Bibliographical reference

A New Approximate Formula for Scattering of Plane Acoustic Waves by a Spherical Obstacle, Subodh K. Sharma and Ratan K. Saha, Acta Acustica **vol. 95** (Number 1), 2009, pp. 1-6

DOI
https://doi.org/10.3813/AAA.918123

ACTA ACUSTICA UNITED WITH ACUSTICA
Vol. 95 (2009) 1 – 6

Scientific Papers
DOI 10.3813/AAA.918123

A New Approximate Formula for Scattering of Plane Acoustic Waves by a Spherical Obstacle

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Summary
A new approximate formula for scattering amplitude of a plane acoustic wave by a sphere is presented. It is based on a similar formula introduced long ago by Hart and Montroll in the context of scattering of light by a sphere. The scattering amplitude is shown to consist of two terms. The first term is related to the scattering amplitude in the modified Born approximation, a variant of the well known Born approximation. The second term constitutes a correction which is of the order of impedance mismatch between the scatterer and the surrounding medium. Numerical comparisons of this formula with exact results have been performed. It is shown that the new approximation betters the modified Born approximation and other variants of the Born approximation as the weak scatterer condition for the validity of Born type approximations is relaxed.

PACS no. 43.20.Fn

1. Introduction

Approximation methods are known to play an important role in the analysis of acoustic scattering. This is because (i) exact solutions are not always possible and (ii) approximate solutions are simple and can provide deeper physical insight into the scattering processes. In many cases, these lead to analytic relations involving particle properties and the quantity to be observed such as scattered intensity or attenuation etc. Therefore, it is not surprising that a number of approximate methods have been developed over the years. Some of these, valid for weak scatterers, are Born approximation [1], long wavelength approximation [1], Rytov approximation [2] and eikonal approximation [3]. The Born approximation (BA) and long wavelength approximation are applicable to scatterers of sizes small or comparable to wavelength of incident wave. For larger obstacles, the Rytov and the eikonal approximations can be employed.

Many attempts have been made in the past to design formulas that better BA results. While variants such as distorted wave Born approximation (DWBA) [4, 5] and the modified Born approximation (MBA) [3] have been devised to improve scattering pattern predictions, at least one attempt has also been made to correct the scattering amplitude by introducing a phase term in DWBA [6] for the purpose of determining extinction using forward scattering theorem. This, of course, is inconsequential in scattering pattern computations and will be mentioned only for the sake of completeness in discussions in this paper. As for validity domains of these modified variants, all three have the same validity domain theoretically and hence can be looked upon as belonging to the same class.

In this paper a new approximate formula is introduced for scattering of acoustic plane waves by a sphere. This has been done by drawing a parallel between scalar light scattering and acoustic wave scattering. In contrast to other variants of BA, such as MBA and DWBA, this approximation goes a step further. It takes higher order corrections into account explicitly. The first term in this formula is related to MBA by a multiplicative factor which depends on size parameter of scatterer and velocity and impedance mismatch between the scatterer and the surrounding medium. This multiplicative factor could be approximated to unity for soft scatterers. The second term is the correction which is expected to broaden the validity of MBA to higher impedance mismatch between scatterer and the surrounding medium. The original approximation scheme, in the context of optics, is due to Hart and Montroll [7] and hence has been referred to as the Hart and Montroll approximation (HMA). We continue to refer analogous approximation in acoustics too as HMA.

This paper is organized as follows. To begin with, relevant formulas in BA, MBA and DWBA have been introduced in section 2. The development of HMA is presented in section 3. Numerical comparison of HMA with exact

Received 19 April 2008,
accepted 16 October 2008.

results are displayed in section 4. Finally, we conclude by summarizing our results in section 5.

2. Scattering by a sphere in the Born approximation and its variants

As is well known, the scattering amplitude in BA for an obstacle of arbitrary shape can be written as [1]

$$\Phi_{ba}(\mathbf{k_i},\mathbf{k_s}) = \frac{k^2}{4\pi}\int_v \left(\gamma_\kappa + \hat{\mathbf{k}}_\mathbf{i}\,\hat{\mathbf{k}}_\mathbf{s}\gamma_\rho\right)\mathrm{e}^{\,\mathrm{i}(\mathbf{k}_i-\mathbf{k}_s)\,\mathbf{r}}\,\mathrm{d}v, \quad (1)$$

