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Acoustical Instability of Inhomogeneous Gas Flows With Distributed Heat Release

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Summary
Acoustical instability of the inhomogeneous heat-emitted gas flow in dependence on the Mach number is investigated. Obtained acoustical equations for perturbations of velocity, density, pressure, and temperature have different forms in inhomogeneous media. Growth increments for these perturbations differ from each other as well. They are significantly dependent on the Mach number and the flow direction. The features of sub- and supersonic acoustics are considered.

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1. Introduction

Over 130 years ago, Rayleigh [1] understood that heating and cooling could create acoustical power "if heat be given to the air at the moment of greatest condensation or be taken from it at the moment of greatest rarefaction". At the present time, Rayleigh's criterion of acoustic instability is investigated for many classes of continuous media including media with the heat release depending on temperature and density, nonequilibrium media, media with longitudinal or transverse gradients etc. and permits to generalize and formalize great number of results obtained for particular cases.

For a wide class of nonequilibrium homogeneous gas media, Rayleigh's criterion of acoustical instability exclusive of thermal conduction and shear viscosity is satisfied under the condition [2, 3]

$$\xi < 0, \tag{1}$$

where ξ is the bulk (second) viscosity coefficient. In earlier works, the conditions for the negative bulk viscosity existence were found in a large number of nonequilibrium media such as a vibrationally excited gas, atomic or molecular nonisothermal plasma, chemical active mixtures, media with nonequilibrium phases (e.g., volcanic magma, atmosphere regions with nonequilibrium condensation etc), interstellar media and stretched horizons of black holes [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

At stationary homogeneous nonequilibrium gas media, the acoustic increment including a shear viscosity coefficient η and a heat conduction coefficient χ can be written in a general form,

$$\alpha = \frac{\omega^2}{2c_s^3(\omega)\rho}\left[\frac{4}{3}\eta + \left(\frac{1}{C_{V\infty}} - \frac{1}{C_{p\infty}}\right)\mu\chi + \xi(\omega)\right], \tag{2}$$

where c_s is the speed of sound of frequency ω, ρ is the gas density, $C_{V\infty}$, $C_{p\infty} = C_{V\infty} + R$ are the molar high-frequency specific heat capacities at constant volume and pressure, respectively, R is the universal gas constant, and μ is the molar mass. In equation (2), the coefficient $\xi(\omega)$ depends on the type of nonequilibrium and a relaxation model.

However, stationary nonequilibrium fluids are usually inhomogeneous. Rayleigh's criterion of acoustical instability is met in different classes of inhomogeneous media, e.g., in the thermoacoustical engines with a steep axial temperature gradient. This gradient is artificially created, e.g., by heating one end of channel and cooling another end [13, 14, 15, 16, 17]. In stationary nonequilibrium media with distributed heating gains there is a temperature gradient connected with the type of the heat removal, e.g., convective cooling in the flow. Investigation of the acoustical stability of flows of the inhomogeneous nonequilibrium gas is important for development of powerful lasers, combustion chambers and plasma aerodynamics. In [7], we considered the acoustical instability of subsonic flows of nonequilibrium gas with the distributed heating gain. In this paper, we will consider the acoustical stability of the one dimensional gas flow with the distributed heating gain for wide range of Mach numbers and an arbitrary dependence of the heating gain on temperature and density. We will obtain the one-dimensional linear acoustical equation of the weak inhomogeneous gas flow with the distributed heating gain and corrections of equations (1), (2) for an axial temperature gradient created by the convective heating removal.

