2} - \frac{\partial\rho_e}{\partial t}.$$

Finally, the dynamic equation for the entropy mode takes a form

$$-\frac{\partial\rho_e}{\partial t} + \frac{\delta}{\gamma-1}\frac{\partial^2\rho_e}{\partial x^2} = \qquad (12)$$

$$\delta\frac{\gamma-3}{2}\rho_a\frac{\partial^2\rho_a}{\partial x^2} - (\gamma-1)\frac{\partial\rho_a}{\partial x}\widehat{A}\rho_a + \delta\frac{3\gamma-5}{2}\left(\frac{\partial\rho_a}{\partial x}\right)^2.$$

The right-hand side of equation (12) exhibits the nonlinear thermoviscous origin of acoustic heating. The instantaneous formulae (11), (12) govern sound and the respective heating in the relaxing fluids, and they linearized versions agree with the set (9). The equations (11), (12) may be obtained by immediate projecting of the system (7) into specific dynamic equations. The formal procedure in the case of standard attenuation $\widehat{A} = \widehat{A}_1$ is discussed in details in [4]. In the particular case $\widehat{A} = \widehat{A}_1$, equation (11) rearranges into the well-known Burgers equation which may be solved exactly with the help of the Cole-Hopf transform making it the linear diffusion equation.

Only quadratic nonlinear terms are kept in the both equations. The formulae (11), (12) are valid over the time domain, where magnitude of rightwards sound density keeps much larger than the entropy mode one. Otherwise, both equations should be corrected in view of the growing influence of entropy mode on the background of sound propagation. Nonlinear interactions make the leftwards sound grow, so it should be involved in considering at the later stages of evolution.

2.2. Splitting of the entropy balance into acoustic and non-acoustic parts

The irreversible growth of integral entropy S is given by the second equation from the set (1). Dimensional total heat release in unit time and mass Q is as follows:

$$Q_{total,dim} = T\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{\mathrm{d}e}{\mathrm{d}t} + \frac{p}{\rho}\frac{\partial v}{\partial x} = \frac{c^3}{\lambda}(Q_e + Q_a)$$
$$= \frac{c_0^3}{\lambda(\gamma-1)}\left((\gamma-3)\delta\rho_a\frac{\partial^2\rho_a}{\partial x^2} + (\gamma-2)\delta\left(\frac{\partial\rho_a}{\partial x}\right)^2 + \delta\frac{\partial^2\rho_a}{\partial x^2} - \frac{\delta}{\gamma-1}\frac{\partial^2\rho_e}{\partial x^2} - \frac{\partial\rho_a}{\partial x}\widehat{A}\rho_a\right), \quad (13)$$

where Q_e, Q_a are dimensionless quantities associating with entropy and acoustic modes,

$$Q_e = -\frac{1}{\gamma-1}\frac{\partial\rho_e}{\partial t} = \frac{\delta}{2(\gamma-1)}\left((\gamma-3)\rho_a\frac{\partial^2\rho_a}{\partial x^2} + (3\gamma-5)\left(\frac{\partial\rho_a}{\partial x}\right)^2 - \frac{2}{\gamma-1}\frac{\partial^2\rho_e}{\partial x^2}\right) - \frac{\partial\rho_a}{\partial x}\widehat{A}\rho_a, \quad (14)$$

$$Q_a = \frac{\delta}{(\gamma-1)}\left(\frac{\partial^2\rho_a}{\partial x^2} - \frac{\gamma-1}{2}\left(\frac{\partial\rho_a}{\partial x}\right)^2 + \frac{\gamma-3}{2}\rho_a\frac{\partial^2\rho_a}{\partial x^2}\right). \quad (15)$$

In the right-hand sides of resulting equalities (14), (15), terms of order δM^2 and quadratic nonlinear terms responsible for weak dispersion are retained. The ratio of entropy mode magnitude and sound amplitude is small, so that all cross and quadratic entropy terms are ignored. In the formula (14), the small term

$$-\frac{2}{\gamma-1}\frac{\partial^2\rho_e}{\partial x^2}$$

in the brackets is included in order to stress that the excess entropy density satisfies the diffusion equation. The origin of this term, as well of the another linear term,

$$\frac{\delta}{\gamma-1}\frac{\partial^2\rho_a}{\partial x^2},$$

in the right-hand side of (15), is linear losses in entropy associating with heat conduction (the term $\chi\dfrac{\partial^2 T}{\partial x^2}$ in the right-hand side of energy balance, the second equation from the set (1)). It is remarkable that the irreversible growth in total entropy in its acoustic part Q_a is proportional exclusively to the thermal conductivity. For weakly nonlinear flows considered in the present investigation, the linear term in the right-hand side of (15) is larger than two last quadratic ones, so that the acoustic part approximately equals

$$Q_a \approx \frac{\delta}{(\gamma-1)}\frac{\partial^2\rho_a}{\partial x^2}. \quad (16)$$

Acoustic excess density in the "source" right-hand sides of equations (13)–(15) must satisfy the dynamic equation (12).

