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Conditions for Bifurcation of the Longitudinal Wave in a Porous Medium

Lin Lin, Zongmu Wang, M. L. Peterson
Department of Mechanical Engineering, University of Maine, Orono, Maine 04469, USA.
Lin.lin@umit.maine.edu; Zongmu.wang@yahoo.com; Michael.peterson@maine.edu

Summary
The critical wave number for the slow longitudinal wave in porous materials has been previously determined using asymptotic methods. However, in some situations, such as porous polymeric materials, the accuracy of the asymptotic assumption behavior is questionable. This paper presents an exact solution for calculation of the critical wave number. The solution is shown to significantly alter the critical wave number for low density highly porous materials such as polymeric porous materials. The potential importance of the measurement of critical wave number is demonstrated by showing the sensitivity of the measure to permeability and the insensitivity of the measure to other input variables.

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1. Introduction

Biot first developed phenomenological models for the propagation of elastic waves in fluid-saturated porous materials [1]. In Biot's work the existence of one shear wave and two longitudinal waves was predicted. These waves were denoted as fast (type one) and slow (type two) waves respectively with reference to their velocity. The slow longitudinal wave has a lower phase velocity and generally higher attenuation than the fast wave. Both the phase velocity and the attenuation of the slow wave are strongly frequency dependent. The highly attenuated slow wave is of general interest because it is a phenomenon that does not have an analog in solid materials. The slow longitudinal wave is characterized by the out of phase movement of the pore fluid and the matrix materials. As others have recently noted, the out of phase motion offers a unique opportunity to study properties of porous materials such as permeability and tortuosity [2, 3].

The first experimental observation of the slow wave was reported by Plona for sintered glass beads [4]. Following Plona's work, extensive consideration has been given to the slow wave in different porous media, especially geophysical materials [5, 6, 7], and human bones [8, 9]. In Biot's theory the slow wave is highly attenuated below some critical frequency. This critical frequency is related to the skin depth of the fluid in the pores. Recent work has focused on finding the critical frequency for specific media [10]. A theoretical analysis to study the reflection and transmission at the interface of the fluid and porous material also was developed by Wu *et al.* [11]. This is the most complete description of the interface properties, but requires a total of eight parameters to describe the material and structure of the porous material. Edelman [12] used asymptotic methods to explain the bifurcation of the waves in a porous material. The slow wave, as expected, does not propagate if the wave number is below a critical wave number. Questions regarding the range of accuracy for the asymptotic method arise for experimental investigators making use of lower acoustic impedance, higher porosity materials. Therefore an exact solution to this problem is of practical use. It is of particular interest to experimentalists who may be searching for the transition frequency in order to probe the permeability characteristics of porous materials.

Permeability is an important parameter in the theory of sound propagation in porous materials at the low frequency range. The permeability is introduced through the description of the viscous coupling between the fluid and the porous structure [2]. Therefore, the ability to measure permeability is extremely useful in studies of acoustical properties of porous materials. Understanding permeability at porous interface is critical to in applications in both biological and engineering system. Examples include efflux and influx of nutrient and contaminants in sediments [13]; transport of percellular fluid into bones [14]; removal of fluid from porous rocks [15]; and determination of the efficiency of a membrane separation among other applications. No method currently exists for in-situ determination of the permeability of a porous material in industrial applications [16]. The measurement of critical wave number provides a unique method to obtain the permeability measurement. This method will provide a in- situ, nondestructive, non-contaminating and low cost method for

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this measurement. Previous researchers have shown that in order to observe the slow wave experimentally, the viscous frequency should be used [10, 17]. The viscous frequency is on the order of 10^4 of the critical frequency.

In this work we consider the exact determination of the bifurcation condition. An exact solution for the critical wave number is developed. An analysis of the reflection coefficient is used to provide a practical method for observing the effects of the slow wave. The experimentally important problems of interest such as permeability are considered for typical geophysical and other materials. These results are useful from a practical perspective since it makes it possible to identify the range of frequencies which must be investigated in order to detect the critical transition frequencies in these materials. In addition, the result presented here should be useful for validating and, if necessary, refining numerical models of the system.

