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Transmission Loss Investigation of Two-Dimensional Thin Mufflers by the Modal Expansion Method Using a New Line Source Model

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Summary

The transmission losses (TLs) of different types of two-dimensional thin mufflers have been investigated by solving the pressure response function using the modal expansion method. These thin mufflers are commonly employed in domestic refrigeration compressors because of the limited space in such compressors. The point source model has convergence problems at points close to the source, whereas the surface source model may generate a non-uniform pressure distribution on the source surface. A new square line source model was implemented in this study. Using this new model, a generic formulation of four-pole parameters by the modal expansion method was developed in two different curvilinear coordinates, enabling TL to be calculated. To verify the proposed line source model, the TL results for a cylindrical thin muffler were compared with those obtained by the three-dimensional finite element method (FEM). The TLs of rectangular and cylindrical mufflers with various inlet/outlet positions and dimensions were investigated. The results demonstrate that the proposed model can determine the effects of higher-order modes on TLs. In addition, comparison with the circular line source model revealed that the proposed square line source model is computationally efficient and can determine TLs with a sufficiently high accuracy.

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1. Introduction

Pressure pulsations in the suction and discharge of compressors are undesirable because they can reduce a compressor's performance, induce pipeline vibrations, and lead to system failure. Reactive mufflers are generally used to attenuate pressure pulsations, since the small holes and porous materials of dissipative mufflers may eventually become clogged with deposits [1]. Large, conventional reactive mufflers are widely employed in large reciprocating compressors used in industrial applications because these compressors have sufficient space to mount large mufflers. However, such large mufflers are difficult to install in domestic refrigeration compressors due to the limited space. Therefore, thin reactive mufflers are commonly used in such compressors [2].

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The formulation of four-pole parameters and the transfer matrix method were first used by Munjal [2] to investigate the performance of reactive mufflers. This method is convenient for investigating plane wave propagation below the cut-on frequency of the muffler because of its computational efficiency and flexibility, although it neglects the effects of higher-order modes of mufflers. Kim and Soedel [3] investigated pressure pulsations in three-dimensional acoustic cavities by accounting for the effects of higher-order modes by utilizing pressure response solutions derived using the modal expansion method. Lai and Soedel [2, 4] applied the same approach to analyse two-dimensional mufflers with uniform and non-uniform thicknesses. Gas pulsations in such mufflers have also been investigated [5, 6]. The abovementioned studies [2, 3, 4, 5, 6] employed point source models to calculate the pressure response functions; however, these models might not converge for sound pressures at points near the point source. To overcome these convergence problems, a

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surface source model [7] has been proposed for calculating the pressure response functions. Kadam and Kim [8] obtained the four-pole parameters of a three-dimensional rectangular muffler using the surface source model and verified the results by experiment. However, the surface source model may generate a highly non-uniform pressure distribution on the source surface [7], which greatly reduces the computational accuracy. Huang and Jiang [9] used a circular line source model to calculate the four-pole parameters of two-dimensional rectangular mufflers. This model gives a more uniform pressure distribution and does not suffer from the convergence problems of the point source model. However, it is computationally intensive.

In the present study, a square line source model, which is more computationally efficient than the circular line source model, is used to calculate the pressure response solutions of two-dimensional mufflers based on the method of modal expansion. The corresponding four-pole parameters and TLs of two thin mufflers with different inlet/outlet positions and dimensions were formulated by the solutions of pressure responses. This method does not suffer from the convergence problems of the point source model. To verify the proposed model, the TL results of a cylindrical thin muffler were compared with those obtained by three-dimensional FEM. In addition, the square surface source model [7], the circular line source model [9], and the proposed square line source model were compared.

2. Formulation of TL by the modal expansion method using a square line source model

Before discussing the formulation of the modal expansion method using the square line source model in Sections 2.1 and 2.2, it is necessary to briefly review the modal expansion method for solving pressure response functions. The transfer matrix defines the relationship between the input and output variables of an acoustic system in the frequency domain. For the acoustic system shown in Figure 1, the equations that relate the transfer matrix and the input/output variables can be expressed as

$$\begin{Bmatrix} Q_1 \\ P_1 \end{Bmatrix} = \begin{Bmatrix} A & B \\ C & D \end{Bmatrix} \begin{Bmatrix} Q_2 \\ P_2 \end{Bmatrix}, \tag{1}$$

where the subscripts 1 and 2 indicate input and output points, respectively. P and Q are the harmonic amplitudes of the acoustic pressure and the volume flow rate, respectively. A, B, C, and D are the four-pole parameters; their equations and definitions are given in [3].

