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Simulation Analysis of Transmission Loss in the Vibrating End Plate of an Expansion Chamber Silencer

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Summary
The boundary element method (BEM) is frequently employed to predict the transmission loss (TL) of an expansion chamber silencer with a rigid walled boundary, while the effect of the vibration of the end plates of the expansion chamber silencer is usually ignored. However, when the airflow or acoustic pressure in a duct is large and the end plate of the expansion chamber silencer is not thick enough, the effect of the vibration of the end plate should not be ignored. In this paper, based on a coupling of the BEM and the finite element method (FEM), the numerical results illustrate the existence of the effect on the TL of the vibrating end plates. Furthermore, numerical experiments also show that for an expansion chamber of constant size, the thicker the end plate, the less influence the vibration of the end plate has on the TL. And this may be helpful for the design of silencer.

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1. Introduction

The acoustic characteristics of sudden area changes in air ducts, such as expansion and contraction, are of importance in the reduction of duct-borne noise in automotive exhaust and ventilation systems [1]. To simplify the study of the performance of expansion silencers, most past studies assumed the use of a rigid duct wall and that the end plates of the expansion chamber would have a negligible effect [2, 3, 4, 5, 6]. Flexible structural layers have recently been introduced into air ducts and silencers for controlling noise [7]. The boundary element method (BEM) has been used to predict the performance of silencers in air duct and automotive exhaust systems. When a silencer is modeled by BEM, the transmission loss can be evaluated by either the conventional four-pole method or the more recently developed three-point method. However, when acoustic pressure or airflow in the duct is large and the expansion chamber silencer end plates are not thick enough, acoustic pressure or airflow can cause the end plates to vibrate. As a result, the value of the transmission loss (TL) is affected by the vibrating end plates of the expansion chamber [8, 9]. In this case, the value of the TL of a muffler cannot be exactly predicted using BEM. Therefore, based on the coupling of the boundary element method (BEM) and the finite element method (FEM), the aim of the present work is to study the transmission loss of an expansion chamber silencer that is due to the vibrating end plates of the expansion chamber.

In this paper, a simple expansion chamber silencer with a large diameter is considered. It is assumed that the diameter of the inlet duct is equal to the diameter of the outlet duct as shown in Figure 1, where L is the length of the expansion chamber, S_1 and S_2 are the cross-section areas of the inlet and outlet air ducts, respectively, of the main expansion chamber, and the expansion ratio $m = S_2/S_1$. The structure of the end plate is shown in Figure 2.

2. Coupling the BEM and FEM Models

A three-dimensional wave equation for the propagation of sound in an acoustics cavity is given by [6]

$$\nabla^2 p(\vec{x}, t) - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}(\vec{x}, t) = \rho_0 \frac{\partial \psi}{\partial t}, \tag{1}$$

where p is the acoustic pressure, c is the speed of the sound wave, ρ_0 is the density of the medium, and ψ is the acoustic source power-flux per unit volume. Based on Laplace transform, the coupled BEM and FEM method can be obtained from [10, 11, 12, 13].

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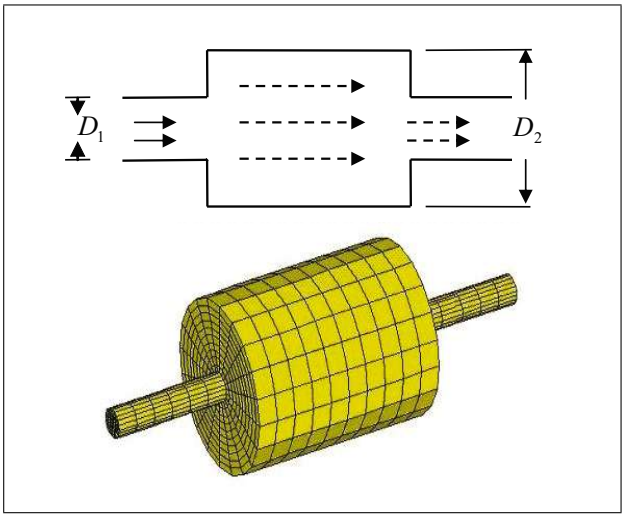

Figure 1. A simple expansion chamber silencer with a chamber.