where $\mathbf{k}_i$ and $\mathbf{k}_s$ are respectively incident and scattered wave vectors, $\gamma_\kappa = (\kappa_1 - \kappa)/\kappa$ and $\gamma_\rho = (\rho_1 - \rho)/\rho_1$ are respectively the normalized mismatch of compressibility and density. The notations κ_1, ρ_1 are for obstacle and κ, ρ are for the surrounding medium. The integration is over the volume v of the scatterer. The most crucial assumption in derivation of (1) is that the unknown field inside the scatterer can be taken to be the same as incident field $\mathrm{e}^{\,\mathrm{i}\mathbf{k}_i\mathbf{r}}$. The integration in (1) can be performed analytically for a homogeneous sphere and one arrives at the following expression for the scattering amplitude [1],

$$\Phi_{ba}(k,\theta) = k^2a^3\left[\gamma_\kappa + \gamma_\rho\cos\theta\right]\left[\frac{j_1(2ka\sin(\theta/2))}{2ka\sin(\theta/2)}\right], \quad (2)$$

where $|\mathbf{k}_i| = |\mathbf{k}_s| = k$, a is the radius of the spherical obstacle and j_1 is the spherical Bessel function of first order. The subscript ba indicates Born approximation. Mathematically, the conditions for the validity of BA can be expressed as

$$|\gamma_\kappa| \ll 1 \qquad |\gamma_\rho| \ll 1, \quad (3)$$

and

$$ka|\gamma_\kappa| < 1 \qquad ka|\gamma_\rho| < 1. \quad (4)$$

The inequalities (4) imply that BA can be valid even for particles large compared to wavelength of the incident wave provided the interaction strength $(ka|\gamma_\kappa|, ka|\gamma_\rho|)$ is still small. A modification over the BA was proposed by Chu *et al.* [4], Stanton *et al.* [5] and Chu and Ye [6]. The modified approximation has been refereed to as the DWBA. In DWBA the field inside the scatterer is assumed to have the same functional form as in BA but replaces $\mathbf{k}_i$ in the phase by $\mathbf{k}_1$, representing a distorted wave. With the corresponding modification in the Green's function, scattering amplitude in the DWBA takes the form

$$\Phi_{dwba}(\mathbf{k_i},\mathbf{k_s}) = \frac{k_1^2}{4\pi}\int_v \left(h^2\gamma_\kappa + \hat{\mathbf{k}}_\mathbf{i}\,\hat{\mathbf{k}}_\mathbf{s}\gamma_\rho\right)\mathrm{e}^{\,\mathrm{i}(\mathbf{k}_i-\mathbf{k}_s)\,\mathbf{r}}\,\mathrm{d}v, \quad (5)$$

where $h = c_1/c$ is sound speed contrast and $|\mathbf{k}_s| = |\mathbf{k}_1| = k/h$. For a sphere, integration in (5) can be performed. The result is [6]

$$\Phi_{dwba}(k,\theta) = k_1^2a^3\left[h^2\gamma_\kappa + \gamma_\rho\cos\theta\right] \cdot \left[\frac{j_1(2k_1a\sin(\theta/2))}{2k_1a\sin(\theta/2)}\right]. \quad (6)$$

The only major difference between Φ_{ba} and Φ_{dwba} is that the wave number $k's$ in large square bracket in (2) are replaced by $k_1's$. Although $k_1 \approx k$ for weak scattering, a slight change in k to k_1 in the argument of the spherical Bessel function has been noted [6] to gives rise to a significant difference in the frequency response of the scattering, especially for large ka.

A somewhat similar approximation has been obtained by Sharma and Saha [3] recently. In this approximation too, as in DWBA, the unknown pressure field within the scatterer is assumed to have the same functional form as in BA. But $\mathbf{k}_i$ is now replaced by $n\mathbf{k}_i$, where $n = c/c_1 = 1/h$ is the mismatch of wave speed in the surrounding medium and in the scatterer. The quantity n is analogue of the refractive index in optics. The expression for the scattering amplitude in this modified Born approximation (MBA) takes the form

$$\Phi_{mba}(k,\theta) = k^2a^3\left[\gamma_\kappa + n\gamma_\rho\cos\theta\right]\frac{j_1(R)}{R}, \quad (7)$$

where $R = ka\sqrt{1+n^2-2n\cos\theta}$. The subscript mba indicates the modified Born approximation.