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2. A model system. Linear high-frequency acoustical equations

The basic system of hydrodynamical equations of one-dimensional non-stationary heat-emitted gas motions along the axis Ox includes equation of continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0, \tag{3}$$

equation of motion

$$\rho\Big(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\Big) = -\frac{\partial p}{\partial x} + \frac{4}{3}\frac{\partial}{\partial x}\Big(\eta\frac{\partial v}{\partial x}\Big), \tag{4}$$

energy equation

$$C_{V\infty}\Big(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x}\Big) - \mu\frac{p}{\rho^2}\Big(\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x}\Big) = Q(T,\rho) + \mu\chi\frac{1}{\rho}\frac{\partial^2 T}{\partial x^2} + \frac{4}{3}\frac{\mu}{\rho}\eta\Big(\frac{\partial v}{\partial x}\Big)^2, \tag{5}$$

and equation of state

$$p = \frac{\rho R T}{\mu}. \tag{6}$$

In equations (3)–(6), v, T, ρ, p are, respectively, the velocity, temperature, density, and pressure, Q is the power of the distributed heating gain per one mole of the gas.

To consider the instability of the investigated inhomogeneous gas flow we will linearize the initial system of the equation. Let us set

$$v(x,t) = v_0(x) + \tilde{v}(x,t), \tag{7}$$
$$p(x,t) = p_0(x) + \tilde{p}(x,t), \tag{8}$$
$$\rho(x,t) = \rho_0(x) + \tilde{\rho}(x,t), \tag{9}$$
$$T(x,t) = T_0(x) + \tilde{T}(x,t), \tag{10}$$

where subscripts 0 indicate unperturbed stationary quantities (background flow), tildes indicate flow disturbances.

At first, let us consider the background flow. The gradients of the stationary flow components are obtained from equations (3)–(6) in forms

$$\frac{\mathrm{d}\rho_0}{\mathrm{d}x} = -\frac{1}{1-\gamma_\infty M_\infty^2}\frac{\rho_0}{T_0}\frac{\mathrm{d}T_0}{\mathrm{d}x}, \tag{11}$$
$$\frac{\mathrm{d}v_0}{\mathrm{d}x} = \frac{c_\infty M_\infty}{T_0(1-\gamma_\infty M_\infty^2)}\frac{\mathrm{d}T_0}{\mathrm{d}x}, \tag{12}$$
$$\frac{\mathrm{d}p_0}{\mathrm{d}x} = -\frac{\rho_0 c_\infty^2 M_\infty^2}{T_0(1-\gamma_\infty M_\infty^2)}\frac{\mathrm{d}T_0}{\mathrm{d}x}, \tag{13}$$

where $\gamma_\infty = C_{p\infty}/C_{V\infty}$ is the high-frequency adiabatic index (the ratio of specific heats), $c_\infty = \sqrt{\gamma_\infty R T_0/\mu}$ is the high-frequency local sound speed, $M_\infty = v_0/c_\infty$ is the local Mach number in the gas flow.

The temperature gradient depends on the heating gain and is equal to

$$\frac{\mathrm{d}T_0}{\mathrm{d}x} = \frac{1-\gamma_\infty M_\infty^2}{C_{p\infty}c_\infty M_\infty(1-M_\infty^2)}\frac{RT_0}{\tau_Q}, \tag{14}$$

where $\tau_Q = RT_0/Q_0$ is the characteristic time of heating.

Using equations (12), (14), we obtain

$$\frac{\mathrm{d}M_\infty}{\mathrm{d}x} = \frac{1+\gamma_\infty M_\infty^2}{2C_{p\infty}c_\infty(1-M_\infty^2)}\frac{R}{\tau_Q}. \tag{15}$$

In equations (11)–(15), we assume that shear viscosity and heat conductivity coefficients are small and have no effect on the background flow. We will consider the continuously differentiable solutions of equations (3)–(6) without shock waves. It follows from equation (15) that we consider the subsonic or the supersonic flows without the transfer across the sound speed.