2. Fundamental relations and dispersion equation

Consider the propagation of the bulk wave through the infinite fluid-saturated porous medium with the porosity of n_0. The mass density of the fluid is ρ_0^{FR}, and ρ_0^{SR} is the mass density of the solid. The two-phase porous medium can be described by the system of mass conservation equations, momentum conservation equations, balance equation of porosity, and constitutive relation equations for linear poroelastic materials as shown in the Appendix. The longitudinal wave propagation is considered. The coupling effect caused by the change of porosity is ignored here, since the influence on the fast and slow waves velocity is much smaller than those due to the relative motion of fluid and solid phases. The dispersion equation for longitudinal waves propagated in a porous medium is then given as [10]:

$$\begin{aligned} &r(\omega^2 - c_f^2 k^2)(\omega^2 - k^2) \\ &+\mathrm{i}\omega\pi\left[(1+r)\omega^2 - k^2(1+rc_f^2)\right] = 0, \end{aligned} \tag{1}$$

where

$$r = \frac{\rho_0^F}{\rho_0^S}, \tag{2}$$

$$c_f = \frac{U^F}{U_{II}^S}. \tag{3}$$

Here ρ_0^F represents the partial mass density of the fluid, and can be calculated by $\rho_0^F = n_0\rho_0^{FR}$, ρ_0^S represents the partial mass density of the solid and $\rho_0^S = (1-n_0)\rho_0^{SR}$. U^F is the velocity of the longitudinal wave in the fluid and U_{II}^S is the velocity of the longitudinal wave in the skeleton. ω is the frequency, k is the wave number and the constant π is related to the porosity n_0, the viscosity of the fluid μ^F and the permeability of the porous medium κ as $\pi = n_0\mu^F/\kappa$. Obviously, here, r, c_f, π and k are all positive coefficients.

3. Bifurcation formulas

3.1. Two quadratic equations and associated discriminants

Equation (1) can be written into a quartic equation of x,

$$\begin{aligned} &x^4 - \left(1+\frac{1}{r}\right)x^3 + \left(1+c_f^2\right)yx^2 \\ &-\left(\frac{1}{r}+c_f^2\right)yx + c_f^2 y^2 = 0, \end{aligned} \tag{4}$$

where

$$x = \frac{\mathrm{i}\omega}{\pi}, \tag{5}$$

$$y = \frac{k^2}{\pi^2}. \tag{6}$$

The quartic equation (4) is transformed to

$$z^4 + pz^2 + qz + s = 0 \tag{7}$$

through the substitution

$$z = x - \frac{1}{4}\left(1+\frac{1}{r}\right), \tag{8}$$

where in equation (7)

$$p = -\frac{3}{8}\left(1+\frac{1}{r}\right)^2 + \left(1+c_f^2\right)y, \tag{9}$$

$$q = -\frac{1}{8}\left(1+\frac{1}{r}\right)^3 + \frac{1}{2}\left(1-\frac{1}{r}\right)\left(1-c_f^2\right)y, \tag{10}$$

$$\begin{aligned} s = &-\frac{3}{256}\left(1+\frac{1}{r}\right)^4 + \frac{1}{16}\left(1+\frac{1}{r}\right) \\ &\cdot\left(1-\frac{3}{r}-3c_f^2+\frac{c_f^2}{r}\right)y + c_f^2 y^2. \end{aligned} \tag{11}$$

Equatikon (7) can then be expressed as the product of the quadratic polynomials with real coefficients

$$\left(z^2 + lz + m\right)\left(z^2 - lz + n\right) = 0, \tag{12}$$

where

$$m + n - l^2 = p, \tag{13}$$

$$l(n-m) = q, \tag{14}$$

$$mn = s. \tag{15}$$

Appropriate combination of equations (13), (14) and (15) yields a resolvent cubic in l^2,

$$l^6 + 2pl^4 + (p^2 - 4s)l^2 - q^2 = 0, \tag{16}$$

When $l^2 = 0$, the left-hand side of equation (16) is less than zero. And when $l^2 \to \infty$, the left-hand side of equation (16) is greater than zero. Hence, the resolvent cubic equation (16) has at least one positive root. Equation (12) for $l \geq 0$ is same as that for $l \leq 0$ except that the positions of the two quadratic polynomials in the equation are reversed. Thus any positive roots of the equation of l may be used in the following result. Two discriminants for two quadratic equations (12) are