Theoretically, the validity domains of (6) as well as (7) are also given by (3) and (4). Thus, numerical comparisons of MBA and the BA were performed by Sharma and Saha [3]. It was demonstrated that MBA leads to a larger applicability domain in comparison to BA. Numerical computations have also been made for $|\Phi_{mba}|^2$ and $|\Phi_{dwba}|^2$ by Saha [8] at scattering angles $\theta = 0, \pi/4$ and π. It was found that at least for near forward scattering MBA performed better than DWBA over a large range of x values. The comparisons were made in the range $\rho_1 = 1.01 - 1.04$, $\kappa_1 = 0.89 - 0.97$ and $ka = 1.0 - 20.0$.

3. Hart and Montroll approximation

The scattering amplitude for the scattering of a plane acoustic wave by a sphere in partial wave analysis can be written as [1]

$$\Phi_{ex}(k,\theta) = \frac{\mathrm{i}}{k}\sum_{m=0}^{\infty}(2m+1)b_mP_m(\cos\theta), \quad (8)$$

where P_m is Legendre polynomial of order m and

$$b_m = \frac{j'_m(x)j_m(y) - \alpha j_m(x)j'_m(y)}{h'_m(x)j_m(y) - \alpha h_m(x)j'_m(y)}. \quad (9)$$

In equation (9), j_m and h_m are respectively the spherical Bessel and Hankel function of order m and primes denote derivative with respect to their arguments. Here $x = ka$, $y = nka$ and $\alpha = n\rho/\rho_1$. Subscript ex in equation (8) refers to exact solution.

For large values of index m, such that $m/x \ll 1$, the spherical Bessel functions can be approximated as [9]

$$j_m(x) \sim \frac{1}{x}\cos\left(x - \frac{m+1}{2}\pi\right),$$
$$j'_m(x) \sim \frac{1}{x}\sin\left(x - \frac{m+1}{2}\pi\right), \quad (10)$$

and

$$h_m(x) \sim \frac{1}{x} e^{\,i\left(x - \frac{m+1}{2}\pi\right)}, \quad h'_m(x) \sim \frac{i}{x} e^{\,i\left(x - \frac{m+1}{2}\pi\right)}. \quad (11)$$

The denominator D of (9) can then be readily expressed as,

$$D = \frac{i}{xy} e^{\,i\left(x-y\right)} \frac{\alpha+1}{2} \left[1 - \frac{\alpha-1}{\alpha+1} e^{2i\left(y - \frac{m+1}{2}\pi\right)}\right]. \quad (12)$$

where $\alpha = \sqrt{\kappa_1 \rho / \rho_1 \kappa}$ is nothing but Z/Z_1 with Z_1 and Z respectively as acoustic impedances of obstacle and surrounding medium. Clearly the contribution of the second term depends on impedance mismatch, α, between obstacle and the surrounding medium. With this approximation for the denominator in place, the scattering amplitude in (8) can be expressed as

$$\Phi_{hma}(\theta) = C \sum_{m=0}^{\infty} (2m+1) \left[j'_m(x) j_m(y) - \alpha j_m(x) j'_m(y) \right] \cdot \left[1 - r(-1)^m e^{2iy}\right] P_m(\cos\theta), \quad (13)$$

where

$$C = \frac{2xy e^{\,ix(n-1)}}{k(\alpha+1)\left[1 - r^2 e^{4iy}\right]}, \quad (14)$$

and $r = (\alpha - 1)/(\alpha + 1)$. The subscript *hma* refers to Hart and Montroll approximation. The infinite sum in (13) can then be carried out by employing the following relations:

$$j'_m(x) j_m(y) - \alpha j_m(x) j'_m(y) = \left[\frac{\partial}{\partial x} - \alpha \frac{\partial}{\partial y}\right] j_m(x) j_m(y), \quad (15)$$

$$\sum_{m=0}^{\infty} (2m+1) j_m(x) j_m(y) P_m(\cos\theta) = \frac{\sin(R)}{R}, \quad (16)$$

and

$$\sum_{m=0}^{\infty} (-1)^m (2m+1) j_m(x) j_m(y) P_m(\cos\theta) = \frac{\sin(\bar{R})}{\bar{R}}, \quad (17)$$

where $\bar{R} = \sqrt{x^2 + y^2 + 2xy\cos\theta}$. The summation leads to the following simple formula for the scattering amplitude,

