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A Bottom Reverberation Model Based on the Coupling Between Parabolic Equation and Scattering Strengths

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Summary
The reverberation is an important issue in underwater acoustics, in particular in shallow water where it has serious consequences on the detection and classification performance of sonar systems. The prediction of the reverberation level is then required and several models have already been developed. In this paper, a shallow-water reverberation model REVPA is presented. On one hand, in order to deal with the forward propagation, a wide-angle parabolic equation model (based on the Fourth order Padé approximation) was developed. On the other hand, interface reverberation (backscattering) phenomena can be well described by scattering strength functions. The REVPA model is based on the coupling between a propagation model and a scattering law by means of a plane-wave decomposition. Whereas any scattering laws and/or bathymetry can be included in the REVPA model, we concentrated, in this paper, on Lambert's law to predict bottom reverberation in the range-invariant case. The simulations results obtained with the REVPA model were compared to the results given by two different models based on the normal-mode theory: for tens of simulations, peak errors ranged from 0.1 to 1 dB for constant sound-speed profiles and from 1 to 2 dB for variable ones.

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1. Introduction

The reverberation is an important issue in underwater acoustics, in particular in shallow water where it has serious consequences on the detection and classification performance of sonar systems. As the "persistent" reverberation is due to both interfaces (air-sea, bottom), our effort will concentrate on this type of reverberation and ignore, at this stage, any volume reverberation. Furthermore, in order to be able to compare to existing models, we will concentrate, in this paper, on the bottom reverberation. Nevertheless, the model developed can be extended to both surface and volume reverberation (for both range-independent and range-dependent cases).

The modeling of the reverberation is faced to the following major challenges:

- the physical phenomena involved are occuring at two very different scales:
  - the scale of the propagation phenomena: hundreds to thousands of meters.
  - the scale of bottom scattering phenomena: tens to hundreds of centimeters (typically half a wavelength).
- the detailed bottom structure and, in particular sub-bottom structure (of high importance in low frequency), is most often unknown and roughly described either by macroscopic topography or qualitative information (sandy, muddy, rocky…) or measured scattering strengths

Various models already exist based on topography descriptions [1, 2] or on scattering strength predictions [3, 4, 5, 6, 7, 8]. The former ones are used mainly to deal with important changes in topography (i.e. sloping bottom), the latter ones have been derived to estimate the variations due to roughness, sediment type, etc. A lot of theoretical and experimental papers deal with detailed microscopic description of scattering by rough surfaces and ocean bottom.

In this paper, we will try to combine both approaches using a plane-wave decomposition of both the incident field and the bottom scattered one (associated with forward and backward scattering). The bottom is described by its bistatic scattering function (for simplicity, the 2D bistatic Lambert's law has been used in the simulations but any other law – either theoretical or experimental, if available – could be used). Both range-dependent or range-independent environments may be considered (as shown in Figure 1).

A parabolic propagation model has been implemented that has been used to model the propagation from the source to the vicinity of the bottom. Parabolic equations

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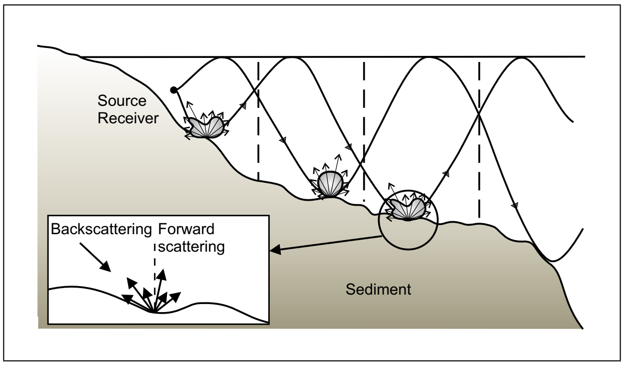

Figure 1. Geometry of range-dependent bottom reverberation.

were used in order to facilitate the future consideration of the range-dependent cases, and/or extend the approach to deep water cases. The incident field is decomposed into plane waves at the scattering interface. For each plane wave, the bottom bistatic scattering function is used for predicting the amplitude and the phase of both forward and backward scattered plane waves, and then the parabolic equation is used to back-propagate the bottom reflected plane waves to the receiver.

The reverberation model we have set up is the so-called REVPA model (REVerberation and Propagation in Acoustics). It has been designed for the simulation of time series. Thus, it is essential that it should not only provide incoherent prediction of reverberation level but has to be fully coherent from beginning to end.