The TL of the system is defined [10] in terms of the four-pole parameters as

$$TL = 20\log_{10}\left[\sqrt{Z_2/Z_1}\,\middle|\, A(Z_2/Z_1) + BZ_1 + C/Z_2 + D\,\middle|\,/2\right], \tag{2}$$

where Z_1 and Z_2 are the characteristic impedances of the inlet and outlet ducts, respectively. For infinitely long ducts without reflection, $Z_1 = \rho c/S_1$ and $Z_2 = \rho c/S_2$. S_1 and S_2 are the cross-sectional areas of the inlet and outlet ducts, respectively. ρ is the fluid density and c is the speed of sound in the fluid.

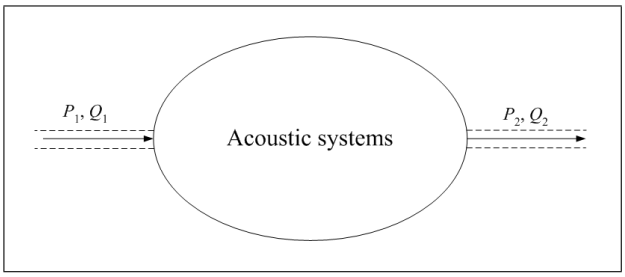


Figure 1. Schematic representation of acoustic system of a muffler.

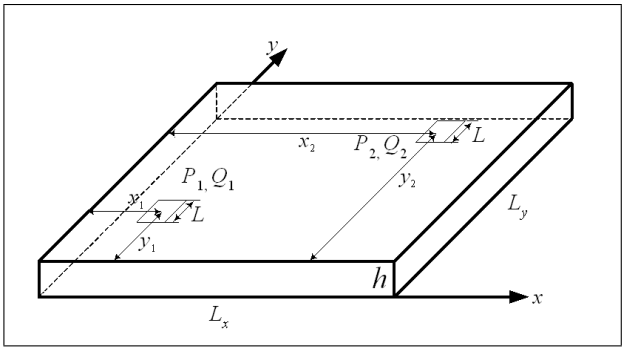


Figure 2. Thin rectangular muffler element in square line source model.

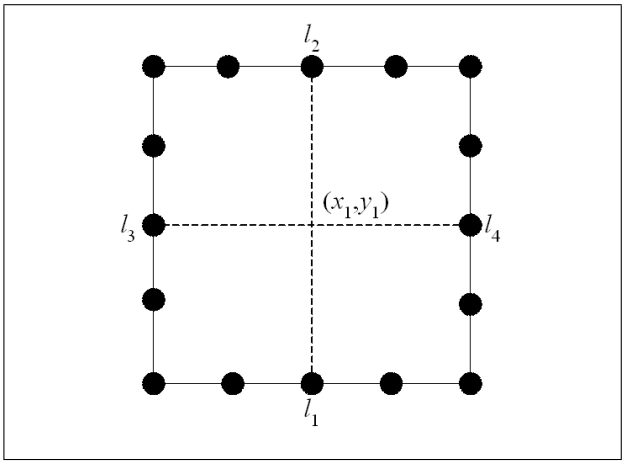


Figure 3. Schematic representation of the square line source model.

2.1. Thin rectangular muffler element

Cartesian coordinates are utilized for the thin rectangular mufflers shown in Figure 2. The wave equation can be written as

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\partial \dot{m}}{\partial t} \tag{3}$$

The dimensions of the muffler are $L_x \times L_y \times h$. The inlet and outlet are located at (x_1, y_1) and (x_2, y_2), respec-

tively. (P_1, Q_1) and (P_2, Q_2) are the harmonic amplitudes of the acoustic pressure and volume flow rate for the inlet and outlet, respectively. With the square line source model shown in Figure 3, the mass flow rate can be expressed as

$$\dot{m} = \sum_{i=1}^{2} \left(\dot{m}_{xi} + \dot{m}_{yi}\right), \tag{4}$$

where

$$\begin{aligned}\dot{m}_{xi} &= \int_{xi} -\frac{\rho Q}{4Lh}\delta(x - x_0)\delta(y - y_0 - L/2)\,\mathrm{d}x \cdot \mathrm{e}^{\mathrm{j}\omega t} \\ &= \frac{\rho Q}{4Lh}\Big[H\big(x - x_0 + L/2\big) - H\big(x - x_0 - L/2\big)\Big] \\ &\qquad \cdot \delta(y - y_0 + (-1)^i L/2) \cdot \mathrm{e}^{\mathrm{j}\omega t}, \end{aligned} \tag{5}$$

$$\begin{aligned}\dot{m}_{yi} &= \int_{yi} -\frac{\rho Q}{4Lh}\delta(y - y_0)\delta(x - x_0 - L/2)\,\mathrm{d}y \cdot \mathrm{e}^{\mathrm{j}\omega t} \\ &= \frac{\rho Q}{4Lh}\Big[H\big(y - y_0 + L/2\big) - H\big(y - y_0 - L/2\big)\Big] \\ &\qquad \cdot \delta(x - x_0 + (-1)^i L/2) \cdot \mathrm{e}^{\mathrm{j}\omega t}, \end{aligned} \tag{6}$$

where $\delta(..)$ is the Dirac function, $H(..)$ is the unit step function, L is the side length of the square line, (x_0, y_0) denotes the coordinates of the center of the source, j is the imaginary unit, and t is time.