$$\Phi_{hma}(\theta) = \frac{2nx^2 a e^{\,ix(n-1)}}{(\alpha+1)} \left[(\gamma_\kappa + n\gamma_\rho \cos\theta) \frac{j_1(R)}{R} - r(\gamma_\kappa - n\gamma_\rho \cos\theta) e^{2iy} \frac{j_1(\bar{R})}{\bar{R}} \right], \quad (18)$$

which is the main result of this paper. In arriving at (18) we have ignored terms of order r^2. We refer to (18) as acoustic analogue of HMA scattering amplitude in optics. For $\alpha = 1$, the second term on the right hand side of (18) vanishes yielding

$$\Phi_{hma1}(\theta) \approx nx^2 a e^{\,ix(n-1)} \left[(\gamma_\kappa + n\gamma_\rho \cos\theta) \frac{j_1(R)}{R} \right]. \quad (19)$$

But for a factor n^2, this approximation yields exactly the same expression for scattered intensity,

$$I(\theta) = |\Phi(\theta)_{hma1}|^2,$$

as does MBA. This explains why MBA yielded excellent results near $\alpha = 1$ in an earlier work [3]. In addition, if (3) and (4) are satisfied, $\exp(ix(n-1))$ and n can be taken as unity and (19) reduces to MBA. Thus, for particles with n close to unity, the second term in (19) can be viewed as a correction term to MBA provided $r < 1$.

Let us now examine more closely the validity of approximations made in arriving at (18). These are, m is large and $m/x \ll 1$. The restrictions imply that scatterer is large compared to the wavelength. At first sight, the use of the approximation $m/x \ll 1$ in writing (10) and (11) may appear questionable because it is well known that significant contributions in the scattering amplitude (8) also arise from m values that are approximately equal to argument x. Nevertheless, the approximation has been used in optics [7, 10] and is found to yield good predictions for scattered intensity at all angles for small particles and at small angles for large particles. In part, the reason for this has been traced to the fact that, apart from the diffraction term, the dominant contribution at forward scattering for a weak scatterer arises from modes close to central incidence i.e., from m values satisfying the condition $x \gg m$ [11]. This suggests a limitation of weak and small angle scattering on the validity of (18). Also as was noted earlier in this section, HMA reduces to MBA for weak scatterers implying that HMA is valid for $x \ll 1$ also despite the initial assumption of large x in the derivation of HMA. A numerical check on the validity of HMA will be made in the next section.

The extinction cross-section is related to the forward scattering amplitude via the relation [1, 6]

$$\sigma_e = \mathrm{Im}\left(\frac{4\pi\Phi(0)}{k}\right), \quad (20)$$

where $\Phi(0)$ is the forward scattering amplitude. As $\Phi(0)$ in BA, DWBA as well as in MBA is purely real, the σ_e obtained from (20) is zero in all three approximations. To rectify this shortcoming, a multiplicative phase term was designed by Chu and Ye [6].

The DWBA, in conjunction with this term, could be cast in the following form:

$$\Phi_{pc-dwba}(\theta, x) = \Phi_{dwba} e^{\frac{2i}{3}[1+\sin(\theta/2)](n-1)x}. \quad (21)$$

The phase term in (21) is designed by extending the exact one dimensional analytic solution to three dimensions in a heuristic manner. A need for more systematic derivation was recognized by authors themselves. In the present derivation the phase term emerges in a natural way. But this phase differs from the one obtained in (21). However, this is not surprising because (18) and (21) are based on entirely different approaches.

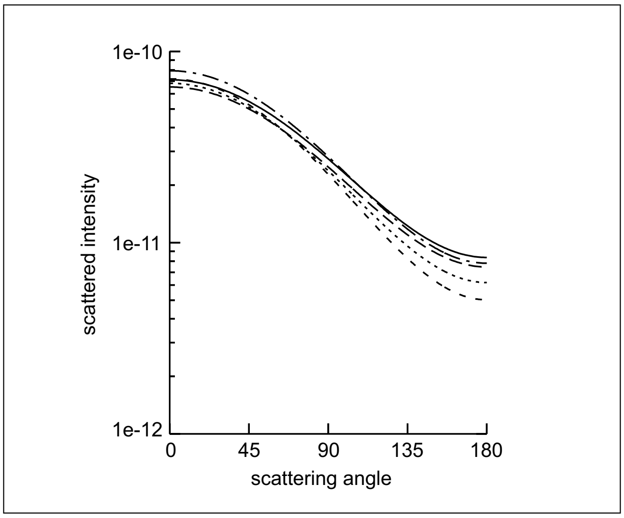

Figure 1. A comparison of exact scattered intensity with that in various approximations for $x = 1, n = 1.099$ and $\alpha = 1.0465$. Solid line: Exact, Large dashed line: BA, Small dashed line: DWBA, Dotted line: MBA, Dash-dot line: HMA.