As it can be readily extended to range-dependent cases, parabolic equation was preferred to coupled modes. Nevertheless, the REVPA model relies on a normal-mode WKB decomposition at intermediate computation steps.

In this paper, we will describe the propagation part of the REVPA model and compare it to existing codes and then extend it to the computation of reverberation level.

2. REVPA: propagation modeling

2.1. Theory

In order to deal with wide propagation angles, the REVPA model is based on the fourth order Padé approximation of the parabolic equation [9, 10, 11]. The complete derivation of parabolic equation being out of the scope of this paper (for more details, see [12]), only the main equations are given below. Considering a constant density, a range-independent and axially symmetric medium, and $exp(-i\omega t)$ time dependence (where ω is the angular frequency), the generalized parabolic equation, in cylindrical coordinates is given as

$$\frac{\partial\psi(r,z)}{\partial r} = \mathrm{i}k_0\left(\sqrt{Q}-1\right)\psi(r,z), \tag{1}$$

where

- r and z are respectively the propagation range and the depth,
- $\psi(r,z)$ is the envelope function defined by the relation $p(r,z) = \psi(r,z)H_0^{(1)}(r)$. $p(r,z)$ and $H_0^{(1)}(r)$ are respectively the acoustic field and the Hankel function of first kind and zero order,
- k_0 is a reference wavenumber, usually defined for a sound speed equal to 1500 m/s,
- $Q = 1+X = 1+(\boldsymbol{n}^2(z)-1)+\frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}$ ($\boldsymbol{n}$ is the refraction index, depth varying).

Using the fourth order Padé approximation ($m = 4$), the acoustic field is defined by

$$\frac{\partial\psi(r,z)}{\partial r} = \mathrm{i}k_0\left[\sum_{j=1}^{4}\frac{a_{j,4}X}{1+b_{j,4}X}\right]\psi(r,z). \tag{2}$$

The coefficients $a_{j,m}$ and $b_{j,m}$ are given by [11]

$$a_{j,m} = \frac{2}{2m+1}sin^2\left(\frac{j\pi}{2m+1}\right),$$
$$b_{j,m} = cos^2\left(\frac{j\pi}{2m+1}\right).$$

Equation (2) is associated with the following boundary and interface conditions:

- pressure release condition at the free surface,

$$p(r,0) = 0,$$

- continuity of pressure and normal velocity at the water/sediment interface,

$$p_w(r,z_i) = p_s(r,z_i),$$
$$\frac{1}{\rho_w}\frac{\partial p_w(r,z_i)}{\partial z} = \frac{1}{\rho_s}\frac{\partial p_s(r,z_i)}{\partial z},$$

  where p_w and p_s represent, respectively, the acoustic fields in the water and in the sediment, ρ_w is the density of the water, ρ_s is the density of the sediment and z_i is the depth of the water/sediment interface,
- Neumann condition on the lower interface of the sediment z_{md},

$$\frac{\partial p_s(r,z_{md})}{\partial z} = 0.$$

The different layers used to model the environment as well as the boundary conditions are shown on Figure 2.

To take the sediment attenuation into account, the sound speed in the sediment, c_s, that is included by means of the refraction index $\boldsymbol{n}$, is considered as a complex number c_s^* [2],

$$c_s^* = \frac{c_s}{1+\mathrm{i}\eta\beta}, \tag{3}$$

where c_s is the sound speed in the sediment without attenuation, $\eta = (40\pi\log_{10}(\mathrm{e}))^{-1}$ and β is the attenuation given in decibels per wavelength.

In the case of a semi-infinite bottom, an artificial layer is added above the lower interface (cf. Figure 2) in order to ensure that no energy is reflected from this boundary [12,

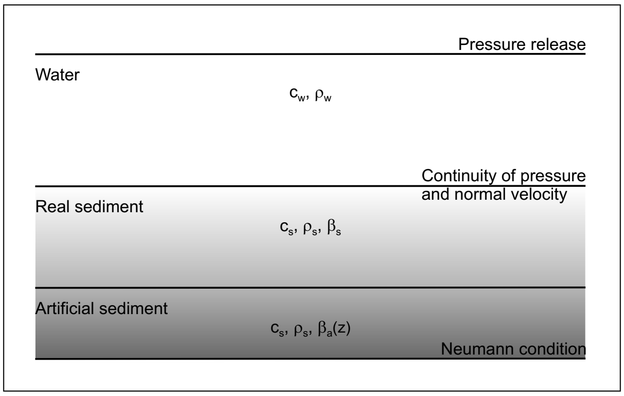

Figure 2. Modeling of the environment.

p. 362]. In this layer, the attenuation is assumed to increase linearly with depth from its sediment value to a maximum one specified by the user (a value of 10 dB/λ was chosen for all the simulations).