$$\Delta_1 = l^2 - 4m, \tag{17}$$

$$\Delta_2 = l^2 - 4n. \tag{18}$$

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3.2. Critical wave number

From equation (5) and equation (8), if frequency ω has four real values, four roots of equation (4) and equation (12) are complex numbers, which require that

$$\Delta_1 < 0, \tag{19}$$
$$\Delta_2 < 0. \tag{20}$$

Thus the critical state determining whether frequency ω has four real values is

$$\Delta_1 = 0, \tag{21}$$
$$\text{and} \quad \Delta_2 = 0. \tag{22}$$

Solving equation (13), equation (15) and equation (22) yields the solutions

$$l = 2\sqrt{-\frac{p}{6} \pm \frac{1}{6}\sqrt{p^2+12s}}, \tag{23}$$
$$m = \frac{p}{2} \pm \frac{1}{2}\sqrt{p^2+12s}, \tag{24}$$
$$n = -\frac{p}{6} \pm \frac{1}{6}\sqrt{p^2+12s}. \tag{25}$$

Substitution of equations (23), (24) and (25) into equation (14) leads to the non-linear equation

$$2\sqrt{-\frac{p}{6} \pm \frac{1}{6}\sqrt{p^2+12s}}\Big(-\frac{2p}{3} \mp \frac{1}{3}\sqrt{p^2+12s}\Big) = q, \tag{26}$$

which can be changed to

$$\left(p^2+12s\right)^3 = \left(p^3 - 36ps + 13.5q^2\right)^2. \tag{27}$$

Substituting equations (9), (10) and (11) into equation (27) leads to the following third order nonlinear equation, which can be solved to obtain the critical value of y, or k^2/π^2:

$$F(y) = Ay^3 + By^2 + Cy + D = 0, \tag{28}$$

where

$$A = 4c_f^2(c_f-1)^4(c_f+1)^4, \tag{29}$$
$$B = -(c_f-1)^2(c_f+1)^2\left[\left(c_f^2+\frac{1}{r^2}\right)\left(c_f^4-10c_f^2+1\right) + 24c_f^2\frac{c_f^2+1}{r}\right], \tag{30}$$
$$C = -2\left(c_f^4+\frac{1}{r^4}\right)\left(c_f^4-4c_f^2+1\right) + \left(\frac{c_f^2}{r}+\frac{1}{r^3}\right)\left(5c_f^6+3c_f^4+3c_f^2+5\right) + \frac{1}{4}\left(\frac{c_f^8+108c_f^6-122c_f^4+108c_f^2+1}{r^2}\right), \tag{31}$$
$$D = -\left(r+1\right)^3\frac{(c_f^2r+1)^3}{r^6}. \tag{32}$$

Here A is positive and D is negative. B and C can be positive or negative based on r and c_f. Equation (28) has at least one positive root since

$$\text{when} \quad y = 0, \quad F(0) = D, \tag{33}$$
$$\text{and} \quad F(0) < 0. \tag{34}$$

Also when $y \to +\infty$,

$$\frac{F(y)}{y^2} = Ay + B - \theta\Big(\frac{1}{y}\Big) \tag{35}$$
$$\text{and} \quad \frac{F(y)}{y^2} > 0. \tag{36}$$

Each positive root of equation (28) corresponds to a bifurcation point. It is shown by equation (34) and equation (36) that at least one bifurcation point exists. The existence of the bifurcation point explains the fact that the slow wave was not observed in some materials. The wave number must be greater than the critical value in order to generate the slow wave.