By implementing the modal expansion method [3], the pressure response at the center of the line source can be obtained using

$$\begin{aligned} &f_1(x, y, \omega) = \\ &\frac{\mathrm{j}\rho c^2 \omega Q}{Lh} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{p_{mn}(x, y)\big(C_x B_y + C_y B_x\big)}{N_{mn}\big[(\omega_{mn}^2 - \omega) + 2\mathrm{j}\omega\omega_{mn}\xi_{mn}\big]}, \end{aligned} \tag{7}$$

where

$$B_x = \begin{cases} \dfrac{2L_x}{m\pi} \sin\dfrac{m\pi L}{2L_x} \cos\dfrac{m\pi x_0}{L_x} & \text{if } m \neq 0, \\ L & \text{if } m = 0, \end{cases} \tag{8}$$

$$B_y = \begin{cases} \dfrac{2L_y}{n\pi} \sin\dfrac{n\pi L}{2L_y} \cos\dfrac{n\pi y_0}{L_y} & \text{if } n \neq 0, \\ L & \text{if } n = 0, \end{cases} \tag{9}$$

and

$$C_x = 2\cos\frac{m\pi x_0}{L_x} \cos\frac{m\pi L}{2L_x}, \tag{10}$$

$$C_y = 2\cos\frac{n\pi y_0}{L_y} \cos\frac{n\pi L}{2L_y}, \tag{11}$$

p_{mn}, ω_{mn}, and ξ_{mn} are the complex pressure modal amplitude, the circular eigenfrequencies, and the damping ratio of the (m, n) mode, respectively. N_{mn} is given by

$$N_{mn} = \begin{cases} L_x L_y/4, & \text{if } m, n = 1, 2, \ldots, \\ L_x L_y/2, & \text{if } m = 0, \\ & \quad n = 1, 2, \ldots, \text{or circulation} \\ L_x L_y, & \text{if } m = n = 0. \end{cases} \tag{12}$$

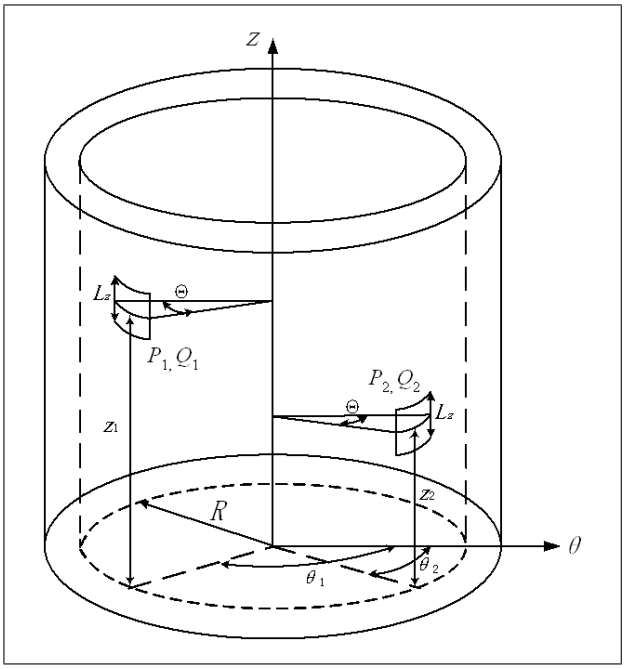


Figure 4. Thin cylindrical muffler element in square line source model.

Because the pressure distribution in the line source model is non-uniform, the pressure response function should be calculated using the average pressure as

$$f_{ii} = \frac{\oint f(x, y, \omega)\,\mathrm{d}l/4L}{Q}. \tag{13}$$

The pressure response function f_{11} is obtained by setting $(x, y) = (x_1, y_1)$ and $Q = 1$ in equation (13). f_{21}, f_{22}, and f_{12} can be obtained in a similar way. Therefore, the four-pole parameters and the TL of a two-dimensional thin rectangular muffler with arbitrarily located inlet and outlet can be obtained using equation (2).

2.2. Thin cylindrical muffler element

Thin cylindrical mufflers (see Figure 4) are frequently used in hermetically sealed refrigeration compressors. In Figure 4, R is the average radius of the thin cylinder, Θ is the included circular angle of the source, and Z is the length of the source in the z direction. The wave equation in cylindrical coordinates is given by

$$\frac{1}{R^2}\frac{\partial^2 p}{\partial\theta^2} + \frac{\partial^2 p}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\partial\dot{m}}{\partial t}. \tag{14}$$