Equation (2) is solved using a finite-difference technique. With this technique, the envelope function $\psi(r, z)$ and its first and second derivatives can be approximated, respectively, by

$$\psi(r,z) \simeq \frac{\tilde{\psi}^n + \tilde{\psi}^{n-1}}{2},$$
$$\frac{\partial\psi(r,z)}{\partial r} \simeq \frac{\tilde{\psi}^n - \tilde{\psi}^{n-1}}{\Delta r},$$
$$\frac{\partial^2\psi(r,z)}{\partial z^2} \simeq \frac{\tilde{\psi}^n_{m-1} - 2\tilde{\psi}^n_m + \tilde{\psi}^n_{m+1}}{\Delta z^2},$$

where Δr and Δz correspond respectively to the horizontal and vertical steps and $\tilde{\psi}^n_m = \tilde{\psi}^j_m$ ($j = 1, 2, 3, 4$ for a fourth order Padé development) is the acoustic field at depth z_m and range r_n for $j = 4$ [13].

These approximations lead to the numerical scheme associated to equation (2),

$$\frac{c_1}{k_0^2\Delta z^2}\tilde{\psi}^n_{m-1} + \left(1 + c_1(\boldsymbol{n}^2-1) - 2\frac{c_1}{k_0^2\Delta z^2}\right)\tilde{\psi}^n_m + \frac{c_1}{k_0^2\Delta z^2}\tilde{\psi}^n_{m+1}, \tag{4}$$
$$= \frac{c_2}{k_0^2\Delta z^2}\tilde{\psi}^{n-1}_{m-1} + \left(1 + c_2(\boldsymbol{n}^2-1) - 2\frac{c_2}{k_0^2\Delta z^2}\right)\tilde{\psi}^{n-1}_m + \frac{c_2}{k_0^2\Delta z^2}\tilde{\psi}^{n-1}_{m+1},$$

where

$$c_1 = b_{j,4} - \frac{\mathrm{i}k_0\Delta r a_{j,4}}{2}, \quad c_2 = b_{j,4} + \frac{\mathrm{i}k_0\Delta r a_{j,4}}{2}.$$

The computation is initialized with one of the following initial sources: modal starter [12], Gaussian source [14] or Greene's source [15].

For the sake of simplicity, the model has been developed, at this stage, to deal only with range-independent environments. However, thanks to the parabolic equation theory, range-dependent environments (and deep water cases) could be considered in the future. In that case, the input parameters (properties of the environment) are updated for every change in the medium i.e. the environment is divided into range-independent cells characterized by their own range-independent sound-speed profile and bottom properties. Furthermore, in the case of a sloping bottom, the continuity conditions have to be modified [16].

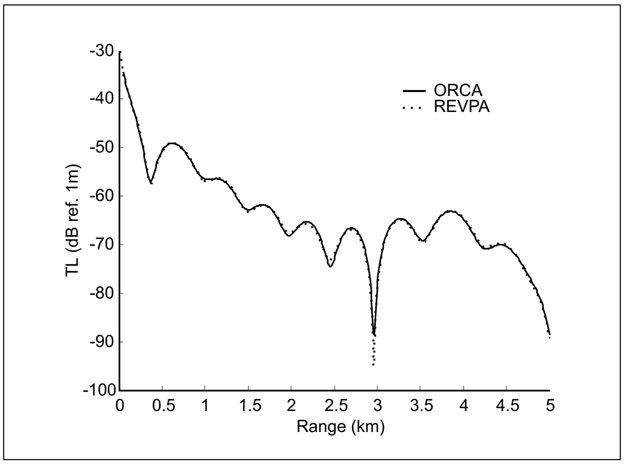

Figure 3. Case 1: Transmission losses predicted with the ORCA and the REVPA models.