The specific part of energy release relating to relaxation processes, follows from (14),

$$Q_{e,relax} = -\frac{\partial\rho_a}{\partial x}\widehat{A}\rho_a. \quad (17)$$

The meaning of "acoustic heating" denotes the isobaric heating, so that it associates with the entropy part, Q_e. It seems useful to write on an expression of the dimensional heat release in unit volume per unit time $Q_{e,dim}$ in terms of dimensional velocity of the rightwards acoustic wave V_a and operator $\widehat{A}$,

$$\left(T\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{e,dim} = Q_{e,dim} = \frac{\chi}{2\rho_0 C_p}\left((\gamma-3)V_a\frac{\partial^2 V_a}{\partial x^2} + (3\gamma-5)\left(\frac{\partial V_a}{\partial x}\right)^2\right) - \frac{1}{\rho_0}\frac{\partial V_a}{\partial x}\widehat{A}V_a. \quad (18)$$

The small term $\delta/(\gamma-1)\,(\partial^2\rho_e/\partial x^2)$ is dropped in (18).

3. Examples on heating caused by stationary sound

The evolution of acoustic heating produced by the weak dispersive process is determined by the formula (17) with an acoustic source satisfying the equation (11). The basic complexity is that general analytical solutions of (11), are not available, so that numerical solutions are necessary in the majority of applications [12, 13, 15, 16].

To investigate the combined effects of relaxation (or the boundary layer) and nonlinearity, it is helpful to ignore standard viscous dissipation, $\widehat{A}_1$, and consider only one relaxation process (or dispersion due to the boundary layer). Note that several relaxation processes take place in many fluids. For example, there are at least two pronounced processes, with typical times $t_{R,1} = 10^{-3}$ s and $t_{R,2} = 1,5\cdot 10^{-5}$ s [17].

The example below deals with the stationary sound waveform in a monorelaxing fluid. The dynamic equation (11) rewritten in the dimensionless retarded time $\tau = t - x$ sounds

$$\frac{\partial\rho_a}{\partial x} - \frac{\gamma+1}{2}\rho_a\frac{\partial\rho_a}{\partial\tau} - \frac{m}{2}\int_{-\infty}^{\tau}\frac{\partial^2\rho_a}{\partial\tau'^2}\exp\left(-(\tau-\tau')/t_R\right)\mathrm{d}\tau' = 0, \quad (19)$$

where t_R denotes the dimensionless relaxation time [7, 13, 15].

A stationary wave is one for which $\partial\rho_a/\partial x = 0$. Integrating of (19) with respect to τ transforms (19) into (20) [13]:

$$\left(\rho_a + \frac{m}{\gamma+1}\right)\frac{\mathrm{d}\rho_a}{\mathrm{d}\tau} + \frac{\rho_a^2}{2t_R} = \text{const}. \quad (20)$$

We consider a density jump $2M$ from $-M$ ahead of the wave front to M behind. Applying the boundary conditions at $x = -\infty$: $\rho_a = -M$, $\mathrm{d}\rho_a/\mathrm{d}\tau = 0$ and $x = \infty$:

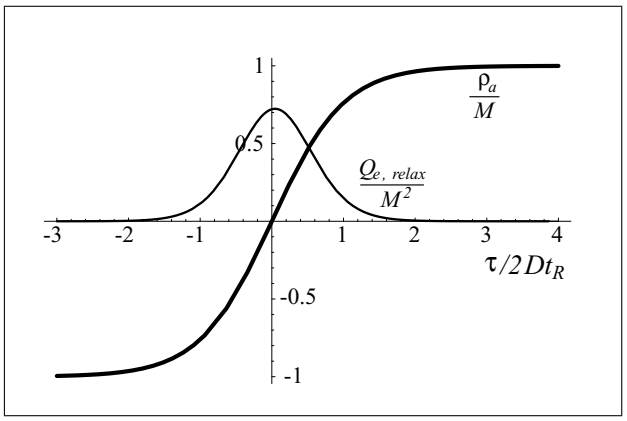

Figure 1. Stationary waveform $\rho_a(\tau)$ (bold line) and correspondent heating $Q_{e,relax}$ caused by a single relaxation process in CO_2 under the normal conditions in accordance to the formulae (17), (22).