If the muffler is connected to ducts with square cross-sections, the relation $R\Theta = Z$ must be satisfied. The mass flow rate of the square line model can be expressed as

$$\dot{m} = \sum_{i=1}^{2} \left(\dot{m}_{zi} + \dot{m}_{\theta i}\right), \tag{15}$$

where

$$\dot{m}_{zi} = \int_{zi} -\frac{\rho Q}{4Zh}\delta(z-z_0)\delta(\theta-\theta_0-(-1)^i\theta/2)\,\mathrm{d}z\dot{\mathrm{e}}^{\mathrm{j}\omega t} = \frac{\rho Q}{4Zh}\Big[H\big(z-z_0+Z/2\big)-H\big(z-z_0-Z/2\big)\Big] \cdot \delta(\theta-\theta_0-(-1)^i\Theta/2)\cdot \mathrm{e}^{\mathrm{j}\omega t}, \quad (16)$$

$$\dot{m}_{\theta i} = \int_{\theta i} -\frac{\rho Q}{4Lh}\delta(\theta-\theta_0)\delta(z-z_0-(-1)^i Z/2)\,\mathrm{d}z\dot{\mathrm{e}}^{\mathrm{j}\omega t} = \frac{\rho Q}{4Lh}\Big[H\big(\theta-\theta_0+\Theta/2\big)-H\big(\theta-\theta_0-\Theta/2\big)\Big] \cdot \delta(z-z_0+(-1)^i Z/2)\,\mathrm{d}z\cdot \mathrm{e}^{\mathrm{j}\omega t}, \quad (17)$$

where (z_0, θ_0) are the coordinates of the centre of the source and h is the muffler thickness. Using the modal expansion method [3], the pressure response function at the centre of the line source can be obtained as

$$f_1(z,\theta,\omega) = \frac{\mathrm{j}\rho c^2\omega Q}{4Zh}\sum_{l=1}^{2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{p_{mnl}(z,\theta)\big(C_\theta B_z + C_z B_\theta\big)}{N_{mnl}\left[(\omega_{mn}^2-\omega)+2\mathrm{j}\omega\omega_{mn}\xi_{mn}\right]}, \quad (18)$$

where

$$B_z = \begin{cases} \dfrac{2L_z}{m\pi}\sin\dfrac{m\pi Z}{2L_z}\cos\dfrac{m\pi z_0}{L_z} & \text{if } m\neq 0,\\ Z & \text{if } m=0,\end{cases} \quad (19)$$

$$B_\theta = \begin{cases} \dfrac{2R}{n}\sin\dfrac{n\Theta}{2}\cos(n\theta_0) & \text{if } n\neq 0 \text{ and } l=1,\\ \dfrac{2R}{n}\sin\dfrac{n\Theta}{2}\sin(n\theta_0) & \text{if } n\neq 0 \text{ and } l=2,\\ R\Theta & \text{if } n=0 \text{ and } l=1,\\ 0 & \text{if } n=0 \text{ and } l=2.\end{cases} \quad (20)$$

$$C_z = 2\cos\frac{m\pi z_0}{L_z}\cos\frac{m\pi Z}{2L_z}, \quad (21)$$

$$C_\theta = 2\cos(n\theta_0)\cos(2n\Theta). \quad (22)$$

Since the natural modes of the cylindrical muffler have no preferred orientation in the θ direction, the parameter l is introduced and the natural modes can be rewritten as [3]

$$p_{mn1} = \cos(n\theta)\cos(m\pi z/L_z), \quad (23)$$

$$p_{mn2} = \sin(n\theta)\cos(m\pi z/L_z), \quad (24)$$

and ω_{mn} is defined as

$$\omega_{mn} = c\sqrt{(n/R)^2+(m\pi/L_z)^2}, \quad (25)$$

where ξ_{mn} is the damping ratio of the (m, n) mode, R is the cylinder radius, L_z is the cylinder height, and N_{mnl} is given by

$$N_{mn1} = \begin{cases} RL_z\pi/2, & \text{if } m,n=1,2,\ldots,\\ RL_z\pi, & \text{if } m=0,\\ & \quad n=1,2,\ldots, \text{or circulation}\\ 2RL_z\pi, & \text{if } m=n=0.\end{cases} \quad (26)$$

$$N_{mn2} = \begin{cases} RL_z\pi/2, & \text{if } m,n=1,2,\ldots,\\ RL_z\pi, & \text{if } m=0,\\ & \quad n=1,2,\ldots, \text{or circulation}\\ 0, & \text{if } m=n=0.\end{cases} \quad (27)$$

Because the pressure distributions of the responses are non-uniform in the line source model [7], the pressure response function can be redefined as

$$f_{ii} = \frac{\oint f(\theta,z,\omega)\,\mathrm{d}l/4Z}{Q}. \quad (28)$$

By obtaining the pressure response functions f_{11}, f_{21}, f_{22}, and f_{12} from equation (27), the four-pole parameters and the TL of two-dimensional cylindrical thin mufflers with different inlet/outlet locations can be calculated using equation (2).

