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Applying the Ambisonics Approach to Planar and Linear Distributions of Secondary Sources and Combinations Thereof

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Summary
Near-field Compensated Higher Order Ambisonics (NFC-HOA) is an approach to the physical synthesis of sound fields. The typical interpretation of the modern NFC-HOA approach is that the sound field to be synthesized and the spatial transfer function of the employed loudspeakers are expanded into series of spherical harmonics in order to determine the loudspeaker driving signals for spherical arrays. Going one level higher in abstraction, NFC-HOA can be interpreted as the single-layer potential solution to the problem, which is solved via a formulation of the synthesis equation in a given spatial frequency domain. The specific spatial frequency domain is selected according to the geometry of the loudspeaker array. For spherical arrays, this spatial frequency domain is the spherical harmonics domain. The presented paper provides an overview over a recent extension of this approach to the employment of planar and linear loudspeaker arrays. In this latter situation, the solution is obtained via a formulation of the synthesis equation in wavenumber domain. The solution is then extended to the employment of combinations of planar or linear arrays of loudspeakers via the Kirchhoff approximation. Along with the solution, a discussion and interpretation of the general procedure and cross-references to other approaches are outlined.

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1. Introduction

The problem of sound field synthesis may be described in words as follows:

> A given ensemble of elementary sound sources shall be driven such that a sound field with given desired physical properties evolves over an extended region.

The employed elementary sound sources will be termed *secondary sources* in the remainder of this paper. In practical implementations, arrays of loudspeakers are used.

In order to facilitate the mathematical treatment and in order to facilitate the exploitation of results that have been achieved in closely related problems such as acoustical scattering [1], the ensemble of secondary sources under consideration will be assumed to be continuous and will therefore be referred to as a *distribution of secondary sources*. If we also assume that the distribution of secondary sources under consideration encloses the target volume where the desired sound field is intended to be synthesized, then the problem of sound field synthesis may be mathematically formulated as

$$S(\mathbf{x}, \omega) = \oint_{\partial\Omega} D(\mathbf{x}_0, \omega) G(\mathbf{x} - \mathbf{x}_0, \omega) \, \mathrm{d}A(\mathbf{x}_0). \qquad (1)$$

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$\partial\Omega$ describes the geometry of the secondary source distribution, $S(\mathbf{x}, \omega)$ denotes the desired sound field, $D(\mathbf{x}_0, \omega)$ the driving function of the secondary source located at $\mathbf{x}_0$, $G(\mathbf{x} - \mathbf{x}_0, \omega)$ the spatio-temporal transfer function of that secondary source, and $dA(\mathbf{x}_0)$ an infinitesimal surface element. Equation (1) will be referred to as *synthesis equation* in the remainder. The objective of sound field synthesis is finding the appropriate secondary source driving signal $D(\mathbf{x}_0, \omega)$ that evokes a given desired sound field $S(\mathbf{x}, \omega)$.

Near-field compensated higher order Ambisonics (NFC-HOA) and Wave Field Synthesis (WFS) constitute the two best known representatives of analytic methods for sound field synthesis. WFS directly implements fundamental physical principles like the Rayleigh integrals or the Kirchhoff-Helmholtz integral [2, 3] and provides an *implicit solution* to (1). The possibilities and limitations of WFS can be deduced in an abstract manner from the physical fundamentals.

NFC-HOA on the other hand was developed from rather intuitive yet physical considerations [4]. The theory was later extended [5] and recently a solid physical interpretation in terms of the *single-layer potential* solution was found [6], which retroactively justifies the approach. The synthesis equation (1) is solved *explicitly* in a transformed domain. For convenience, we use the term *Ambisonics* in the remainder in order to refer to NFC-HOA.

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As pointed out in [7], the Ambisonics solution to the problem of sound field synthesis exhibits a strong relationship to the concept of *Nearfield Acoustical Holography* (NAH) [8]. In the latter approach, the velocity or the sound pressure on the surface of a sound source are determined from the sound pressure on a specific contour. As in Ambisonics, the underlying integral equation is solved in a transformed domain.

Ambisonics formulations such as [5, 6, 9, 10] provide solutions only for spherical and circular distributions $\partial\Omega$ of secondary sources. The Fredholm operator theory, which is applied in the single-layer potential solution, theoretically enables the employment of arbitrarily shaped, simply enclosing distributions $\partial\Omega$ [11, 12]. Although the solutions to such more complicated contours are mathematically well understood, the required basis functions are only available for simple geometries like prolate spheroids and similar.

The present paper starts with a review of a state-of-the-art formulation of Ambisonics employing spherical secondary source distributions. Then, the special cases of planar and linear secondary source distributions $\partial\Omega$ of infinite extent based on an adaptation of the Ambisonics approach is treated. The obtained solution is extended to the employment of combinations of planar or linear secondary source distributions via application of the Kirchhoff approximation.

The presented approach has been initially proposed in [13]. Here, the work from [13, 14, 15, 16] is reviewed from a conceptual point of view.