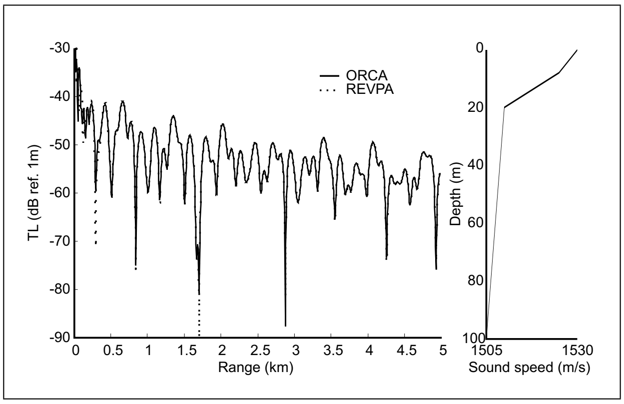

Figure 4. Case 2: Transmission losses predicted with the ORCA and the REVPA models.

2.2. Validation of the propagation code

The REVPA model has been benchmarked by comparison with the normal-mode model ORCA [17] for constant and depth-dependent sound-speed profiles. The results are presented in Figures 3 and 4, for either an homogeneous environment (referred as case 1) or an environment characterized by a downward refracting sound-speed profile (referred as case 2). The different environmental parameters used for the simulations are summarized in Tables I and II.

Both figures show good agreement between the REVPA and the ORCA models (maximum error less than 0.5 dB out of the destructive interference zones). After the validation of the propagation part, the REVPA model was then coupled to scattering models.

Table I. Environmental parameters used for simulations.

	Case 1	Case 2
Source type[1]	Modal [12]	Greene [15]
Propagation range (km)	5	5
Source depth (m)	50	50
Frequency (Hz)	100	100
Water depth (m)	400	100
Sound speed in water (m/s)	1500	see Table II
Water density (kg/m^3)	1000	1000
Thickness of real sediment layer (m)	200	200
Sound speed in sediment (m/s)	1550	1800
Sediment density (kg/m^3)	1000	1000
Attenuation in sediment (dB/λ)	0.7	0.2
Thickness of artificial layer (m)	300	200
Maximum attenuation in artificial layer (dB/λ)	10	10

Table II. Sound-speed profile used for simulation: case 2.

Depth (m)	Sound speed (m/s)
0	1530
8	1525
20	1510
100	1505

3. REVPA: reverberation modeling

3.1. Scattering strength

The backscattering phenomenon was included by means of scattering strength laws. Several empirical or theoretical scattering functions have been determined to describe surface/bottom reverberation/scattering due to roughness, bubbles, etc. [18, 19, 20, 21, 22, 23, 24, 25, 26]. Any bistatic scattering function can be introduced in the REVPA model, but only Lambert's law was used for the simulations. It is defined as

$$SS_s(\theta_i, \theta_s) = 10\log_{10}\left(\mu\right) + 10\log_{10}\left(\sin\theta_i \sin\theta_s\right), \quad (5)$$

where θ_i and θ_s represent respectively the incident and scattering angles and μ is a constant depending on the sediment type, $10\log_{10}(\mu)$ is equal to [27]

- −15 dB for gravel,
- −22 dB for sand,
- −29 dB for mud and clay.

3.2. Plane-wave decomposition - reverberation modeling

As the used scattering coefficients are defined for plane waves, a plane-wave decomposition was performed at each range step in order to include the reverberation phenomena into the propagation model. Such decomposition can be achieved using two main techniques: Fourier decomposition and Single Value Decomposition (WKB approximation associated with SVD). Discrete Fourier transform has been associated with the parabolic equation theory [7] in incoherent computations of the reverberation levels. The extension to our case (coherent) has shown several oscillations problems that we could not overcome [28]. As the field computed with the parabolic equation could be associated to mode decomposition, we have used WKB approximation for the estimation of the plane-wave coefficients. With this approximation, each normal mode was written as a summation of an upward and a downward propagating plane waves. For an $exp(-i\omega t)$ time dependence, the modal function $\Phi_m(z)$ is defined as [12]

$$\begin{aligned}\Phi_m(z) &= \frac{A_m}{\sqrt{k_{zm}(z)}}\left[\exp\left(\mathrm{i}\int_{z_{tp}}^{z} k_{zm}(z)\,\mathrm{d}z - \theta_{tp}/2\right)\right.\\ &\quad\left. + \exp\left(-\mathrm{i}\int_{z_{tp}}^{z} k_{zm}(z)\,\mathrm{d}z + \theta_{tp}/2\right)\right] \qquad (6)\\ &= A_m\vartheta_m(z),\end{aligned}$$

where

- A_m is the amplitude of mode m,
- z_{tp} is the depth of the turning point or reflection depth of mode m, the former being defined by $k_{rm} = k(z_{tp})$ (with $k(z)$ the wavenumber in the water column),
- θ_{tp} the phase at the turning point (equal to $\pi/2$ for interior turning points and π if the turning point is at the surface),
- $k_{zm}(z) = \sqrt{k^2(z) - k_{rm}^2}$ is the vertical wavenumber.