3. Improved Four-pole Method

For calculating the TL, the four-pole method improved has the advantage of retaining the transfer matrix of the silencer, which contains important parameters when the silencer is connected to another silencer or to other components in the exhaust system.

A silencer with an inlet and an outlet can be represented by a linear acoustic four-pole network,

$$\begin{bmatrix} p_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ u_2 \end{bmatrix}, \tag{2}$$

where A, B, C and D are the four-pole parameters. Let $A^* = p_1|_{u_1=1,u_2=0}$, $B^* = p_1|_{u_1=0,u_2=1}$, $C^* = p_1|_{u_1=1,u_2=0}$ and $A^* = p_1|_{u_1=0,u_2=1}$, the four-pole parameters in equation (124) can be obtained for p_1 and v_1 in terms of p_2 and v_2. Doing so yields

$$A = A^*/B^*, \qquad B = B^* - A^*D^*/C^*, \tag{3}$$

$$C = 1/C^*, \qquad D = -D^*/C^*. \tag{4}$$

With the four-pole parameters available, the TL can then be calculated using the following equation [14]:

$$\mathrm{TL} = 20\log\left[\frac{1}{2}\left|A + \frac{B}{z_0} + Cz_0 + D\right|\right]. \tag{5}$$

4. Numerical Simulation

4.1. Influence of Different Thicknesses of End Plate on the TL

The end plate will cause sympathetic vibrations with sound wave propagation in the expansion chamber rigid walls, and a part of the sound wave will be diffused. In this case, instead of BEM method, the BEM and FEM are coupled to obtain an accurate count for the TL.

A simple expansion chamber silencer (an exhaust silencer of 10 kW diesel for test, JINNIU brand, made in China [15]) with a chamber length of $L = 40$ cm, inlet (outlet) duct diameter $D_1 = 7.18$ cm, main expansion chamber diameter $D_2 = 30.48$ cm, and an expansion ratio $m = S_2/S_1 = 18$ is assumed. The assumed material properties for the vibrating end plates are obtained from references as follows:

Youngs modulus $E = 1.0 \cdot 10^{11}$ N/m^2, Poisson's ratio $\mu = 0.3$, density ot the end plate $\rho = 7.0 \cdot 10^3$ kg/m^3, air density $\rho_a = 1.225$ kg/m^3, speed of sound in air $c_a = 340$ m/s, and thickness of the end plate $h = 1.27$ cm.

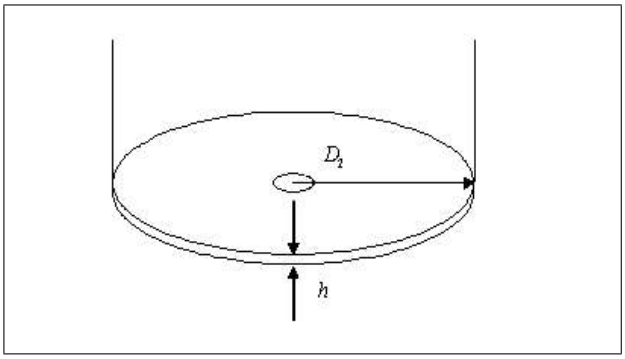

Figure 2. End plate structure.

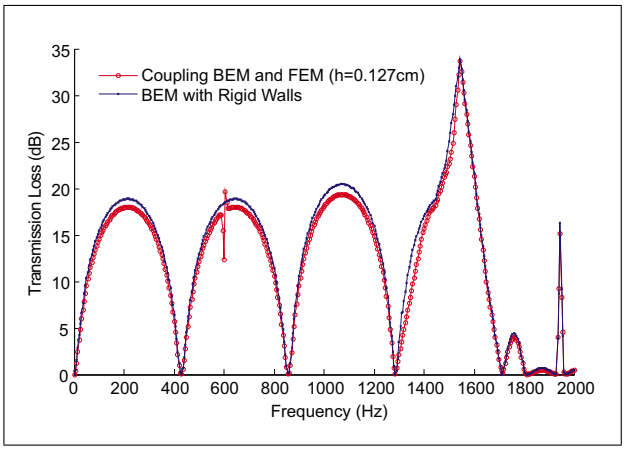

Figure 3. Numerical Result obtained by Coupling BEM and FEM.