2. The Ambisonics solution for spherical secondary source distributions

For illustration purposes, this section reviews state-of-the-art formulations of Ambisonics such as [9, 6, 10], which constitute the single-layer potential solution. A comparable solution based on the underlying inverse problem can be found in [6, 17].

At first stage, these approaches assume a continuous spherical distribution of secondary sources. This continuous distribution is then discretized in order to find the loudspeaker driving signals. The discretization operation is not treated in this paper.

The synthesis equation (1) reformulated for a spherical secondary source contour of radius R centered around the coordinate origin is given by [9]

$$S(\mathbf{x},\omega) = \int_0^{2\pi}\int_0^{\pi} D\left(\mathbf{x}_0,\omega\right) G\left(\mathbf{x}-\mathbf{x}_0,\omega\right) \sin\beta_0 \,\mathrm{d}\beta_0 \,\mathrm{d}\alpha_0, \quad (2)$$

with $\mathbf{x}_0 = R\left[\cos\alpha_0 \sin\beta_0 \sin\alpha_0 \sin\beta_0 \cos\beta_0\right]^T$. α_0 and β_0 denote the azimuth and colatitude of the position of the secondary source under consideration. In order that (2) is valid $G\left(\mathbf{x}-\mathbf{x}_0,\omega\right)$ has to be invariant with respect to rotation around the coordinate origin [9].

Equation (2) can be interpreted as a convolution along the surface of a sphere so that the convolution theorem

$$\mathring{S}_n^m(r,\omega) = 2\pi R^2 \sqrt{\frac{4\pi}{2n+1}}\, \mathring{D}_n^m(\omega)\cdot \mathring{G}_n^0(r,\omega), \quad (3)$$

employing the spherical harmonics expansion coefficients of $S(\mathbf{x},\omega)$, $D\left(\mathbf{x},\omega\right)$, and $G\left(\mathbf{x}-\mathbf{x}_0|_{r=R},\omega\right)$ respectively applies [18]. Note that (3) corresponds to the *mode-matching* applied in the traditional Ambisonics formulation, e.g. [19].

In the interior domain, a sound field $S(\mathbf{x},\omega)$ can be represented by the spherical harmonics coefficients $\mathring{S}_n^m(r,\omega)$ or coefficients $\breve{S}_n^m(\omega)$ respectively as

$$S(\mathbf{x},\omega) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \underbrace{\breve{S}_n^m(\omega)\, j_n\left(\frac{\omega}{c}r\right)}_{=\mathring{S}_n^m(r,\omega)} Y_n^m(\beta_0,\alpha_0), \quad (4)$$

with $Y_n^m(\beta_0,\alpha_0)$ denoting the n-th order m-th degree spherical harmonic [8]. Eq. (4) also holds for the spherical harmonics expansion coefficients $\mathring{G}_n^m$ of the spatio-temporal transfer function $G(\mathbf{x},\omega)$. More precisely, as a direct consequence of the definition of the integration in (2), $G(\mathbf{x},\omega)$ represents the spatio-temporal of the secondary source positioned at $(\alpha_0 = 0, \beta_0 = 0)$, i.e. at the north pole of the sphere.

After reordering (3), it can be shown that [9]

$$\mathring{D}_n^m(\omega) = \frac{1}{2\pi R^2}\sqrt{\frac{2n+1}{4\pi}}\,\frac{\breve{S}_n^m(\omega)}{\breve{G}_n^0(\omega)}. \quad (5)$$

The secondary source driving function $D(\alpha_0,\beta_0,\omega)$ for three-dimensional synthesis of a desired sound field with expansion coefficients $\breve{S}_n^m(\omega)$ is then

$$D(\alpha_0,\beta_0,\omega) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \underbrace{\frac{1}{2\pi R^2}\sqrt{\frac{2n+1}{4\pi}}\,\frac{\breve{S}_n^m(\omega)}{\breve{G}_n^0(\omega)}}_{=\mathring{D}_n^m(\omega)} \cdot Y_n^m(\beta_0,\alpha_0). \quad (6)$$

Typically, the secondary sources are assumed to be monopole sources, e.g. [20, 21]. In principle, any secondary source transfer function that does not exhibit zeros in the spherical harmonics domain can be handled in the presented approach [22]. In practical applications the summation in (6) is not performed over an infinite number of addends but is truncated at a specific value $N-1$ depending on the desired properties of the synthesized sound field [16]. One speaks then of $(N-1)$-th order synthesis.

A formulation of the Ambisonics approach employing circular distributions of secondary sources with a three-dimensional spatial transfer function - so-called *2.5-dimensional synthesis* - can be found in [9].

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3. The Ambisonics-like solution for planar secondary source distributions

As mentioned in section 1, it has been shown that the Ambisonics approach as reviewed in section 2 constitutes the *single-layer potential* solution to sound field synthesis for the special case of spherical distributions of secondary sources [6, 16]. Equation (2) can be understood as a compact Fredholm operator of zero index [11, 23, 21]. The general solution is obtained by expanding the operator and the desired sound field into a series of orthogonal basis functions and performing a comparison of coefficients (i.e. a *mode-matching*). For the spherical secondary source distribution treated in section 2, these orthogonal basis functions are given by the spherical harmonics $Y_n^m(\beta_0, \alpha_0)$ and the convolution theorem (3) constitutes a mode-matching operation.