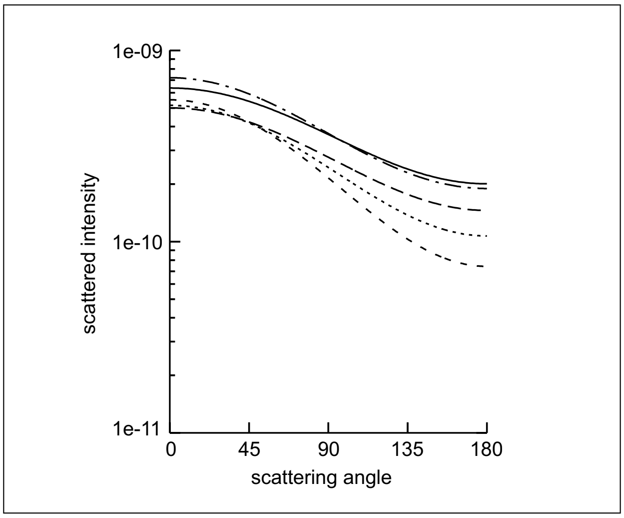

Figure 2. Same as Figure 1 but for $n = 1.255$ and $\alpha = 1.195$.

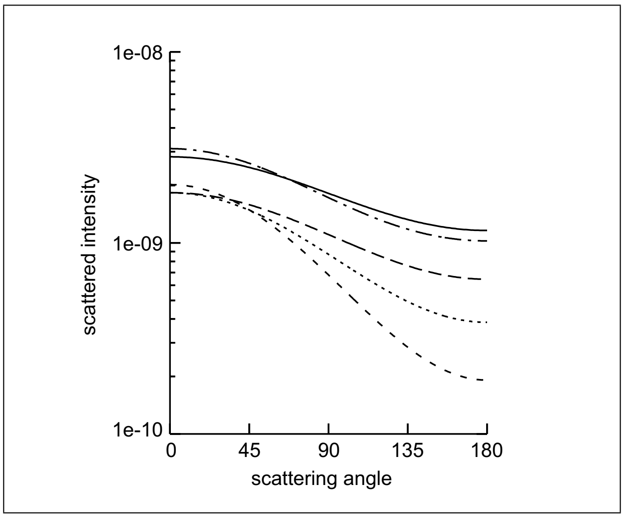

Figure 3. Same as Figure 1 but for $n = 1.449$ and $\alpha = 1.38$.

4. Numerical comparisons

In this section scattered intensities in BA, DWBA, MBA and HMA and imaginary part of scattering amplitude in PCDWBA and HMA have been compared numerically with exact computations for a sphere and results have been presented for some representative values of x, α and n. For scattered intensity PCDWBA is same as DWBA. The scattered intensity has been defined as $I(\theta) = |\Phi(\theta)|^2$. In all computations in this paper, k has been taken to be $8159.980918\,\mathrm{m}^{-1}$. This is a typical value for biomedical tissues which was used in our earlier computations [3] also.

Figure 1 shows a plot of $I(\theta)$ against θ for $x = 1$, $\rho_1/\rho = 1.05$ and $\kappa_1/\kappa = 1.15$. These values of ρ_1/ρ and κ_1/κ are equivalent to $\alpha = 1.04654$ and $n = 1.09886$. It can be seen from Figure 1 that for impedance and velocity mismatch of this order, all approximations perform satisfactorily at small scattering angles. The DWBA has a slight edge over others. At backward angles, HMA performs best with BA a close second. Errors are comparatively larger in MBA and DWBA.

The value of κ_1/κ is increased to 1.5 in Figure 2 while ρ_1/ρ and x remain unchanged. Corresponding α and n are $\alpha = 1.19523$ and $n = 1.25499$. Except for HMA, the error is significantly larger in all other approximations in comparison to those in Figure 1. That the role of correction term in (18) is important is clearly evident in Figure 2. The HMA performs best at all scattering angles. Even for back-scattering HMA yields excellent results. These features of HMA become even more prominent in Figure 3 where we have plotted scattering patterns for same values of x and ρ_1/ρ but κ_1/κ is further increased to 2.0. Corresponding values of α and n are now 1.38013 and 1.44914 respectively.