For the TL analysis of this type of muffler element with rigid shell walls, a mesh of 196 triangular boundary elements and 52 triangular finite elements, and the coupled BEM-FEM model are utilized. As maximal noise frequency ranges of diesel engine about 100–600 Hz, we consider the range of frequencies to be about 0–1200 Hz. In fact, the coupled BEM-FEM model can be applied to that much higher frequencies are analyzed. Figure 3 shows the comparison between the numerical results obtained from the BEM-FEM model (for rigid shell walls and vibrating end plate) and the BEM model (rigid walls including the end plate and the shell walls).

It can be seen from Figure 3 that there is a minor point at a frequency of about 600 Hz. The dip peak in the TL curve at around 600 Hz is due to the first plate symmetric end plate structural resonant mode. In fact, the exact

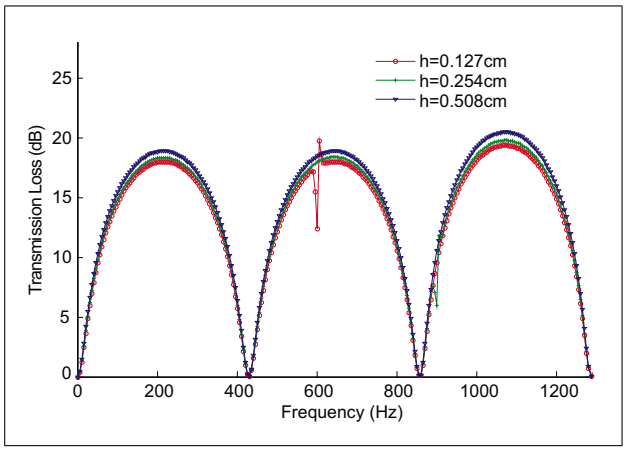

Figure 4. Influence of different thicknesses of vibrating end plates on the TL.

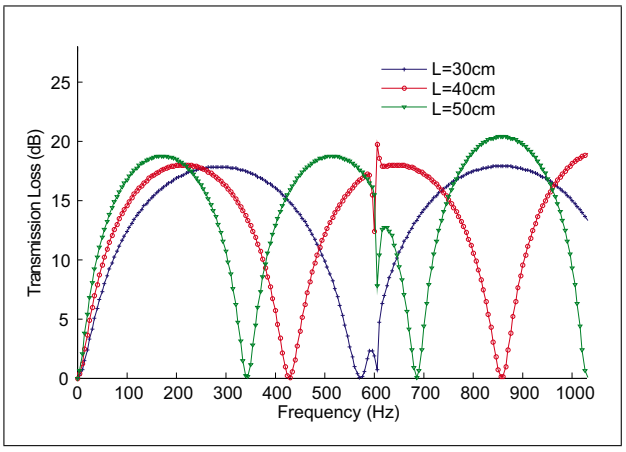

Figure 5. Influence of the vibrating end plates of different lengths on the TL.

value of this minor point is 602 Hz [16]. By using Lee's FEM [17], the approximate value of the minor point can be found to be 606 Hz. The position of the minor point can also be confirmed to be 605 Hz by the BEM-FEM coupled model. Compared with the results of BEM method, the curve of coupled BEM and FEM method illustrate the existence of the effect on the TL of the vibrating end plates. Furthermore, in order to analyze the influence of the vibrating end plate on the TL, a series of numerical experiments were carried out. First, the inhesion frequencies of the end plate were obtained in different thicknesses using the BEM-FEM coupled approach. The computing results of the inhesion frequencies in different thickness are found. When the thickness of the plate is $h = 0.127$ cm, and $h = 0.254$ cm, the inhesion frequencies of the end plate are 602 Hz, and 901 Hz.