In order to find the explicit solution to the synthesis equation for planar contours (i.e. in order to find the Ambisonics solution), we assume a boundary that consists of a disc Ω_0 and a hemisphere Ω_{hemi} of radius r_{hemi} as depicted in Figure 1 [8]. As we let $r_{\text{hemi}} \to \infty$, the disc Ω_0 becomes an infinite plane and the volume under consideration becomes a half-space. We term the latter *target half-space*. We additionally invoke the Sommerfeld radiation condition [8] in order to assure that no contributions to the desired sound field originate from infinity (where the hemispherical boundary is).

For convenience, we assume the boundary of our target half-space (i.e. the secondary source distribution) to be located in the x-z-plane, and we assume the target half-space to include the positive y-axis as illustrated in Figure 2. The sound field $S(\mathbf{x}, \omega)$ synthesized by the infinite uniform planar secondary source distribution is then given by [13, 15]

$$S(\mathbf{x}, \omega) = \iint_{-\infty}^{\infty} D(\mathbf{x}_0, \omega) \cdot G(\mathbf{x} - \mathbf{x}_0, \omega)\, \mathrm{d}x_0\, \mathrm{d}z_0, \quad (7)$$

where $\mathbf{x}_0 = [x_0\ 0\ z_0]^T$. In order that (7) holds, $G(\mathbf{x}-\mathbf{x}_0, \omega)$ has to be invariant with respect to translation along the planar secondary source contour. Integrals like (7) are termed *Fredholm integrals* [12]. Note the resemblance of (7) to the Rayleigh integrals [8].

As with spherical secondary source contours, (7) can interpreted as a convolution along the secondary source contour. In this case it is a two-dimensional convolution along the spatial dimensions x and z respectively. The according convolution theorem is given by

$$\tilde{S}(k_x, y, k_z, \omega) = \tilde{D}(k_x, k_z, \omega) \cdot \tilde{G}(k_x, y, k_z, \omega), \quad (8)$$

which relates the involved quantities in wavenumber domain [24]. Comparing (8) with [8, Eq. (3.2), p. 90] emphasizes the strong relation of the present formulation of the problem of sound field synthesis to NAH, which was mentioned in section 1 for the Ambisonics formulation.

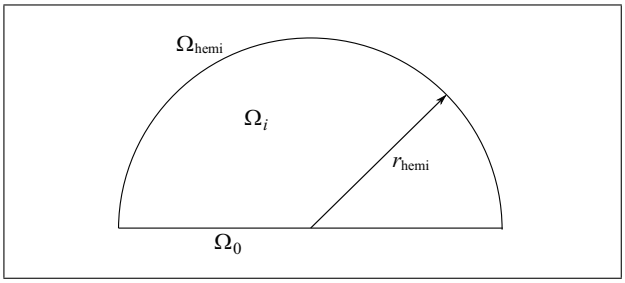


Figure 1. Illustration of a boundary consisting of a disc Ω_0 and a hemisphere Ω_{hemi}.

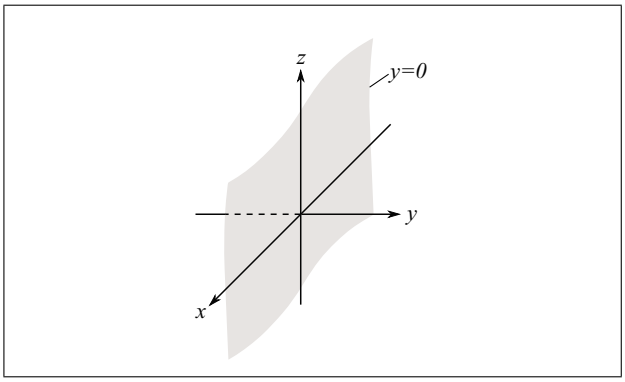


Figure 2. Illustration of the setup of a planar secondary source distribution located along the x-z-plane. The secondary source distribution is indicated by the gray shading and has infinite extent. The target half-space is the half-space bounded by the secondary source distribution and containing the positive y-axis.

Though, the wavefield propagator is applied in opposite direction in the present case.

The secondary source driving function is given in wavenumber domain by

$$\tilde{D}(k_x, k_z, \omega) = \frac{\tilde{S}(k_x, y, k_z, \omega)}{\tilde{G}(k_x, y, k_z, \omega)}, \quad (9)$$

and in time-frequency domain by [13, 15]

$$D(x, z, \omega) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \frac{\tilde{S}(k_x, y, k_z, \omega)}{\tilde{G}(k_x, y, k_z, \omega)} \cdot \mathrm{e}^{-\mathrm{i}k_x x} \mathrm{e}^{-\mathrm{i}k_z z}\, \mathrm{d}k_x\, \mathrm{d}k_z. \quad (10)$$

Note that (9) represents a filtering of the desired sound field in wavenumber domain similarly to what is performed in NAH [8].