In equation (6), the first exponential term represents downward propagating waves and the second term upward propagating ones, for z increasing in depth. From equation (6), it appears that the use of WKB approximation with the parabolic equation, requires that the number of normal modes and the associated horizontal wavenumbers are known. In the case of constant sound speed, these parameters are given by the modal starter [12, p. 342-343]. For depth varying sound-speed profiles, they have to be calculated using a normal-mode model.

For a given range, the acoustic field $\psi(z)$ can be written as a sum of normal modes,

$$\psi(z) = \sum_{n=1}^{N}\Phi_n(z) = \sum_{n=1}^{N} A_m\vartheta_n(z), \qquad (7)$$

where N is the total number of propagating modes.

The incident field was determined using the parabolic equation, the amplitudes of each incident plane wave are calculated solving a linear system of equations (deduced from equation 7) by means of an SVD (Singular Value Decomposition [29]). Associating a scattering coefficient (such as the Lambert's law) to each incident plane wave

[1] In order to compare the models, a modal starter that does not take evanescent waves into account, was used for Case 1, whereas a Greene source that takes both propagating and evanescent waves into account, was used for Case 2.

leads to the calculation of the amplitudes of each scattered plane wave at angle θ_s,

$$A_s(\theta_s, z) = \sum_{\theta_i} \Big(A_i(\theta_i) \sqrt{m_{s,b}(\theta_i, \theta_s)} \exp(\pm i z k_0 \sin\theta_i) \Big), \quad (8)$$

where $A_s(\theta_s, z)$ and $A_i(\theta_i)$ represent respectively the amplitudes of the scattered and incident plane waves, θ_i and θ_s the incident and scattering angles, $m_{s,b}$ is the scattering coefficient (defined as $m_{s,b} = \mu sin\theta_i sin\theta_s$), and k_0 the reference wavenumber.

A phase was also associated to each scattered plane wave,

$$\phi(z, \theta_s) = \exp(\pm i z k_0 \sin\theta_s). \quad (9)$$

The sign of the phase is related to its direction of propagation (the positive sign being associated to downward propagating waves).

For a given scattering spot (given range), the amplitude and the phase of each incident and scattered plane wave were determined. Then the reverberated field was computed at the interface and back-propagated to the source using reciprocity principle (the losses between the source and the scattering area are equal to the losses between the scattering area and the receiver).

In order to compare to other existing models that take into account the range resolution [4], δr, the surface of the insonified spot on the bottom (incident footprint) was integrated for every spot range r_s: $\delta A = 2\pi r_s \delta r$. The final reverberated field, p_{rev}, was then obtained,

$$p_{rev}(r, z_r) = \sqrt{\delta A}\, H_0^{(1)}(k_0 r) \cdot \sum_{\theta_s} \big[A_s(\theta_s) A_i(\theta_s) \exp\big(\pm i k_0 z_r \sin\theta_s \big) \big], \quad (10)$$

where z_r is the receiver depth.

3.3. Simulation results

Some bottom reverberation simulations results obtained with the REVPA model are presented in this paragraph. They are compared with results obtained with two other models based on normal-mode theory. For the case of constant sound speed, a model based on the equations proposed by Brekhovskikh [30] has been developed and used [13]. This model is a very simple normal-mode one that can handle only depth- and range-independent environments. For depth varying sound-speed profile, REVPA results were compared to simulations realized using the existing R-Snap model [6]. For all simulations, the reverberation phenomena were included in the propagation model by means of the Lambert's law, with $10 \log_{10} \mu = -27$ dB, which represents a sandy bottom.

Table III. Environmental parameters used for the simulations presented on Figure 5: constant sound-speed profile.