$\rho_a = M$, $\mathrm{d}\rho_a/\mathrm{d}\tau = 0$, yield for the integration constant $M^2/2t_R$ and results after separating of variables and integrating to the expression for τ,

$$\tau = t_R \ln \frac{(1+\rho_a/M)^{D-1}}{(1-\rho_a/M)^{D+1}}, \tag{21}$$

where $D = m/[(\gamma+1)M]$ measures the ratio of relaxation effects to nonlinear effects.

In the limit of weak nonlinearity $D \gg 1$ the solution (21) can be inverted analytically,

$$\rho_a(\tau) = M \tanh(\tau/2Dt_R), \tag{22}$$

Analysis deals with excess density though it may be replaced by acoustic pressure in all formulae below. The reason for that is link of p_a and ρ_a established by $\psi_{a,1}$ from the set (8). For illustration, the parameters correspondent to a single molecular relaxation taking place in carbon dioxide CO_2 under the normal conditions are used: $c_0 = 258,9\,\mathrm{m/s}$, $c_\infty = 271\,\mathrm{m/s}$ with a relaxation time $5\cdot10^{-6}\,\mathrm{s}$ [1]. Dispersion reveals mostly at the angular frequency $\omega = 0,2\,\mathrm{MHz}$, that corresponds to the characteristic scale $\lambda \approx 1,3\,\mathrm{mm}$. Sound absorption in the vicinity of this frequency overestimates the classical values (accounting for shear viscosity and thermal conductivity) approximately twenty times. Ratio of specific heats is $\gamma = 4/3$. Requiring $D \gg 1$ ($D \approx 5,5$) is satisfied by quantity $M = 0,01$. Acoustic heating, analytically estimated, and correspondent stationary waveform are shown in Figure 1. The large values of acoustic heating are hardly expected in this concrete illustration in spite of low Mach numbers and relaxation ($m \approx 0,1$). Larger nonlinearity and relaxation, larger acoustic heating.

Acoustic heating caused by weak dispersion following propagation of a single waveform may be studied approximately by help of the stationary pulse. That means that effects of nonlinearity and dispersion are considered only by the equation governing heating, but not by the dynamic equation for the sound itself. Let the exemplary excess

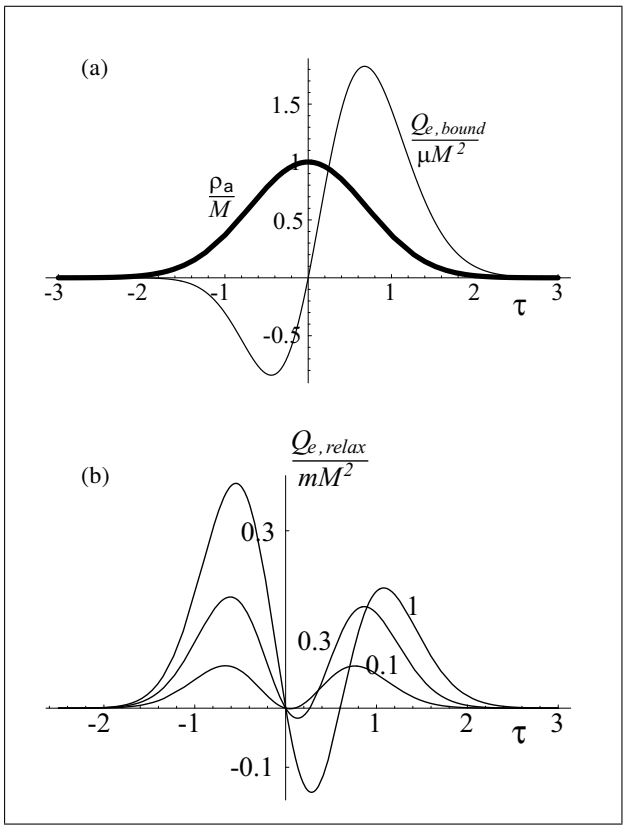

Figure 2. a: Stationary acoustic waveform $\rho_a(\tau)$ (bold line) and heating $Q_{e,bound}$ caused by a boundary viscous layer, $\mu = 2\sqrt{\eta'/\pi}/R'$, where $R' = R/\lambda$, $\eta' = \eta/(\rho_0 c_0 \lambda)$ are dimensionless tube radius and shear viscosity, λ is characteristic waveform width. b: Heating caused by a single relaxation process with different dimensionless characteristic times t_R $(0.1, 0.3, 1)$. Calculations are in accordance to the formula (17), with the acoustic excess dimensionless density (23).