From (9) and (10) it is obvious that the driving signal is essentially yielded by a division in spatial frequency domain. We therefore refer to the presented approach as *Spectral Division Method* (SDM).

In order that $\tilde{D}(k_x, k_z, \omega)$ and $D(x, z, \omega)$ are defined, $\tilde{G}(k_x, y, k_z, \omega)$ may not exhibit zeros. This is indeed the case for secondary monopoles. If this requirement is not fulfilled in practical situations, regularization can be applied in order to ensure a good behavior of the inverse of $\tilde{G}(\cdot)$. In the presented approach, the regularization can be

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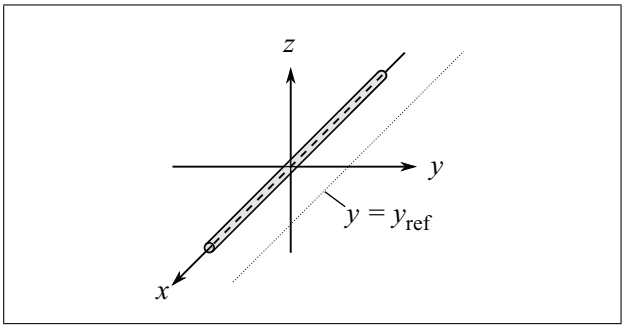


Figure 3. Illustration of the setup of a linear secondary source situated along the x-axis. The secondary source distribution is indicated by the grey shading and has infinite extent. The target half-plane is the half-plane bounded by the secondary source distribution and containing the positive y-axis. The thin dotted line indicates the reference line (see text).

applied to individual spatial frequencies k_x and k_z respectively, which allows for a physically more meaningful regularization than in spatial domain like e.g. in [25].

Equation (10) suggests that $D(x, z, \omega)$ is dependent on the distance y of the receiver to the secondary source distribution since y is apparent on the right hand side of (10). It can be shown that under certain circumstances, y does indeed cancel out making $D(x, z, \omega)$ independent from the location of the receiver. Refer to [15, 16] for a further treatment.

An alternative solution for planar distributions of secondary monopole sources is presented in [17].

4. The Ambisonics-like solution for linear secondary source distributions

Planar secondary source contours like the one treated in section 3 will be rarely implemented in practice due to the enormous amount of loudspeakers necessary. Typically, sound field synthesis systems employ linear arrays or a combination thereof and aim at synthesis in the horizontal plane. The application of SDM to linear secondary source distributions is presented in this section. For convenience, the secondary source distribution is assumed to be located along the x-axis, thus $\mathbf{x}_0 = [x_0\ 0\ 0]^T$. Refer to Figure 3.

For this setup, the synthesis equation is given by [13, 15]

$$S(\mathbf{x}, \omega) = \int_{-\infty}^{\infty} D(\mathbf{x}_0, \omega) \cdot G(\mathbf{x} - \mathbf{x}_0, \omega)\, \mathrm{d}x_0. \qquad (11)$$

Equation (11) can be interpreted as a convolution along the x-axis and the convolution theorem [24]

$$\tilde{S}(k_x, y, z, \omega) = \tilde{D}(k_x, \omega) \cdot \tilde{G}(k_x, y, z, \omega) \qquad (12)$$

holds. The secondary source driving function $\tilde{D}(k_x, \omega)$ in wavenumber domain is thus given by

$$\tilde{D}(k_x, \omega) = \frac{\tilde{S}(k_x, y, z, \omega)}{\tilde{G}(k_x, y, z, \omega)}. \qquad (13)$$

In the above derivation, we intentionally assumed $D(x, \omega)$ to be exclusively dependent on x because x is the only degree of freedom in the position of the secondary sources. However, generally $D(x, \omega)$ will be dependent on the position of the receiver. This is mathematically reflected by the fact that y and z do not cancel out in (13) [15, 16].

It is not surprising that we are not able to synthesize arbitrary sound fields over an extended area since we are dealing with a secondary source distribution that is neither infinite in two dimensions nor does it enclose the target volume [8].

In the present case, the secondary source setup will only be capable of creating wave fronts that propagate away from it. We will treat this circumstance in an intuitive way in the following. Refer to [15] for a rigorous derivation.

The propagation direction of the synthesized sound field can generally only be correct inside one half-plane bounded by the secondary source distribution. We term this half-plane *target half-plane*. The synthesized sound field anywhere else in space is a byproduct whose properties are determined by the secondary source driving function $D(x, \omega)$ and the radiation characteristics of the secondary sources in the respective direction. For convenience, we aim at synthesizing a given desired sound field inside that half of the horizontal plane that contains the positive y-axis. We therefore set $z = 0$.

However, above considerations do not affect the dependence of the driving function on y. In other words, even inside the target half-plane the synthesized sound field will generally only be correct on a line parallel to the x-axis at distance $y = y_{\mathrm{ref}}$ [15]. At locations off this reference line (and off the target half-plane), the synthesized sound field generally deviates from the desired sound field in terms of amplitude, propagation direction, and evanescent components [15].