3. Estimation of TL by three-dimensional FEM

To verify the TL results for thin curved mufflers obtained by the square line source model, a numerical model based on acoustic FEM was developed. The FEM software COMSOL [11] was used to calculate the TL of a two-dimensional thin cylindrical muffler (depicted in Figure 4). A complete three-dimensional model of this muffler element was created with dimensions $L_z = 0.185$ m, $R = 0.07$ m, $h = 0.01$ m, and $Z = 0.01$ m. The inlet and outlet pipes are located at $(z, \theta) = (0.0925, 0)$ and $(0.0925, \pi)$, respectively. The refrigerant used in [2] is applied here; it has $\rho = 6.04\,\mathrm{kg/m^3}$ and $c = 162.9$ m/s. The TLs are calculated using the particle velocities and the inlet and outlet pressures obtained by FEM [10]. TL is given by

$$\mathrm{TL} = 20\log_{10}\left|\frac{p_{\mathrm{in}}+\rho c u_{\mathrm{in}}}{2\rho c u_{\mathrm{out}}}\right|, \quad (29)$$

where p_{in} is the average pressure amplitude at the inlet, and u_{in} and u_{out} are the average particle velocity amplitudes at the inlet and outlet, respectively. A mesh of 9714 tetrahedral elements is used to calculate the TL of the cylindrical thin muffler. This corresponds to about 18 node points per wavelength of the sound at the highest frequency used in this study (2000 Hz). We confirmed that further refinement of the mesh does not give any noticeable difference in the TL results. The radiation plane wave boundary condition [12] is applied to the inlet pipe to prevent pressure reflection. The impedance on the boundary at the outlet is specified to simulate approximately the anechoic termination condition. To make the pressure uniform across the section and to minimize the error involved in the averaging process, two very short pipes were added to the inlet/outlet ports of the cylindrical thin muffler's FEM model, as suggested by Zhou and Kim [7]. The TL calculation is conducted from 10 to 2000 Hz with an interval of 10 Hz. To compare the computational efficiencies, the TLs for this case were also calculated by the modal expansion method with a square line source model. Computations by both methods were performed on the same PC (CPU: Intel® Core™2 2.33 GHz, RAM: 3.5 GB). The CPU time for the modal expansion method using the square line source model for 200 frequency steps was 62 s, whereas the FEM model had a computational time of 297 s (from which

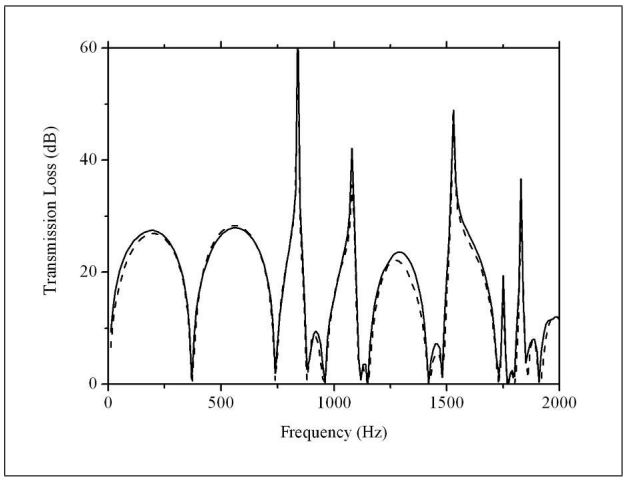


Figure 5. Sound transmission loss of a cylindrical thin muffler with dimensions $L_z = 0.185$ m, $R = 0.07$ m, $h = 0.01$ m, and $Z = 0.01$ m. Inlet is at $(z_1, \theta_1) = (0.0925, 0)$ and outlet is at $(z_2, \theta_2) = (0.0925, \pi)$. —— modal series expansion method using square line source model, - - - - - finite element method.

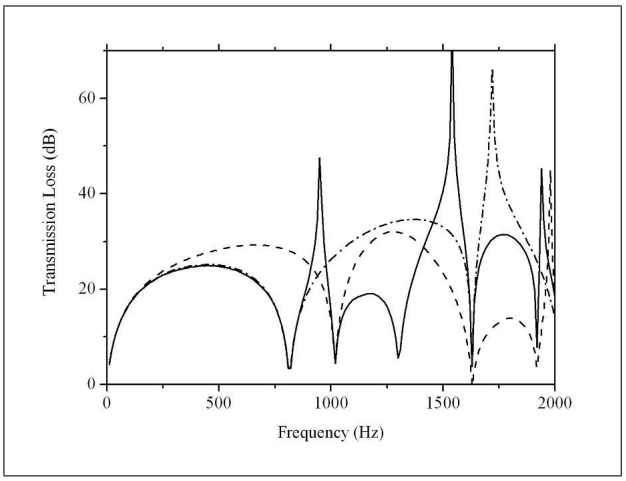


Figure 6. Sound transmission loss of a thin rectangular muffler with dimensions $L_x = 0.1$ m and $L_y = 0.08$ m. —— inlet is at $(x_1, y_1) = (0.02, 0.03)$ and outlet at $(x_2, y_2) = (0.08, 0.07)$. - - - - - inlet is at $(x_1, y_1) = (0.05, 0.03)$ and outlet at $(x_2, y_2) = (0.05, 0.07)$. – · – · – inlet is at $(x_1, y_1) = (0.02, 0.04)$ and outlet at $(x_2, y_2) = (0.08, 0.04)$.

the pre- and post-processing times have been excluded). Thus, the modal expansion method with the square line source model has a higher computational efficiency than the FEM.