Source type	modal
Propagation range (km)	5
Source depth (m)	50
Frequency (Hz)	100
Water depth (m)	100
Sound speed in water (m/s)	1500
Water density (kg/m^3)	1000
Thickness of real sediment layer (m)	150
Sound speed in sediment (m/s)	1800
Sediment density (kg/m^3)	2000
Attenuation in sediment (dB/λ)	0.36
Thickness of artificial layer (m)	200
Max. attenuation in artificial layer (dB/λ)	10

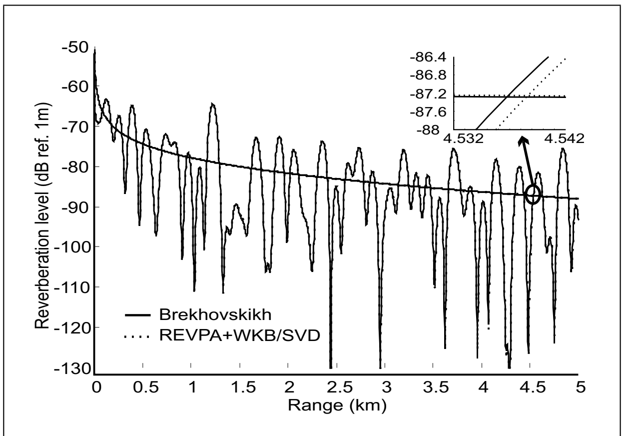

Figure 5. Bottom reverberation level (coherent and incoherent) predicted with a normal-mode model based on Brekhovskikh equations and the REVPA model.

3.3.1. Depth-independent sound-speed profile

The environmental parameters used for the simulations are presented in Table III.

The backscattered acoustic field at the receiver has been computed for both coherent and incoherent cases. The results are presented in Figure 5.

For both cases, the benchmarking is excellent as the error on the reverberation level is smaller than 0.1 dB even in interference areas. As the error is hardly visible (plots overlay) a zoom has been added in the Figure 5 (top right) in order to emphasize the match. After the validation, in this very simple case, the REVPA model was then benchmarked for depth variable sound-speed profiles.

3.3.2. Depth variable sound-speed profile

The bottom backscattered field was estimated using both the REVPA and the R-Snap models. A downward refracting sound-speed profile was considered. The environmental parameters are summarized in Tables IV and V.

Only incoherent calculations were used for benchmarking with R-Snap. The results are presented in Figure 6.

Figure 6 shows less agreement than Figure 5. Nevertheless, the error was about 1–2 dB. In addition, the error is an

Table IV. Environmental parameters used for the simulations presented on Figure 6: depth variable sound-speed profile.

Source type	Greene
Propagation range (km)	5
Source depth (m)	50
Frequency (Hz)	100
Water depth (m)	130
Sound speed in water (m/s)	see Table V
Water density (kg/m^3)	1000
Thickness of real sediment layer (m)	170
Sound speed in sediment (m/s)	1800
Sediment density (kg/m^3)	1000
Attenuation in sediment (dB/λ)	0.7
Thickness of artificial layer (m)	200
Max. attenuation in artificial layer (dB/λ)	10

Table V. Sound-speed profile used for the simulations presented on Figure 6.

Depth (m)	Sound speed (m/s)
0	1521
36	1519
38	1514
50	1511
130	1508

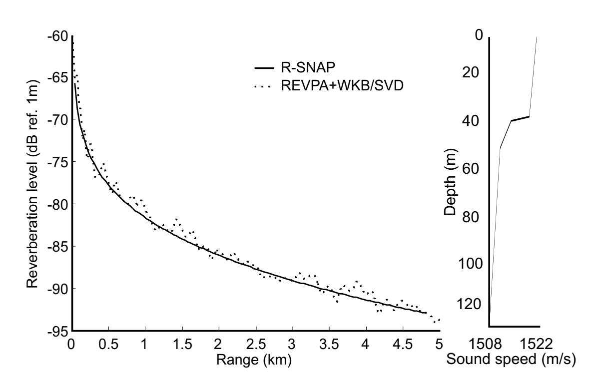

Figure 6. Bottom reverberation level predicted with the R-Snap and the REVPA models.

oscillating one in the latter case. After running numerous trials in several configurations and comparing the results, the source of error was finally identified: it is mainly due to the definition of turning points in the WKB approximation. Whereas higher frequencies should remove this problem, it seems clear that, in future versions, the accuracy of computation of the modal functions in the vicinity of the turning points has to be improved.