Equation (28) can then be reduced to the form

$$F(t) = t^3 + at + b = 0, \tag{37}$$

where

$$a = \frac{1}{3}\left[3\Big(\frac{C}{A}\Big) - \Big(\frac{B}{A}\Big)^2\right], \tag{38}$$
$$b = \frac{1}{27}\left[2\Big(\frac{B}{A}\Big)^3 - 9\Big(\frac{B}{A}\Big)\Big(\frac{C}{A}\Big) + 27\Big(\frac{D}{A}\Big)\right], \tag{39}$$
$$t = y + \frac{B}{3A}. \tag{40}$$

Since

$$\frac{b^2}{4}+\frac{a^3}{27} = \frac{(c_f^2r^2-1)^2\left[(8c_f^6+3c_f^4+3c_f^2+1)(c_f^2r^2+1)\right]^3}{442368c_f^8r^{11}(c_f^2-1)^{12}} + \frac{-r(c_f^8+76c_f^6-282c_f^4+76c_f^2+1)}{442368c_f^8r^{11}(c_f^2-1)^{12}} \tag{41}$$

and

$$\frac{b^2}{4}+\frac{a^3}{27} > 0. \tag{42}$$

Hence, there will be only one real root for equation (37). This real root is positive as shown above. Therefore, equation (28) also has one and only one positive root, which is

$$y = \sqrt[3]{-\frac{b}{2}+\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}} + \sqrt[3]{-\frac{b}{2}-\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}} - \frac{B}{3A}, \tag{43}$$

giving the critical wave number as

$$k_{cr} = \sqrt{y}\pi. \tag{44}$$

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As mentioned in reference [12], this dimensionless equation (44) reads in physical variables as:

$$k_{cr} = \sqrt{y}\frac{\pi}{\rho_0^S U_{II}^S}. \tag{45}$$

It is important to note that the critical wave number can thus be determined based on knowledge of four parameters of the porous structure: the velocity of the longitudinal wave in the skeleton, the mass density of the solid, the total porosity of the structure, and the permeability of the material.

4. Numerical examples

To compare the results with other published data, the water-saturated properties of the sandstone used for the asymptotic computations [12] was considered. The physical constants for the sandstone are: porosity $n_0 = 0.2$, density of the fluid $\rho_0^{FR} = 1000\,\mathrm{kg/m^3}$, density of the solid phase $\rho_0^{SR} = 2650\,\mathrm{kg/m^3}$. The velocities of the longitudinal wave in fluid and solid skeleton are $U^F = 1450\,\mathrm{m/s}$ and $U_{II}^S = 4450\,\mathrm{m/s}$, respectively and $\pi \approx 4 \cdot 10^7\,\mathrm{kg/m^3 s}$. The critical wave number evaluated for the above parameters is $k_{cr} = 70.27/\mathrm{m}$ using the exact solution shown above. The corresponding result of $k_{cr} = 70.22/\mathrm{m}$ is obtained using an asymptotic solution.

The exact solution may be applied to any material which is of interest. Human cancellous bone is an important topic of inquiry in which Biot's theory has proven important. Consider a bone sample designated M1 by Fellah *et al.* [8] with the following characteristics: porosity $n_0 = 0.83$, fluid density $\rho_0^{FR} = 1000\,\mathrm{kg/m^3}$, solid density $\rho_0^{SR} = 1960\,\mathrm{kg/m^3}$, velocities of the longitudinal wave in fluid is $U^F = 1450\,\mathrm{m/s}$. The velocity of the longitudinal wave in the solid skeleton is calculated to be $U_{II}^S = 3952\,\mathrm{m/s}$. With the parameter $\pi \approx 3 \cdot 10^4\,\mathrm{kg/m^3 s}$, the critical wave number is about 0.02/m, which is comparable to the critical frequency calculated by Hughes *et al.* [10].

Another important porous material is porous cellulose acetate material used in filtration. A PVDF membrane manufactured by Millipore Co. is considered here. The properties of the membrane are: porosity $n_0 = 0.75$, fluid density $\rho_0^{FR} = 1000\,\mathrm{kg/m^3}$, solid density $\rho_0^{SR} = 1780\,\mathrm{kg/m^3}$. Velocity of the longitudinal wave in the fluid and the solid skeleton are $U^F = 1450\,\mathrm{m/s}$ and $U_{II}^S = 1844\,\mathrm{m/s}$, respectively. The difference in the calculation of critical wave number evaluated for the above parameters is 33.6% using the exact solution and the asymptotic method. It is evident from this example of a low acoustic impedance, highly porous material that the asymptotic method is suitable for finding critical wave numbers for many porous materials, but limits exist for materials with higher permeability and low acoustic impedance.