Linearized system (3)–(6) has a form

$$\rho_0\frac{\partial \tilde{v}}{\partial x} + \frac{\mathrm{d}\rho_0}{\mathrm{d}x}\tilde{v} + \frac{\partial \tilde{\rho}}{\partial t} + v_0\frac{\partial \tilde{\rho}}{\partial x} + \frac{\mathrm{d}v_0}{\mathrm{d}x}\tilde{\rho} = 0 \tag{16}$$

$$\rho_0\frac{\partial \tilde{v}}{\partial t} + \rho_0 v_0\frac{\partial \tilde{v}}{\partial x} + \rho_0\frac{\mathrm{d}v_0}{\mathrm{d}x}\tilde{v} - \frac{4}{3}\frac{\partial}{\partial x}\Big(\eta\frac{\partial \tilde{v}}{\partial x}\Big) + v_0\frac{\mathrm{d}v_0}{\mathrm{d}x}\tilde{\rho} + \frac{\partial \tilde{p}}{\partial x} = 0, \tag{17}$$

$$\Big(C_{V\infty}\rho_0\frac{\mathrm{d}T_0}{\mathrm{d}x} - RT_0\frac{\mathrm{d}\rho_0}{\mathrm{d}x}\Big)\tilde{v} - \frac{8}{3}\mu\eta\frac{\mathrm{d}v_0}{\mathrm{d}x}\frac{\partial \tilde{v}}{\partial x} + \Big(C_{V\infty}v_0\frac{\mathrm{d}T_0}{\mathrm{d}x} - \rho_0\frac{\partial Q}{\partial \rho} - Q_0\Big)\tilde{\rho} - RT_0\frac{\partial \tilde{\rho}}{\partial t} - RT_0 v_0\frac{\partial \tilde{\rho}}{\partial x} + C_{V\infty}\rho_0\frac{\partial \tilde{T}}{\partial t} + C_{V\infty}\rho_0 v_0\frac{\mathrm{d}\tilde{T}}{\mathrm{d}x} - \Big(Rv_0\frac{\mathrm{d}\rho_0}{\mathrm{d}x} + \rho_0\frac{\mathrm{d}Q}{\mathrm{d}T}\Big)\tilde{T} - \mu\chi\frac{\partial^2 \tilde{T}}{\partial x^2} = 0, \tag{18}$$

$$\tilde{p} = \frac{\rho_0 R\tilde{T} + RT_0\tilde{\rho}}{\mu}. \tag{19}$$

It is well-known, that the solution of linearized fundamental hydrodynamics equations system (16)–(19) contains several modes. In this work, we will discuss stability of acoustical modes only. One acoustical mode propagates along the flow in the positive direction of axis Ox and another mode propagates in the negative direction. Relatively the fixed coordinate system, these modes contra propagate in the subsonic flow and propagate in one direction in the supersonic flow. This difference must be taken into account to obtain the acoustical increment in moving media.

Let us study the propagation of an acoustical perturbation of frequency ω in the weakly inhomogeneous flow, when

$$\frac{|1 \pm M_\infty|c_\infty}{\omega}\frac{1}{T_0}\left|\frac{\mathrm{d}T_0}{\mathrm{d}x}\right| = \varepsilon \ll 1, \tag{20}$$

where ε is the first order of smallness quantity. The upper sign corresponds to the acoustical mode propagating along the flow, and the lower sign corresponds to the acoustical mode propagating in the opposite direction. Condition (20) can be reduced to

$$\omega\tau_Q \sim \varepsilon^{-1} \gg \frac{R}{C_{p\infty}M_\infty}\left|\frac{1-\gamma_\infty M_\infty^2}{1 \mp M_\infty}\right|. \tag{21}$$

Conditions (20), (21) mean that the temperature gradients for such high-frequency acoustical perturbations are

the first order of smallness quantities, the second spatial derivative of the stationary temperature is the second order of smallness quantity etc. Also, we suppose

$$\frac{\eta}{c_\infty^2\rho_0\omega}\sim\varepsilon,\qquad \frac{R\chi}{C_{V\infty}c_\infty^2\omega}\sim\varepsilon. \tag{22}$$