The presented results are very simple and not characteristic of the complex situations encountered *in situ*. However, they correspond to first steps in the development of the reverberation model. For range-independent environment, both incoherent and coherent calculations can be performed with this model, the latter one being required to obtain time series simulations. Whereas the extension of the propagation part of the model to range-dependent cases is readily available thanks to the parabolic equation, the reverberation calculations require to determine the horizontal wavenumbers, i.e. to use a normal-mode model, at each range the environmental properties change. As such a method can constitute a strong limitation of the model, another procedure could be considered based on a Taylor series expansion of the transcendental equation, for a Pekeris waveguide,

$$\tan\left(\sqrt{\left(\frac{\omega}{c_w}\right)^2 - k_r^2 h}\right) = -\frac{\rho_s}{\rho_w}\frac{\sqrt{\left(\frac{\omega}{c_w}\right)^2 - k_r^2}}{\sqrt{k_r^2 - \left(\frac{\omega}{c_s}\right)^2}}$$

where
- ω is the angular frequency,
- c_w is the sound speed in the water,
- k_r is the horizontal wavenumber,
- h is the water depth,
- ρ_s is the sediment density,
- ρ_w is the water density,
- c_s is the sound speed in the sediment.

Once the horizontal wavenumbers are determined at the source range, an expansion of the previous equation should allow to determine the horizontal wavenumbers for different values of the environmental parameters (depth, sound speed, etc). However, such a procedure has to be studied in order to determine its limitations.

4. Conclusion

A new model (REVPA) has been developed for the computation of reverberation level at long ranges in shallow water. While using "conventional" methods, the REVPA model has focused on the coupling problem allowing scales separation and the introduction of any bottom (or surface) scattering law. It is then based on the coupling of a wide-angle parabolic equation method (fourth order Padé) to the bottom bistatic scattering function through a plane-wave decomposition. To handle incoherent as well as coherent calculations, this decomposition is based on a Single Value Decomposition.

Several tens of simulations have been conducted for several configurations. Some representative cases have been used for benchmarking the model for simple geometries (range-independent) before extending it to more complex ones (range-dependent bottom (topography and type) and range variable velocity profile).

The benchmarking has shown good performance of the REVPA model (maximum error on all simulations: 0.1 to 1 dB) for constant sound-speed profile. Nevertheless, the REVPA performances are still good but less satisfactory for a continuously varying sound velocity profile (error: 1–2 dB). In addition, the error looked ”periodic”. Thanks to several simulations for different configurations, the source of error was identified: the integration of turning points in the WKB approximation while carrying out plane-wave decomposition.

The presented results correspond to first steps in the development of a complete reverberation model. It has been designed from the beginning to handle range varying cases and general bottom (or surface) scattering functions. Simulations have been conducted in the range invariant situation in order to compare the REVPA results with the ones obtained in the published literature.

The model might be readily extended to the range variable case.

A relevant enhancement for future versions is the improvement of plane-wave decomposition, by improving the computation accuracy of the modal functions in the vicinity of the turning points. The accuracy of the current model should also improve with frequency as the turning point phase becomes better approximated as $\pi/2$ and the depth of the turning point become better defined.

Future step is the comparison of REVPA model, in the range-dependent case, either to existing models or preferably to experimental data (first in controlled tank conditions and then in real sea situations). The major difficulty, at sea, will be the access to enough experimental and ground truth data for using as model inputs: sound velocity profiles in the water column and experimental bottom bistatic scattering function. The model calibration on tank data will be an essential intermediate step.

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References

[1] M. D. Collins: The rotated parabolic equation and sloping ocean bottoms. J. Acoust. Soc. Am. **87** (1990) 1035–1037.

[2] M. D. Collins, E. K. Westwood: A higher-order energy-conserving parabolic equation for range-dependent ocean depth, sound speed, and density. J. Acoust. Soc. Am. **89** (1991) 1068–1075.

[3] H. P. Bucker, H. E. Morris: Normal-mode reverberation in channels or ducts. J. Acoust. Soc. Am. **44** (1968) 827–828.

[4] D. D. Ellis: A shallow-water normal-mode reverberation model. J. Acoust. Soc. Am. **97** (1995) 2804–2814.

[5] D. M. Fromm, B. J. Orchard, S. N. Wolf: Range-dependent, normal-mode reverberation model for bistatic geometries. – In: Ocean reverberation. D. D. Ellis, J. R. Preston, H. G. Urban (eds.). Kluwer Academic, 1993, 155–160.