In the above figures we have shown the effect of increase of α on the accuracy of various approximations. However, in doing so we have simultaneously increased the value of n also. A typical graph showing the effect of increase in α for a fixed n can be seen in Figure 4. The n value here is same as in Figure 1 but α has increased to 1.08798 from 1.04654 with $\rho_1/\rho = 1.01$ and $\kappa_1/\kappa = 1.19554$. It is evident that the performance of HMA relative to other approximations is better in Figure 4 in comparison to that in Figure 1.

Clearly, HMA constitutes a notable advance over other approximations at all angles when $(\alpha - 1)$ is not very small.

For large value of x, $\bar{R}$ is a large number as long as $\cos\theta$ remains positive ($\theta = 0 - 90^0$). Hence, the contribution of correction term is negligible for near forward angles. Thus, HMA and MBA are not expected to yield very different values for large particles at small scattering angles. At $\theta = \pi/2$, $\bar{R} = R$ and at scattering angles greater than this $\bar{R}$ starts decreasing and R starts increasing. This means that at backward angles the correction term could even become the dominant contribution if r is not very small. Figure 5 shows a comparison of scattered intensi-

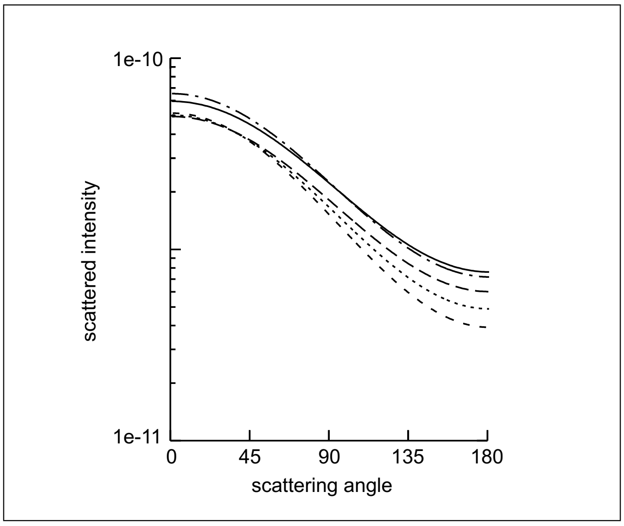

Figure 4. Same as Figure 1 but for α increased to 1.088.

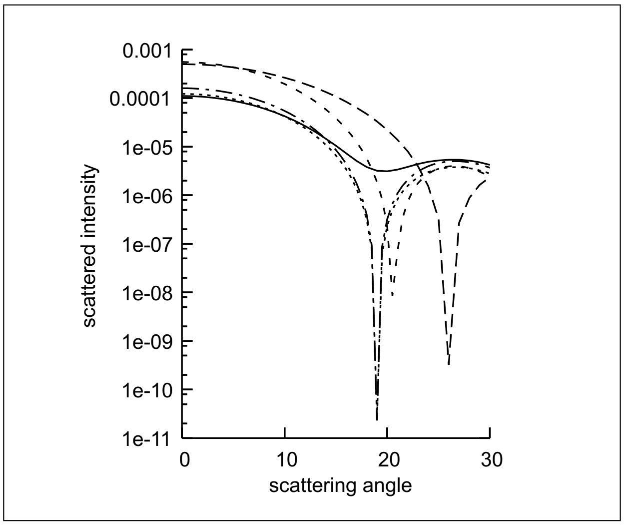

Figure 6. Same as Figure 2 but for x = 10.0.

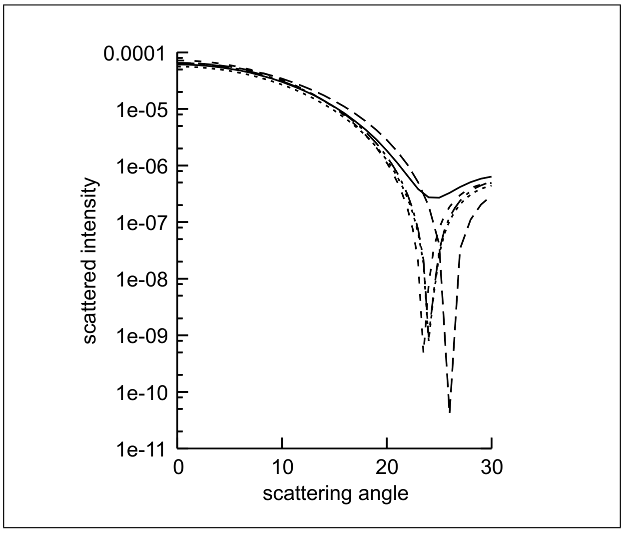

Figure 5. Same as Figure 1 but for x = 10.0.