The present situation, i.e. the employment of secondary sources with a three-dimensional spatio-temporal transfer function for two-dimensional synthesis and all resulting properties of the synthesized sound field are termed *2.5-dimensional synthesis*, e.g. [26]. This 2.5-dimensional synthesis exhibits special properties as discussed below. Interestingly, we are not aware of an according formulation of NAH for such a situation. Recall that there exists a strong relationship between NAH and three-dimensional sound field synthesis.

In order to simplify the mathematical treatment, we restrict the validity of equations (11)–(13) to our reference line in the target half-plane, i.e. $z = 0$ and $y = y_{\mathrm{ref}}$.

Equation (13) is then given by

$$\tilde{D}(k_x, \omega) = \frac{\tilde{S}(k_x, y_{\mathrm{ref}}, 0, \omega)}{\tilde{G}(k_x, y_{\mathrm{ref}}, 0, \omega)}. \qquad (14)$$

Performing an inverse Fourier transform with respect to k_x on (14) yields the driving function $D(x, \omega)$ in temporal spectrum domain as

$$D(x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\tilde{S}(k_x, y_{\mathrm{ref}}, 0, \omega)}{\tilde{G}(k_x, y_{\mathrm{ref}}, 0, \omega)} \mathrm{e}^{-\mathrm{i}k_x x}\, \mathrm{d}k_x. \qquad (15)$$

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In order for $D(x, \omega)$ to be defined, $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$ may not exhibit zeros. As with planar contours, regularization can be applied in practice in order to ensure a good behavior of the inverse of $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$. An example of SDM employing secondary sources with complex radiation properties can be found in [27].

Note that a solution comparable to [27] has been presented for WFS in [28]. Unlike the SDM solution, the latter constitutes an approximation even on the reference line where the SDM is exact. Alternatively, numerical solutions based on the theory of multiple-input-multiple-output (MIMO) systems have been proposed in order to compensate for loudspeaker radiation characteristics and influence of the listening room (loudspeaker radiation characteristics are part of the latter). Approaches suitable for linear loudspeaker arrays are e.g. [25, 29, 30, 31].

For purposes of illustration of the basic properties of the presented approach, we treat the scenario of a monochromatic virtual plane wave of frequency ω_{pw} that propagates along the x-y-plane in direction θ_{pw} synthesized by a continuous linear distribution of secondary monopoles. When secondary monopoles are assumed, the inverse of $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$ is well defined and no regularization is required.

Considering above described referencing, the secondary source driving function can be shown to be [15]

$$\tilde{D}_{\text{pw}}(k_x, \omega) = \frac{4\text{i} \cdot \text{e}^{-\text{i}k_{\text{pw},y} y_{\text{ref}}}}{H_0^{(2)}\left(k_{\text{pw},y} y_{\text{ref}}\right)} \cdot 2\pi\delta(k_x - k_{\text{pw},x})\, 2\pi\delta(\omega - \omega_{\text{pw}}), \tag{16}$$

and

$$D_{\text{pw}}(x, \omega) = \frac{4\text{i} \cdot \text{e}^{-\text{i}k_{\text{pw},y} y_{\text{ref}}}}{H_0^{(2)}\left(k_{\text{pw},y} y_{\text{ref}}\right)} \cdot \text{e}^{-\text{i}k_{\text{pw},x} x} 2\pi\delta(\omega - \omega_{\text{pw}}) \tag{17}$$

respectively, with

$k_{\text{pw},x} = k_{\text{pw}} \cos\theta_{\text{pw}}$ and $k_{\text{pw},y} = k_{\text{pw}} \sin\theta_{\text{pw}}$.

Transferred to the time domain and formulated for broadband signals, (17) reads

$$d_{\text{pw}}(x, t) = f(t) *_t \hat{s}\left(t - \frac{x}{c}\cos\theta_{\text{pw}} \sin\phi_{\text{pw}} - \frac{y_{\text{ref}}}{c}\sin\theta_{\text{pw}} \sin\phi_{\text{pw}}\right). \tag{18}$$

$f(t)$ denotes a filter with frequency response

$$F(\omega) = \frac{4i}{H_0^{(2)}\left(k_{\text{pw},y} y_{\text{ref}}\right)}, \tag{19}$$

the asterisk $*_t$ denotes convolution with respect to time, and $\hat{s}(t)$ the time domain signal that the plane wave carries. Thus, the time domain driving signal for a secondary source at a given location is yielded by applying a delay and a filter to the time domain input signal.

The transfer function $F(\omega)$ of the filter exhibits a slope of approximately 3 dB per octave and may be interpreted as a half-order highpass. $F(\omega)$ is exclusively dependent on the propagation direction of the desired plane wave and on the amplitude reference distance y_{ref}. It is therefore equal for all secondary sources and it is sufficient to perform the filtering only once on the input signal before distributing the signal to the secondary sources. The delay is dependent both on the propagation direction of the desired plane wave and on the position of the secondary source. It therefore has to be performed individually for each secondary source. Note the strong resemblance of this implementation scheme to the implementation of WFS [3, 15].