We will solve system (16)–(19) by the method of step-by-step approximations. Let the perturbations be

$$\tilde v(x,t) = v^{(1)}(x,t)+v^{(2)}(x,t)+\dots, \tag{23}$$
$$\tilde p(x,t) = p^{(1)}(x,t)+p^{(2)}(x,t)+\dots, \tag{24}$$
$$\tilde\rho(x,t) = \rho^{(1)}(x,t)+\rho^{(2)}(x,t)+\dots, \tag{25}$$
$$\tilde T(x,t) = T^{(1)}(x,t)+T^{(2)}(x,t)+\dots, \tag{26}$$

where superscripts mean orders of smallness:

$$v^{(1)}/c_\infty\sim p^{(1)}/p_0\sim\rho^{(1)}/\rho_0\sim T^{(1)}/T_0\sim\varepsilon, \tag{27}$$
$$v^{(2)}/c_\infty\sim p^{(2)}/p_0\sim\rho^{(2)}/\rho_0\sim T^{(2)}/T_0\sim\varepsilon^2. \tag{28}$$

Setting equations (23)–(26) in equations (3)–(6) and taking into account conditions (20)–(22), we have the set of equations for perturbations of the fist order smallness,

$$\begin{aligned}
\frac{\partial\rho^{(1)}}{\partial t}+v_0\frac{\partial\rho^{(1)}}{\partial x}+\rho_0\frac{\partial v^{(1)}}{\partial x} &= 0,\\
C_{V\infty}m\left(\frac{\partial^2 v^{(1)}}{\partial t^2}+2v_0\frac{\partial^2 v^{(1)}}{\partial t\,\partial x}+v_0^2\frac{\partial^2 v^{(1)}}{\partial x^2}\right)&\\
-C_{p\infty}T_0\frac{\partial^2 v^{(1)}}{\partial x^2} &= 0,\\
T_0\frac{\partial v^{(1)}}{\partial x}+C_{V\infty}\left(\frac{\partial T^{(1)}}{\partial t}+v_0\frac{\partial T^{(1)}}{\partial x}\right) &= 0,\\
\rho_0\left(\frac{\partial v^{(1)}}{\partial t}+v_0\frac{\partial v^{(1)}}{\partial x}\right)+\frac{\partial p^{(1)}}{\partial x} &= 0.
\end{aligned} \tag{29}$$

Using (29), we obtain the following relations between these perturbations:

$$\frac{\partial}{\partial t} = -\left(v_0\pm c_\infty\right)\frac{\partial}{\partial x}, \tag{30}$$
$$\rho^{(1)} = \pm\frac{\rho_0}{c_\infty}v^{(1)}, \tag{31}$$
$$T^{(1)} = \pm\frac{T_0}{C_{V\infty}c_\infty}v^{(1)}, \tag{32}$$
$$p^{(1)} = \pm c_\infty\rho_0 v^{(1)}. \tag{33}$$

Relation (30) is valid for perturbations $\sim\varepsilon$, when inhomogeneity is not important.

The set of equations for perturbations of the second order smallness has a form

$$\begin{aligned}
\frac{\partial\rho^{(2)}}{\partial t}+v_0\frac{\partial\rho^{(2)}}{\partial x}+\rho_0\frac{\partial v^{(2)}}{\partial x}+\theta_1 &= 0,\\
\rho_0\frac{\partial v^{(2)}}{\partial t}+\rho_0 v_0\frac{\partial v^{(2)}}{\partial x}+\frac{\partial p^{(2)}}{\partial x}+\theta_2 &= 0,\\
RT_0\frac{\partial\rho^{(2)}}{\partial t}+RT_0v_0\frac{\partial\rho^{(2)}}{\partial x}-C_{V\infty}\rho_0\frac{\partial T^{(2)}}{\partial t}&\\
-C_{V\infty}\rho_0v_0\frac{\partial T^{(2)}}{\partial x}+\theta_3 &= 0,\\
RT_0\rho^{(2)}+R\rho_0T^{(2)}-\mu p^{(2)} &= 0,
\end{aligned} \tag{34}$$