Figure 5 shows that the TLs of a cylindrical thin muffler calculated by the square line source model agree well with those obtained by the FEM model, especially at low frequencies. The effects of higher-order modes could also be identified using the proposed model. Small deviations are found from the TL results at frequencies above 1000 Hz because the analytical model of the modal expansion method for the cylindrical muffler is two-dimensional in the present study, whereas that of the FEM method is inherently three-dimensional. Another reason for such deviations could be truncation errors, as the modal expansion method adopts a finite number of modes in simulating the contributions of an infinite number of modes.

4. TLs of two-dimensional mufflers by the square line model

4.1. Rectangular thin mufflers with various inlet/outlet locations

The pressure response functions of thin rectangular mufflers can be obtained using equations (7) and (13). The TL can be calculated using equation (2). A muffler with dimensions $L_x = 0.1$ m, $L_y = 0.08$ m, and $h = 0.01$ m was tested in this study. Figure 6 illustrates the effects of varying the inlet/outlet locations on the TLs of thin rectangular mufflers. Three pairs of inlet/outlet locations are considered in this section: $(x_1, y_1)/(x_2, y_2) = (0.02, 0.03)/(0.08, 0.07)$, $(0.05, 0.03)/(0.05, 0.07)$, and $(0.02, 0.04)/(0.08, 0.04)$. The results reveal that the inlet/outlet locations have a noticeable influence on the TLs of mufflers. Higher TLs at lower frequencies in the range 174 Hz to 910 Hz can be obtained when the inlet/outlet ports are located on the

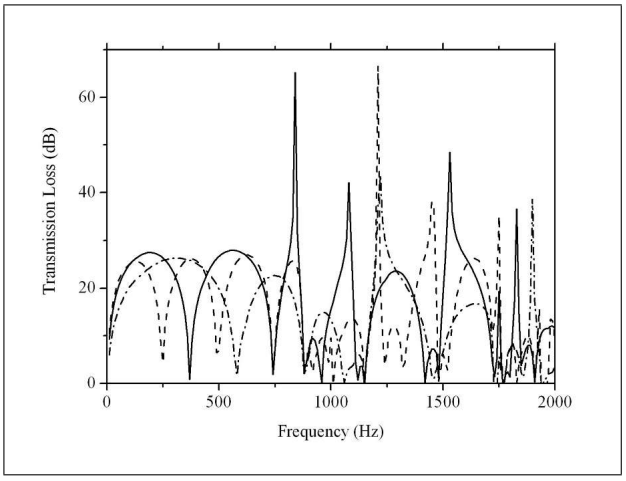


Figure 7. Sound transmission loss of cylindrical thin mufflers with various radii. Inlet is at $(z_1, \theta_1) = (0.0925, 0)$ and outlet is at $(z_2, \theta_2) = (0.0925, \pi)$; $L_z = 0.185$ m. —— $R = 0.07$ m. - - - - - $R = 0.105$ m. – · – · – $R = 0.045$ m.

nodal lines of the mode: (1, 0) and (0, 1) at $x = 0.05$ m and $y = 0.04$ m, respectively. Thus, the TL can be increased if the inlet and/or outlet are located on the nodal lines of the muffler, as the resonances of such modes cannot be simulated.

4.2. Cylindrical thin muffler element with various radii

TL analysis of two-dimensional cylindrical thin mufflers with different dimensions was performed. To analyse the effects of different radii, L_z was kept constant. The TL results are shown in Figure 7. The first dips (at eigenfrequencies of 576 Hz for $R = 0.045$ m, 470 Hz for $R = 0.07$ m, and 247 Hz for $R = 0.105$ m on the TL curves) all correspond to the resonances of mode (1, 0), which shift

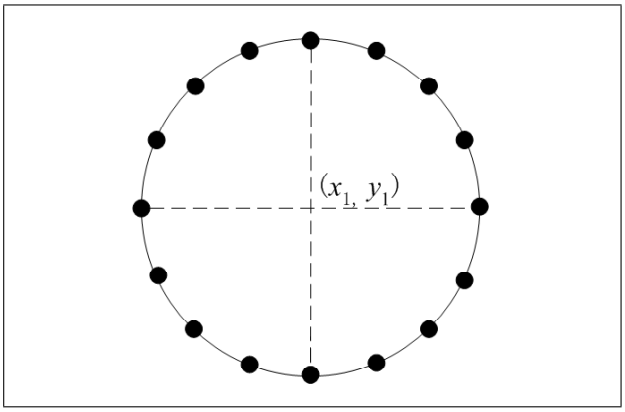


Figure 8. Schematic representation of the circular line source model.