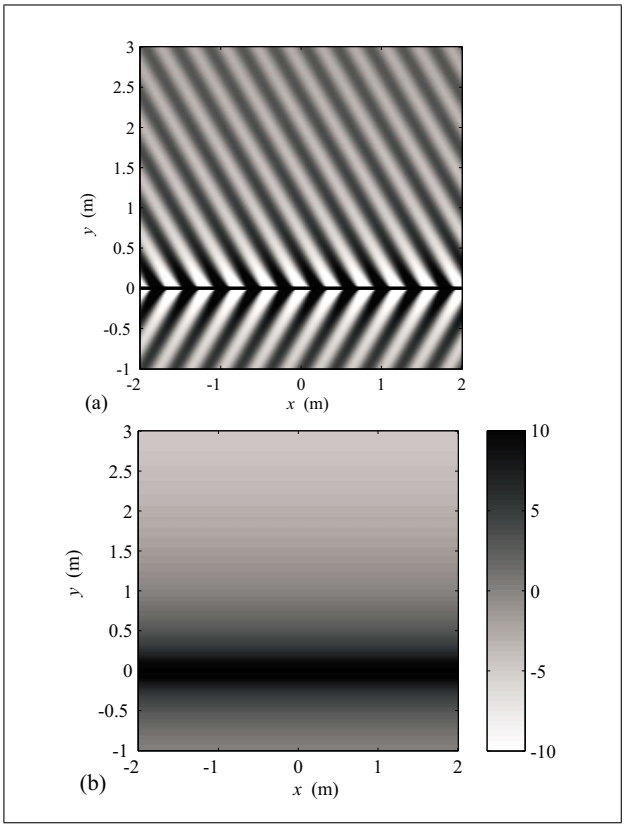


Figure 4. A cross-section through the horizontal plane of the sound pressure $S_{\text{pw}}(\mathbf{x}, \omega)$ of a continuous linear distribution of secondary point sources synthesizing a virtual plane wave of $f_{\text{pw}} = 1000\,\text{Hz}$ and unit amplitude with propagation direction $\theta_{\text{pw}} = \pi/4$ referenced to the distance $y_{\text{ref}} = 1.0$ m. The secondary source distribution is indicated by the grey line. (a) $\Re\{S_{\text{pw}}(\mathbf{x}, \omega)\}$, (b) $20 \cdot \log_{10}|S_{\text{pw}}(\mathbf{x}, \omega)|$; the values are clipped as indicated by the colorbar.

Figure 4 depicts simulations of above treated scenario of a virtual plane with propagation direction $\theta_{\text{pw}} = \frac{\pi}{4}$. It can be seen from Figure 4a that the wave fronts are indeed straight. As evident from Figure 4b, the synthesized sound field though exhibits an amplitude decay of approximately 3 dB for each doubling of the distance to the secondary source distribution. This circumstance is characteristic for 2.5-dimensional synthesis [9].

5. The Ambisonics-like solution for combinations of planar or linear secondary source distributions

In the present section, the results from section 3 and 4 are exploited in order to find an approximate solution for

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combinations of planar or linear secondary source distributions.

As pointed out in [6], it is helpful to interpret sound field synthesis by considering the equivalent problem of scattering of sound waves at a sound-soft object whose geometry is identical to that of the secondary source distribution. Sound-soft objects exhibit ideal pressure release boundaries, i.e. a homogeneous Dirichlet boundary condition is assumed.

When the wavelength λ of the wave field under consideration is much smaller than the dimensions of the scattering object and when the object is convex the so-called *Kirchhoff approximation* or *physical optics approximation* can be applied [1]. The surface of the scattering object is divided into a region that is *illuminated* by the incident wave, and a *shadowed* area. The problem under consideration is then reduced to far-field scattering off the illuminated region whereby the surface of the scattering object is assumed to be locally plane. The shadowed area has to be discarded in order to avoid an unwanted *secondary diffraction*, which constitutes a virtual reflection of the synthesized sound field from the secondary source contour [1]. The convexity is required in order to avoid scattering of the scattered sound field.

For such small wave lengths, any arbitrary simply connected convex enclosing secondary source distribution may also be assumed to be locally plane. Consequently, when the driving function (10) for planar secondary source distributions is applied in such a scenario, a high-frequency approximation of the driving function is obtained when only those secondary sources are driven that are located in that region that is illuminated by the virtual sound field. A similar argumentation may be deployed in conjunction with linear secondary source distributions.

The better the assumptions of the physical optics approximation are fulfilled, most notably the wave length under consideration being significantly smaller than the dimensions of the secondary source distribution, the smaller is the resulting inaccuracy. A further analysis of this inaccuracy may be found in [32, 16].