density of the progressing rightwards sound is a single Gauss pulse:

$$\rho_a(\tau) = M \exp\left(-\tau^2\right). \tag{23}$$

The acoustic heating in Figure 2 is evaluated accordingly to the formula (17) with correspondent dispersive operator, $\widehat{A}_2$ or $\widehat{A}_3$ from (3).

The asymmetry of curves of $Q_{e,relax}$ are more pronounced for the larger values of t_R. That may be concluded immediately on the base of equation (17), because for small t_R, $\widehat{A}_2$ is almost proportional to $t_R\mathrm{d}/\mathrm{d}\tau$ (plus small dispersion term $O(t_R^2)$). The shape of curves of $Q_{e,relax}$ (achieves maximum for negative τ) and $Q_{e,bound}$ (achieves maximum for positive τ) reflects the dispersion features of operators $\widehat{A}_2$ and $\widehat{A}_3$. In contrast to relaxation dispersion, tube wall dispersion produces phase speeds smaller than the free-space phase speed c_0. Note that in spite of positive and negative areas of $Q_{e,relax}$ and $Q_{e,bound}$, summary heat produced in unit time and unit cross-section, $\int Q_e \mathrm{d}\tau$ (the value which equals to the ratio of excess and static temperatures after the sound passing), is always positive.

4. Conclusions

Acoustic heating is governed in fact by the two formulae, one describing the distortions of sound itself due to nonlinear, thermoviscous and dispersive phenomena (11), and the second one describing the correspondent heating (17). Both equations are instantaneous, the first one is well-known from many sources (see, for example, [1, 7, 17]). It may be derived by means of splitting of conservation equations in differential form into specific parts, by use of features of acoustic and non-acoustic types of fluid motion. The advance in the theory of acoustic heating is the instantaneous dynamic equation (17) governing acoustic heating caused by weak dispersion, including relaxation and Stoke's boundary layer effect.

It is important to stress the assumptions and ordering procedures on which the equations (11), (17) are based. Approximate equations are thought as series in powers of small parameters. The first is the Mach number M, in order for the differential Navier-Stokes system to be valid. The second is characteristic ratio of magnitudes of acoustic and non-acoustic (entropy) motions, the value varying with time. In the frames of traditional acoustics, sound is presupposed to be dominant, so the heating is secondary, induced by sound. Standard attenuation due to thermal conduction δ should be small, as well as the dispersive parameter.

General analytic solutions of equation (11), are not available. That is the reason to consider possibly simple sound waveforms in the Sec.3. The example of single relaxation process taking place in carbon dioxide CO_2 under normal conditions deals with the stationary waveform in the limit of weak nonlinearity. In the second example of boundary layer dispersion, analytic solutions of (11), even without the free-stream losses, are not available, so that one should resort to numerical solutions. As a rough approximation, the single stationary Gaussian pulse (23) stands as acoustic source. So, influence of nonlinearity and dispersion on sound itself are eliminated, but they are considered as the necessary origins of the respective heating.

It may be easily concluded, that the model equation for acoustic heating caused by an acoustic beam propagating in the positive direction of axis x, is also governed in the leading order by equation (17), with dimensionless excess acoustic density ρ_a satisfying the equation, analogous to the Khokhlov-Zabolotskaya-Kuznetsov one with account of weak dispersion:

$$\frac{\partial \rho_a}{\partial t} + \frac{\partial \rho_a}{\partial x} + \frac{\gamma+1}{2}\rho_a \frac{\partial \rho_a}{\partial x} + \frac{\varepsilon}{2}\int \Delta_\perp \rho_a \mathrm{d}x + \frac{1}{2}\left(\widehat{A} - \delta\frac{\partial}{\partial x}\right)\frac{\partial \rho_a}{\partial x} = 0, \tag{24}$$

where $\varepsilon\Delta_\perp$ denotes the dimensionless Laplacian, transversal to the direction x, ε is a small diffraction parameter expressing ratio of the longitudinal and transversal scales of perturbation [18]. The limits of integration must be determined accordingly to the physical meaning of the problem.

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