to lower frequencies when the radius is increased. However, dips corresponding to the longitudinal modes (0, 2) and (0, 4) along the z-axis (at eigenfrequencies of 895 Hz and 1790 Hz, respectively) are unchanged when the radius is increased. The longitudinal modes (0, 1) and (0, 3) at 447 Hz and 1342 Hz, respectively, are suppressed in this case due to the inlet/outlet ports being located on the corresponding nodal lines in the vertical direction.

5. Discussion

5.1. Comparison of square and circular line models

All the above TL calculations are based on the square line source model, which was constructed based on the concept of the circular line source model proposed in [9]. Figure 8 shows a schematic representation of the circular line model. The pressure responses determined by the circular line source model are more accurate than those determined by the square line source model due to the pressure distribution being more uniform in the former model. This finding can be explained by the perfect symmetry of the circular line model [9].

To investigate the performance of the two line models, the TLs of a two-dimensional rectangular muffler are calculated using both models on the same PC. Figure 9 compares the TL results obtained using the circular and square line source models. There are no obvious differences in the TL results obtained by these two line models over the frequency range used in the calculation. Although the square line model is not perfectly symmetric, it has almost the same accuracy as the circular line model. As for the computational time, the CPU time for the circular line source model was 432 s, whereas that for the square line source model was 65 s for the same number of modes. The difference in the computational times of the two models can be explained by the different mathematical models they employ. The circular line source model requires more computational time because the numerical integration in equation (17) in [9] must be implemented for each mode when calculating the pressure response functions. Therefore, the square line model is more computationally efficient than

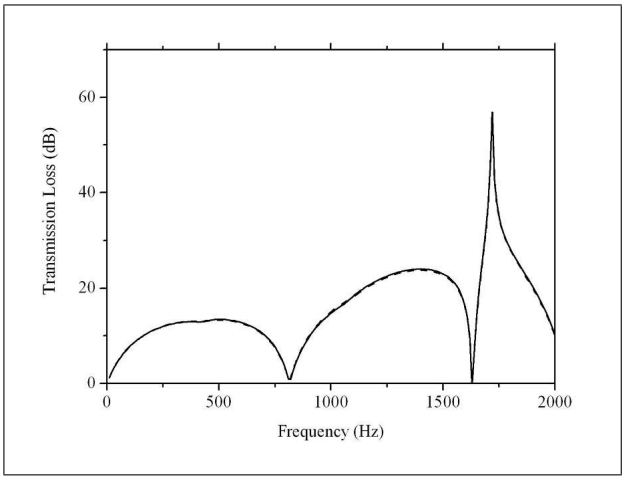


Figure 9. Sound transmission loss of a thin rectangular muffler calculated using the circular and square line models. $L_x = 0.1$ m, $L_y = 0.08$ m, $h = 0.01$ m, and $L = 0.01$ m. Inlet is at $(x_1, y_1) = (0.02, 0.04)$ and outlet is at $(x_2, y_2) = (0.08, 0.04)$. —— square line model, - - - - - circular line model.

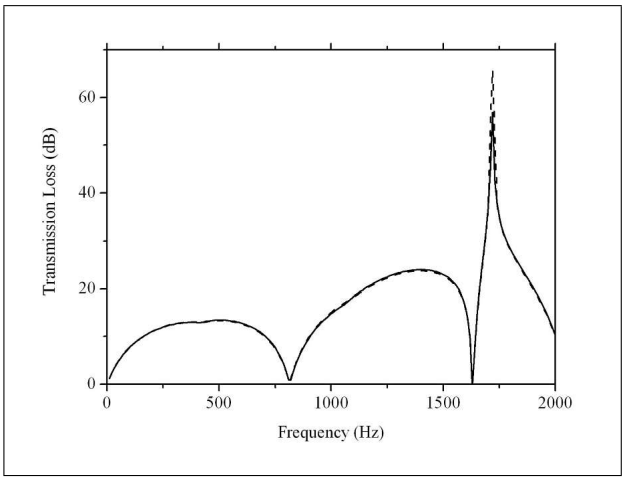


Figure 10. Sound transmission loss of a thin rectangular muffler calculated using the square surface and line models. $L_x = 0.1$ m, $L_y = 0.08$ m, $h = 0.01$ m, and $L = 0.01$ m. Inlet is at $(x_1, y_1) = (0.02, 0.04)$ and outlet is at $(x_2, y_2) = (0.08, 0.04)$. —— square line model, - - - - - circular line model.

the circular line model and it is sufficiently accurate for calculating TLs.