The illuminated area can be straightforwardly determined via geometrical considerations as indicated in Figure 5. For a virtual plane wave, the illuminated area is bounded by two lines parallel to the propagation vector $\mathbf{k}_{\mathrm{pw}}$ of the plane wave passing the secondary source distribution in a tangent-like manner.

If the proper tangent on the boundary of the illuminated area is not parallel to $\mathbf{k}_{\mathrm{pw}}$ or is not defined (like the boundary of a planar distribution of finite size), a degenerated problem is considered (case **A** in Figure 5). That means, the illuminated area is incomplete and artifacts have to be expected. The perceptual prominence of such spatial truncation artifacts can be reduced by the application of *tapering*, i.e. an attenuation of the secondary sources towards the edges of the illuminated area. Tapering is a well-established technique in the context of WFS [26].

It has been shown that the illuminated area does not need to be smooth. Corners are also possible with only little additional error introduced [33]. This is also evident from Figure 6a, which shows the synthesis of a virtual plane wave by a rectangular distribution of monopoles driven with SDM. A secondary source spacing of $\Delta x = 0.1$ m is assumed. The secondary source driving function was deduced from (17) via an appropriate translation and rotation of the coordinate system. Since this scenario constitutes 2.5-dimensional synthesis, the synthesized sound field does exhibit an undesired amplitude decay.

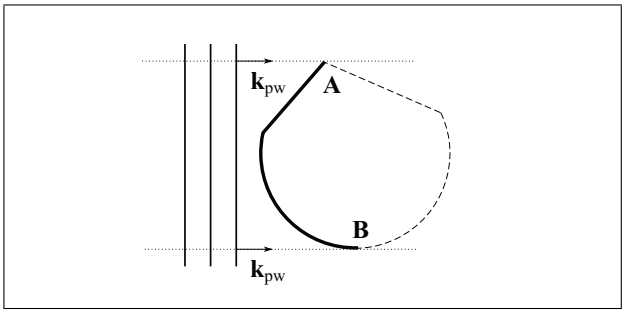


Figure 5. Secondary source selection for a virtual plane wave with propagation direction $\mathbf{k}_{\mathrm{pw}}$. Thick solid lines indicate the area that is illuminated by the virtual sound field. The illuminated area corresponds to the active secondary sources. The dashed line indicates the shadowed part of the secondary source distribution. The two dotted lines are parallel to $\mathbf{k}_{\mathrm{pw}}$ and pass the secondary source distribution in a tangent-like manner. In case **A** tapering may be applied, in case **B** not.

In the case of a piecewise linear secondary source distribution the synthesized sound field is referenced to contour that is also piecewise linear. However, as a consequence of the application of the Kirchhoff approximation, the synthesized sound field is then only an approximation of the desired sound field even on the reference contour.

Figure 6b depicts the synthesis of a virtual plane wave by a circular distribution of monopoles driven with SDM. Again, a secondary source spacing of $\Delta x = 0.1$ m is assumed. This circular distribution may be interpreted as a combination of linear sections of infinitesimal length. Due to the different geometry of the secondary source contour, the amplitude decay of the synthesized sound field is different to the one in Figure 6a. Note that the required numerical accuracy in an implementation of the presented solution is significantly lower than for the Ambisonics solution (6), which involves special functions like Legendre polynomials and sphercial Hankel functions, which are numerically critical at high orders n.

6. Comparison to Wave Field Synthesis

WFS [2] constitutes the best-known analytical sound field synthesis approach besides Ambisonics. WFS is based on Rayleigh's first integral formula, which enables the calculation of the loudspeaker driving signals from the directional gradient of the sound field to be synthesized. Unlike Ambisonics, WFS does thus not explicitly solve the synthesis equation but constitutes an implicit solution derived from specific physical relationships.

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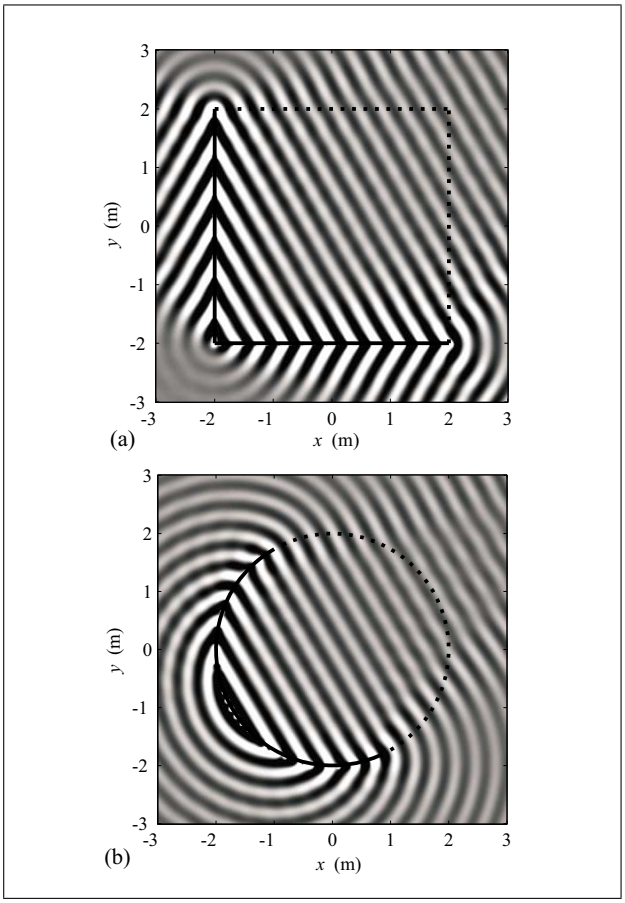