Reflection and transmission of elastic waves at an interface between fluid and fluid-saturated porous solids is important from the point of view of a number of applications in ultrasonic evaluation. When the wave number is higher than the critical wave number, the slow wave propagates in the porous material although attenuation may be high. The propagation of the slow wave will partition the energy at the interface even if the wave only propagates a short distance. As a result, a significant decrease in the reflection coefficient is observed above critical wave number. This effect is evident in the transition of the reflection coefficient shown by Wu *et al.* [11].

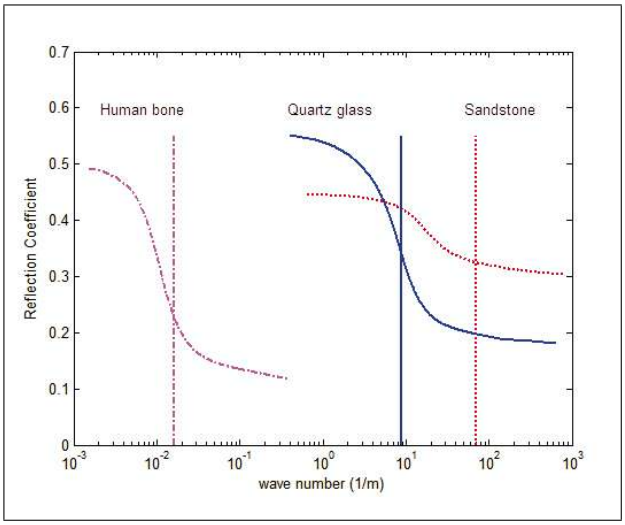


Figure 1. The reflection coefficient is shown as a function of wave number for quartz glass, human bone and sandstone. The exact solution is represented as vertical lines.

Figure 1 shows the calculated reflection coefficient of the quartz glass micro-fiber material [4] as a function of wave number. Human bone [8] and sandstone [12] are also calculated and shown in figure 1 for comparison. The calculated critical wave number from the exact solution is shown as a vertical line. The result from sandstone shows significant error. However, the reflection coefficient based on Wu *et al.* [11] requires more properties than the original data [12] used for asymptotic calculations. So, the other parameters including: bulk modulus of the skeleton, shear modulus of the skeleton, pore size, and tortuosity were estimated based on Carcione *et al.* [18] and Johnson *et al.* [19].Therefore, the error is mostly a result of uncertainty caused by limited knowledge of porous structure.

5. Sensitivity test

Numerical tests can be performed to show the sensitivity of material parameter using the exact solution. The sensitivity test can be done by varying the single parameter by 20% while the values of other properties are kept constant. Porosity is an important parameter of any porous materials. Porosity is a primary concern since the total porosity of any sample tested may not be well characterized. The total porosity may also vary spatially in a sample in some materials such as bone. The numerical result shows when the porosity increases 20%, the value of k_{cr} increases 0.53%. Thus the sensitivity to this measurement is quite small.

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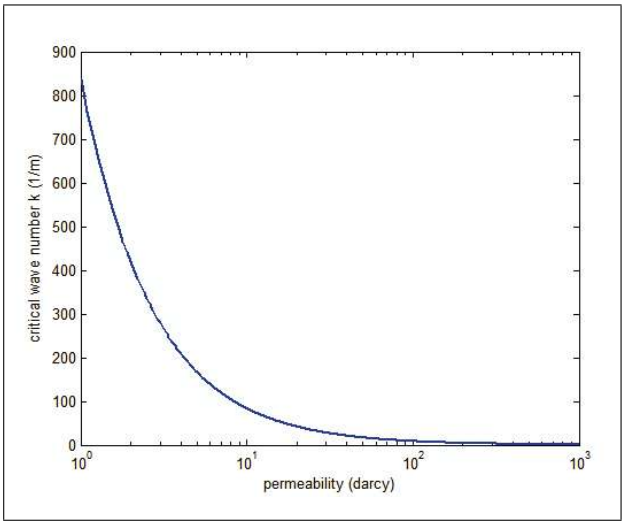


Figure 2. The critical wave number decreases with the increase of the permeability.