[6] K. D. LePage: Bottom reverberation in shallow water: coherent properties as a function of bandwidth, waveguide characteristics, and scatter distributions. J. Acoust. Soc. Am. **106** (1999) 3240–3254.

[7] H. G. Schneider: Surface loss, scattering, and reverberation with the split-step parabolic wave equation model. J. Acoust. Soc. Am. **93** (1993) 770–781.

[8] R. Zhang, G. Jin: Normal-mode theory of average reverberation intensity in shallow water. J. Sound. Vib. **119** (1987) 215–223.

[9] A. Bamberger, B. Engquist, L. Halpern, P. Joly: Parabolic wave equation approximations in heterogeneous media. SIAM J. Appl. Math. **48** (1988) 99–128.

[10] A. Bamberger, B. Engquist, L. Halpern, P. Joly: Higher order paraxial wave equation approximations in heterogeneous media. SIAM J. Appl. Math. **48** (1988) 129–154.

[11] M. D. Collins: Applications and time-domain solution of higher-order parabolic equations in underwater acoustics. J. Acoust. Soc. Am. **86** (1989) 1097–1102.

[12] F. B. Jensen, W. A. Kuperman, M. B. Porter, H. Schmidt: Computational ocean acoustics. American Institutes of Physics, New York, 2000.

[13] G. Bouchage: Modélisation de la réverbération basse fréquence par petits fonds. Ph.D. thesis, Lyon, 2002.

[14] F. D. Tappert: The parabolic approximation method. – In: Wave propagation in underwater acoustics. J. B. Keller, J. S. Papadakis (eds.). Springer-Verlag, New-York, 1979, 224–287.

[15] R. R. Greene: The rational approximation to the acoustic wave equation with bottom interaction. J. Acoust. Soc. Am. **76** (1984) 1764–1773.

[16] M. B. Porter, F. B. Jensen, C. M. Ferla: The problem of energy conservation in one-way models. J. Acoust. Soc. Am. **89** (1991) 1058–1067.

[17] E. K. Westwood, C. T. Tindle, N. R. Chapman: A normal mode model for acousto-elastic ocean environments. J. Acoust. Soc. Am. **100** (1996) 3631–3645.

[18] R. P. Chapman, J. H. Harris: Surface backscattering strengths measured with explosive sound sources. J. Acoust. Soc. Am. **34** (1962) 1592–1597.

[19] F. S. Henyey: Acoustic scattering from ocean microbubble plumes in the 100 Hz to 2 kHz region. J. Acoust. Soc. Am. **90** (1991) 399–405.

[20] A. N. Ivakin, Y. P. Lysanov: Theory of underwater sound scattering by random inhomogeneities of the bottom. J. Acoust. Soc. Am. **27** (1981) 61–64.

[21] E. Y. T. Kuo: Wave scattering and transmission at irregular surfaces. J. Acoust. Soc. Am. **36** (1964) 2135–2142.

[22] B. F. Kur'yanov: The scattering of sound at a rough surface with two types of irregularity. Sov. Phys. Acoust. **8** (1963) 252–257.

[23] B. E. McDonald: Echoes from vertically striated subresonant bubble clouds: a model for ocean surface reverberation. J. Acoust. Soc. Am. **89** (1991) 617–622.

[24] J. C. Novarini, J. W. Caruthers: A simplified approach to backscattering from a rough seafloor with sediment inhomogeneities. IEEE J. Oceanic Eng. **23** (1998) 157–166.

[25] J. A. Ogilvy: Theory of wave scattering from random rough surfaces. Institute of Physics Publishing, Bristol, 1992.

[26] M. Schulkin, R. Schaffer: Backscattering of sound from the sea surface. J. Acoust. Soc. Am. **36** (1964) 1699–1703.

[27] M. Gensane: A statistical study of acoustic signals backscattered from the sea bottom. IEEE J. Oceanic Eng. **14** (1989) 84–93.

[28] G. Bouchage, K. D. LePage: A shallow-water reverberation PE model. Acta Acustica united with Acustica **88** (2002) 638–641.

[29] W. H. Press, B. P. Flannery, S. K. Teutolski, W. J. Vetterling: Numerical recipes. Second edition. University Press, Cambridge, 1986.

[30] L. M. Brekhovskikh, Y. P. Lysanov: Fundamentals of ocean acoustics. Second edition. Springer-Verlag, Berlin, 1991.