5.2. Comparison of square line model and square surface model

To prevent convergence problems, the dimension of the source geometry should be equal to or one less than that of the muffler element [7]. Therefore, the square surface source model can be used to calculate TLs of two-dimensional mufflers. Since the surface model might produce a highly non-uniform pressure distribution [7], the averaging process may give dubious results. To compare the performance of the proposed model with that of the surface model, the TLs of a two-dimensional rectangular muffler are calculated using both models. Figure 10

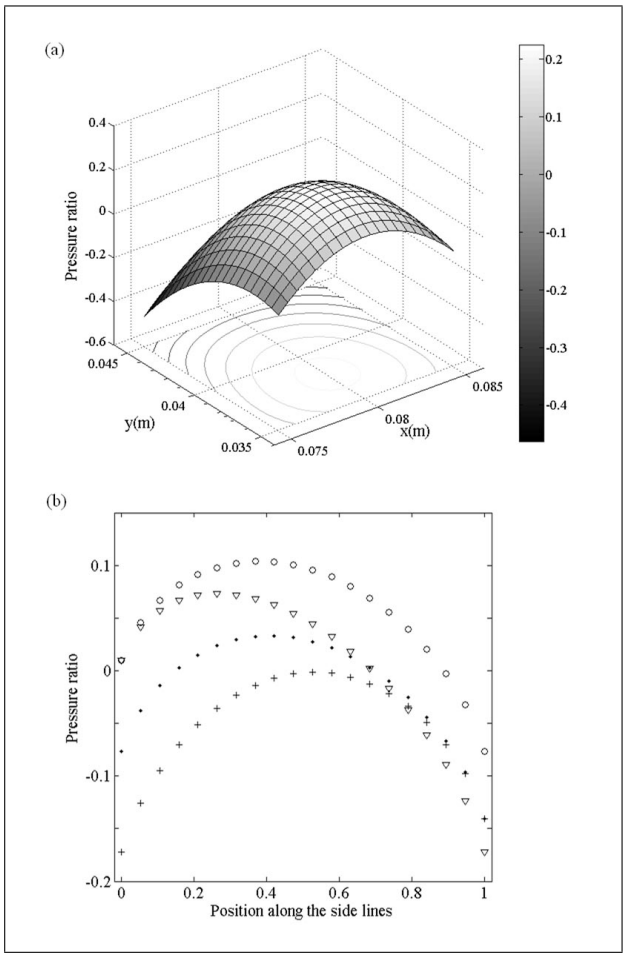


Figure 11. Pressure difference ratios on the inlet port at 1720 Hz obtained using: (a) square surface model, (b) square line model $\triangledown$ l_1; $\bullet$ l_2; $\circ$ l_3; $+$ l_4.

demonstrates that the TL results obtained by the two models have almost the same accuracies at most frequencies. However, there is a noticeable difference in the TL results in Figure 10 at a frequency of 1720 Hz. The following discussion demonstrates that this difference is due to the non-uniform pressure distribution in the surface source model. The pressure ratio λ (defined by equation (19) in [9]) is applied here. The pressure ratios at 1720 Hz for the two models were calculated; the pressure ratio of the surface model varied in the range -0.45 to 0.22, whereas that of the line model varies from -0.18 to 0.11 (Figure 11). Thus, a more uniform pressure distribution is obtained using the square line model. Since both line and surface sources can be regarded as consisting of an infinite number of point sources, the pressure response on the source can be regarded as the contribution of these point sources. If all these point sources have a symmetrical arrangement, the pressure response distribution will be more uniform, as discussed in [9]. Compared with the surface model, the distribution of point sources in the square line model is more symmetrical, which indicates that this new line model can result in improved calculation precision.

As for computation time, the CPU time was 83 s for the surface model and 65 s for the line model. Both models used the same number of modes (40,000). The difference in their computational times can be explained by the different mathematical models they employ. The surface model has a longer computational time because the pressure over the whole square surface needs to be calculated to obtain the average value, whereas the line model only needs to calculate the pressure on the sides of a square. Therefore, the new line model is more convenient than the surface model.

6. Conclusions

The modal expansion method with a new square line source model has been applied in the present study to calculate the TLs of two-dimensional thin mufflers. This new model does not suffer from the convergence problems of the point source model for points close to the source. The pressure response functions of two thin mufflers (rectangular and cylindrical mufflers) were obtained using the square line source model. The four-pole parameters were obtained and the TLs of mufflers with different inlet/outlet locations and of mufflers with different dimensions were calculated. The effects of higher-order modes in these two-dimensional mufflers could be analysed using the proposed model. The results obtained for a two-dimensional thin cylindrical muffler using the proposed model were validated by comparison with the results obtained using FEM. The square line model was more computationally efficient than FEM. The square line model is sufficiently accurate in most cases and has a higher computational efficiency than the circular line model because of its simpler mathematical formula. The proposed square line model has a higher computational efficiency and accuracy than the conventional square surface model.

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