Figure 4 shows the influence of different thicknesses of vibrating end plates on the transmission loss, and the positions (605 Hz, 902 Hz) of the minor points are shown. Figure 4 shows that for an expansion chamber of constant size, the thicker the end plate, the less influence the vibrating end plates have on the TL. The amplitude (difference between the maximum value and the minimum value) of the TL for the minor points where $h = 0.127$ cm and $h = 0.254$ cm is 7.32 dB and 5.81 dB, respectively, and the amplitude of the TL for the skipping point where $h = 0.508$ cm cannot be seen. For an expansion chamber of constant size, the value of the TL when $h = 0.508$ cm is very close to the rigid end plate and the rigid walls case (shown in Figure 3).

4.2. Influence of Different Lengths of Expansion Chamber on the TL

The influence of different lengths ($L = 30$ cm, 40 cm, 50 cm) of expansion chamber on the TL is studied. Consider the case where the thickness of the vibrating end plate , main expansion chamber diameter $D_2 = 30.48$ cm and an expansion ratio $m = S_2/S_1 = 187$. The transmission loss in vibrating end plates is similarly calculated using the BEM-FEM model. The influence of different lengths of the expansion chamber on the TL is shown in Figure 5.

It can be seen from Figure 5 that, for different lengths, there is a minor point at a frequency of about 605 Hz. The amplitudes (difference between the maximum value and the minimum value) of the TL for the minor points where $L = 30$ cm, 40 cm, 50 cm are 5.30 dB, 7.32 dB and 7.73 dB, respectively. For a constant diameter of the expansion chamber and constant thickness of the end plate, the longer the length of the expansion chamber, the more influence the vibration of the end plate has on the TL.

4.3. Influence of Different Diameters of Expansion Chamber on the TL

The influence of different expansion ratios ($m = S_2/S_1 = 13, 18, 23$) of the expansion chamber on the TL is studied. We assume the chamber length of $L = 40$ cm, inlet (outlet) duct diameter $D_1 = 7.18$ cm and different expansion ratios $m = S_2/S_1 = 13, 18, 23$, that is, different main expansion chamber diameters $D_2 = 25.89$ cm, 30.48 cm, 34.43 cm. Evaluated by the BEM-FEM coupled model, the inhesion frequencies of the end plate when the expansion ratio is $m = S_2/S_1 = 13, 18, 23$, are 920 Hz, 605 Hz and 455 Hz, respectively. It can be seen from Figure 6, that the amplitude (difference between the maximum value and the minimum value) of the TL for the minor points where $m = S_2/S_1 = 13, 18, 23$ is 4.73 dB, 7.32 dB and 8.24 dB, respectively. Similarly, Figure 6 shows that for an expansion chamber of constant length, thickness of the end plate and constant inlet (outlet) duct diameter constant, the bigger the expansion ratio of the expansion chamber, the more influence the vibrating end plates have on the TL.

5. Conclusion

Based on the coupled BEM and FEM method, this paper discussed the prediction of the transmission loss in the vibrating end plates of an expansion chamber silencer. Numerical experiments illustrate the existence of the effect

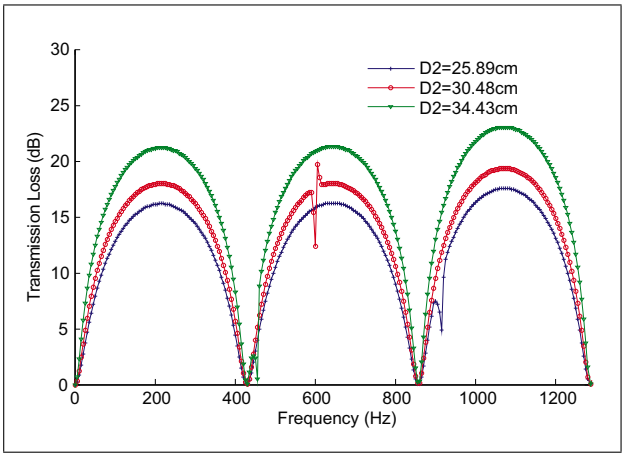

Figure 6. Influence of vibrating end plates of different diameters on the TL.