Porous materials have complicated structures, in most cases, the modulus of the frame may have to be estimated. This is particularly true in polymeric materials where the cross-link density of the material may change the modulus but is not well characterized in the porous material. The velocity of the longitudinal wave in the skeleton also affects the critical wave number. Numerical results shows with the increase of 20% in the velocity in the skeleton results in a decrease in the critical wave number of only 0.51%. It is clear that the critical wave number is then quite insensitive to the frame modulus or the velocity in the skeleton.

Permeability however is a key feature of interest in geophysical applications and filtration performed using a porous material. Unlike the small effect caused by porosity and the velocity in skeleton, the critical wave number decreases by approximately 16.7% when the permeability of the porous material increases 20%. Thus it appears that measuring the critical wave number would be a suitable technique for measuring the permeability of a porous material. Figure 2 shows the relationship of the critical wave number with the permeability of a cellulose acetate material used in filtration.

6. Conclusions

The conditions for bifurcation of the Biot waves in a porous medium are investigated analytically in this work. An exact solution for the critical wave number is given. The exact solution is compared to a previously published asymptotic solution and another analytical model. The exact solution is shown to be well suited to making calculations in all materials including those where the acoustic velocity in the skeleton is similar to the velocity in the fluid. A general relationship is developed which will allows the critical wave number of the slow wave to be used to measure permeability, a phenomena of technological interest in porous materials.

Appendix

The propagation of the bulk wave through the infinite fluid-saturated porous medium can be described by the system of equations as follows:
Mass conservation equations:

$$\begin{aligned} &\frac{\partial \rho^F}{\partial t} + \mathrm{div}\left(\rho^F v^F\right) = 0, \\ &\frac{\partial \rho^S}{\partial t} + \mathrm{div}\left(\rho^S v^S\right) = 0, \end{aligned} \tag{A1}$$

where ρ^F represents the partial mass density of the fluid, which can be calculated by $\rho^F = n\rho^{FR}$. ρ^S represents the partial mass density of the solid, and $\rho^S = (1-n)\rho^{SR}$. ρ^{FR} is the mass density of the fluid, ρ^{SR} is the mass density of the solid, and n is the porosity. v^F and v^S are the velocity vectors, and represent the velocity of fluid and solid phases respectively.
Momentum conservation equations:

$$\begin{aligned} &\rho^F \frac{\partial v^F}{\partial t} - \mathrm{div}T^F + \pi\left(v^F - v^S\right) = 0, \\ &\rho^S \frac{\partial v^S}{\partial t} - \mathrm{div}T^S + \pi\left(v^F - v^S\right) = 0, \end{aligned} \tag{A2}$$

where T^F and T^S are the partial stress tensors of fluid and solid phases; and the constant π has the relation with the initial porosity n_0, the viscosity of the fluid μ^F, and the permeability of the porous medium κ as $\pi = n_0\mu^F/\kappa$.
Balance equation of porosity:

$$\frac{\partial n}{\partial t} + n_0 \mathrm{div}\left(v^F - v^S\right) = -\frac{n - n_0}{\tau}, \tag{A3}$$

where τ is the relaxation time of porosity.
Constitutive relations for linear poroelastic materials:

$$\begin{aligned} p^F &= p_0^F + K\left(\rho^F - \rho_0^F\right), \\ T^F &= -p^F - \beta\left(n - n_0\right), \\ T^S &= T_0^S + \lambda^S I_1\left(e^S\right) + 2\mu^S e^S + \beta\left(n - n_0\right), \end{aligned} \tag{A4}$$

where p^F is the pore pressure, K represents the compressibility of the fluid, p_0^F and ρ_0^F are the initial values of the pore pressure and the fluid mass density. β is the coupling constant, λ^S and μ^S are the Lamé constants of the solid skeleton, and $I_1(e^S)$ is the first invariant of the small deformation tensor.

Identification of the parameters of this model in terms of the Biot parameter, and the assumption of ignoring the coupling constant have been discussed in the Appendix of reference [12].

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