Figure 6. A cross-section through the horizontal plane of the sound pressure $S_{\mathrm{pw}}(\mathbf{x}, \omega)$ synthesized by different secondary monopole distributions synthesizing a virtual plane wave of $f_{\mathrm{pw}} = 1000\,\mathrm{Hz}$ and unit amplitude with propagation direction $\theta_{\mathrm{pw}} = \frac{\pi}{6}$ referenced to $y_{\mathrm{ref}} = 1.0$ m. Solid lines indicate the illuminated area; dotted lines indicate the shadowed area. (a) Rectangular secondary distribution, (b) Circular secondary source distribution.

In its initial formulation, WFS assumes a planar secondary source distribution in the three-dimensional case and a linear one in the 2.5-dimensional case. This circumstance allows a direct comparison of WFS to SDM presented in section 3 and 4. Due to the practical relevance, we restrict further considerations to the case of linear secondary source distributions.

A very convenient property of WFS is the fact that the driving signals can be comfortably calculated via the directional gradient of the virtual sound field, which is feasible even for complicated virtual sound fields like moving sources and alike [34] where the SDM and NFC-HOA solutions have not been found.

For simple virtual sound fields like plane and spherical waves, SDM and WFS are indeed very similar. The fundamental theoretical benefit of SDM is the fact that it provides an exact solution on the reference line. 2.5-dimensional WFS on the other hand, constitutes an approximation even on the reference line [35]. This exact property of SDM allows for specific applications and especially insights that are not possible using WFS:

- SDM employs a formulation of the synthesis equation in wavenumber domain, e.g. (12). This representation is very convenient since it allows for a straightforward discrimination of the involved propagating and evanescent sound fields. Such knowledge can be used to investigate the synthesis of focused virtual sound sources where evanescent components can be significant [36].
- The formulation of the desired sound field in wavenumber domain allows for the generation of quiet zones of various geometrical shapes [37], which can not be achieved using other representations.

Additionally, the exact property of SDM has been exploited in order to optimize the performance of WFS in the following ways:

- It has been shown that 2.5-dimensional WFS suffers from systematic amplitude errors additional to those amplitude errors due to the 2.5-dimensionality [15, 16]. So far, this error has only been investigated for the synthesis of plane waves were it depends on the propagation direction of the plane wave. For plane waves propagating perpendicular to the secondary source distribution no error arises. An attenuation of several dB arises for plane waves propagating almost parallel to the secondary source distribution.
- The derivation of the loudspeaker driving signals for 2.5-dimensional WFS involves two separate stationary phase approximations. As a consequence, the driving signals are only correct when both the virtual source under consideration as well as the receiver are sufficiently far away from the secondary source contour. When a virtual source is located in the vicinity of the secondary source contour, a systematic coloration is introduced [35]. In the worst case, a lowpass slope of 3 dB per octave is imposed on the magnitude spectrum of the input signal. Note that coloration also arises in the low-frequency range when linear secondary source distributions of finite length are employed [35]. None of the synthesis methods discussed in this article can directly compensate for this.

7. Conclusions

We have presented an overview over the problem of sound field synthesis and analytical explicit solutions thereto. The best-known representative of the latter is Near-field Compensated Higher Order Ambisonics (NFC-HOA). The Spectral Division Method (SDM) for sound field synthesis using planar or linear distributions of secondary sources was developed starting with a general formulation. SDM can be interpreted as an extension of the NFC-HOA approach, which is restricted to spherical and circular secondary source contours. While the Ambisonics solution is obtained in spherical harmonics domain, the SDM solution is obtained in wavenumber domain.

The Kirchhoff approximation was then applied in order obtain an approximate solution for combinations of planar and linear secondary source distributions based on the

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SDM solution. Curved secondary source distributions constitute a limiting case, which may be interpreted as a combination of planar or linear sections of infinitesimal length.

The operator describing sound field synthesis using planar and linear secondary source distributions is not compact. This fact leads to continuous spatial frequencies. Spherical contours as in the Ambisonics context employ a compact operator so that only discrete spatial frequencies (i.e. *spatial modes*) evolve [12].

Finally, a comparison between SDM and WFS is given, which discusses selected theoretical and practical properties. It has been shown that SDM provides drawbacks mainly with respect to complexity of the implementation and solutions have only been available for selected scenarios. Though, it has also been shown that SDM provides specific benefits the aesthetical potential of which are currently under investigation by the authors.

Whether or not SDM can provide benefits for listeners that are off the target plane in 2.5D synthesis can not be judged.

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