on the TL of the thickness of the vibrating end plates. It can also be seen from the numerical results that the minor points of transmission loss are related to the inhesion frequencies of the end plate. The numerical results show that the transmission loss in the vibrating end plates is also affected by the thickness of the end plate, the diameter of the expansion chamber, and the length of the expansion chamber. For an expansion chamber of constant size, the thicker the end plate, the less influence the vibration of the end plate has on the TL. It should be pointed out that the range of frequency where the sudden change of TL caused by the vibrating end plates is very small, and the effect might be negligible. Nevertheless, our investigation may be helpful for the better design of silencer. In addition, when calculating the TL value of a silencer, in comparison to the BEM method with the assumption of rigid walls, the use of the BEM-FEM coupling method can reduce the errors that arise.

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References

[1] B. S. Sridhara, M. J. Crocker: Review of theoretical and experimental aspects of acoustical modeling of engine exhaust systems. Journal of the Acoustical Society of American **95** (1994) 2363–2370.

[2] S. Bilawchuk, K. R. Fyfe: Comparison and implementation of the various numerical methods used for calculating transmission loss in silencer systems. Applied Acoustics **64** (2003) 903–916.

[3] L.-J. Yeh, Y.-C. Chang, M.-C. Chiu, G.-J. Lai: GA optimization on multi-segments muffler under space constraints. Applied Acoustics **65** (2004) 521–543.

[4] A. Cummings: Low frequency acoustic transmission through the walls of rectangular ducts. Journal of Sound and Vibration **61** (1978) 327–345.

[5] L. Huang: Modal analysis of a drumlike silencer. Journal of the Acoustical Society of America **112** (2002:) 2014–2025.

[6] A. Selamet, M. B. Xu, I.-J. Lee, N. T. Huff: Analytical approach for sound attenuation in perforated dissipative silencers. Journal of the Acoustical Society of America **115** (2004) 2091–2099.

[7] S. Ramamoorthy, K. Grosh, T. G. Nawar: Structural acoustic silencers-design and experiment. Journal of the Acoustical Society of America **114** (2003) 2812–2824.

[8] A. Cummings: End plate vibration in exhaust silencer chambers. Journal of Sound and Vibration **30** (1973) 367–372.

[9] C. Young, M. Crocker: Finite element acoustical analysis of complex muffler systems with and with wall vibration. Noise Control Engineering **9** (1977) 86–93.

[10] K. M. Mourand: The application of the boundary element method to predict the response of a differential operator model for a coupled visco elastic-acoustic system. Ph.D. Thesis, North Carolina State University, 1990.

[11] S. V. Sorokin, J. B. Nielsen, N. Olhoff: Green's matrix and the boundary integral equation method for the analysis of vibration and energy flow in cylindrical shells with and without internal fluid loading. Journal of Sound and Vibration **271** (2004) 815–847.

[12] A. Cummings, R. J. Astley: Finite element computation of attenuation in bar-silencer and comparison with measured data. Journal of Sound and Vibration **196** (1996) 351–369.

[13] T. Tsuji, T. Tsuchiya, Y. Kagawa: Finite element and boundary element modeling for the acoustic wave transmission in mean flow medium. Journal of Sound and Vibration **255** (2002) 849–866.

[14] M. L. Munjal: Acoustics of ducts and mufflers. Wiley-Inter science, New York, 1987.

[15] J. Li, S. Wang, L. Wu: Test and study on noise reduction of silencer (in Chinese). Machinery Design and Manufacture **8** (2005) 120–121.

[16] R. D. Blevins: Formulas for natural frequency and mode shape. Van Nostrand Reinhold Co., 1979.

[17] C. M. Lee, L. H. Royster, R. D. Ciskowski: Coupling of FEM and BEM for the analysis of a structure-acoustic-cavity system. – In: Boundary Elements XV. Computational Mechanics Publications, Southampton, 1993.