ties in various approximations against exact scattered intensity for $x = 10.0$, $n = 1.09886$ and $\alpha = 1.04654$ at small scattering angles. Scattered intensity is depicted only up to 30 deg to maintain clarity in the figure. As expected MBA and HMA yield identical results. Again, as in case for $x = 1$, all approximations yield reasonable results at small scattering angles. However, the HMA or MBA yield the best results in the sense that they reproduce the positions of maxima and minima most accurately. This is noteworthy because the positions of minima and maxima are related to the size of the scatterer. The minima are a result of first zero of the Bessel function which occurs when its argument is 4.49. As θ increases, errors in all the approximations increase. The results for a higher mismatch for the same value of x are shown in Figure 6. Here $\alpha = 1.19523$ and $n = 1.25499$. All the features noted from Figure 5 become more explicit in this figure. We noted that HMA replicates, although only qualitatively, even the back-scattering lobe for large particles. This is not the case for other approximations.

Imaginary parts of the scattering amplitude in the forward direction predicted by PCDWBA and HMA have been compared with exact computations for a large number of ρ_1/ρ and κ_1/κ values to test the potential use of (20). The values of x were limited to $x \leq 20$ as in the work of Chu and Ye [6]. We find that the percent errors in approximations are not very small. Nevertheless, there are regions of x, n and α values for which PCDWBA works well. On the other hand, there are also domains of x, n and α values for which HMA yields better agreement with exact results. However, a clear cut identification of domains in which use of one approximation or other is suitable for imaginary part of scattering amplitude doe not seem possible in a straightforward manner.

5. Conclusions

To summarize, a new approximate formula (referred to as HMA) is presented in this paper to describe the scattering of acoustic plane waves by a sphere. This formula is acoustic analogue of Hart and Montroll formula of optical scattering. It is fundamentally different from previous modifications of BA in that it takes into account higher order impedance mismatch corrections and its validity is not limited by inequalities (3) and (4).

The HMA has been contrasted numerically with exact computations for various values of size and impedance and velocity mismatch parameters. For completeness, the Born approximation, the distorted wave Born approximation and the modified Born approximation have also been included in the comparison. Following conclusions can be drawn from the comparisons of scattered intensities. For small scatterers i.e., for $x < 1$, all approximations examined in this paper are reasonably good at small scattering angles when the scattering is weak. The best among these is the DWBA. At large scattering angles different approximations yield widely different results. The HMA yield best predictions and gives very good results even at backward angles. As α increases, the HMA is found to yield the best agreement with exact scattered intensities at all scattering angles. In relation to other approximations, HMA is much better even at backward angles.

For larger particles, it is found that (i) In the region where $\cos\theta$ is close to 1, $\bar{R}$ is much larger than R. Thus, the effect of second term in HMA is negligible at small scattering angles. In this region HMA and MBA do not yield significantly different results. Here, both the approximations are found to be equally good for first few maxima and minima. (ii) From $\theta = 90\,\text{deg}$ to $\theta = 180\,\text{deg}$, $\cos\theta$ varies between 0 and -1. In this angular domain, the second term can become the dominant term at backward angles if r is not very small. None of the approximations yields good results at backward angles, though HMA predictions are closest to exact results and even mimic the back scattering lobe qualitatively.

The phase term obtained in HMA differs from that obtained in PCDWBA. This is not surprising because the two derivations differ in approach. While the PCDWBA has been based on some heuristic arguments, the HMA has been derived systematically starting from the exact solution for scattering by a homogeneous sphere. From the numerical comparisons of imaginary part of the forward scattered amplitude, it is concluded that one needs to be rather cautious in using (20) in conjunction with HMA or PCDWBA.

Finally, it may be mentioned that although we restricted ourselves to a spherical scatterer in this paper, a formula on same lines can be easily derived for an infinite long cylinder in an identical way. This problem, in the context of optical scattering, has been addressed by Sharma *et al.* [10].

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