where

$$\theta_1 = \frac{\mathrm{d}\rho_0}{\mathrm{d}x}v^{(1)}+\frac{\mathrm{d}v_0}{\mathrm{d}x}\rho^{(1)}, \tag{35}$$
$$\theta_2 = v_0\frac{\mathrm{d}v_0}{\mathrm{d}x}\rho^{(1)}+\rho_0\frac{\mathrm{d}v_0}{\mathrm{d}x}v^{(1)}-\frac{4}{3}\eta\frac{\partial^2 v^{(1)}}{\partial x^2}, \tag{36}$$
$$\begin{aligned}
\theta_3 = &-C_{V\infty}v_0\frac{\mathrm{d}T_0}{\mathrm{d}x}\rho^{(1)}-C_{V\infty}\rho_0\frac{\mathrm{d}T_0}{\mathrm{d}x}v^{(1)}\\
&+v_0\frac{\mathrm{d}\rho_0}{\mathrm{d}x}RT^{(1)}+RT_0\frac{\mathrm{d}\rho_0}{\mathrm{d}x}v^{(1)}+Q_0\rho^{(1)}\\
&+\rho_0\frac{\partial Q}{\partial\rho}\rho^{(1)}+\rho_0\frac{\partial Q}{\partial T}T^{(1)}+\mu\chi\frac{\partial^2 T^{(1)}}{\partial x^2}
\end{aligned} \tag{37}$$

By $\vec\varphi$ we denote the vector of perturbations

$$\vec\varphi = \begin{bmatrix}\tilde v\\ \tilde p\\ \tilde\rho\\ \tilde T\end{bmatrix} \tag{38}$$

Using equations (30)–(33), after cumbersome transformations of equations (34)–(37), we obtain with an accuracy of $\sim\varepsilon^2$ four closed equations for different projections of vector $\vec\varphi$,

$$\begin{aligned}
&\frac{\partial^2\tilde\varphi}{\partial t^2}+2v_0\frac{\partial^2\tilde\varphi}{\partial t\,\partial x}+v_0^2\frac{\partial^2\tilde\varphi}{\partial x^2}-c_\infty^2\frac{\partial^2\tilde\varphi}{\partial x^2}\\
&-\frac{1}{\rho_0}b\left(\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial x}\right)\frac{\partial^2\tilde\varphi}{\partial x^2}+2c_\infty\alpha_\infty\left(\frac{\partial\tilde\varphi}{\partial t}+v_0\frac{\partial\tilde\varphi}{\partial x}\right)\\
&+v_0\frac{K_1}{1-\gamma_\infty M_\infty^2}\frac{T_{0x}}{T_0}\left(\frac{\partial\tilde\varphi}{\partial t}+v_0\frac{\partial\tilde\varphi}{\partial x}\right)\\
&+c_\infty^2\frac{K_2}{1-\gamma_\infty M_\infty^2}\frac{T_{0x}}{T_0}\frac{\partial\tilde\varphi}{\partial x} = 0.
\end{aligned} \tag{39}$$

Here,

$$\alpha_\infty = -\frac{R}{2c_\infty C_{p\infty}}\left(\frac{\rho_0}{RT_0}\frac{\partial Q}{\partial\rho}+\frac{1}{C_{V\infty}}\frac{\partial Q}{\partial T}\right), \tag{40}$$
$$b = \frac{4}{3}\eta+\frac{R\mu\chi}{C_{p\infty}C_{V\infty}}, \tag{41}$$
$$K_1 = \begin{bmatrix}\gamma_\infty+M_\infty^2\\ (1+\gamma_\infty)(1+M_\infty^2)\\ 3+\gamma_\infty+M_\infty^2-\gamma_\infty M_\infty^2\\ \gamma_\infty-1+(\gamma_\infty+1)M_\infty^2\end{bmatrix}, \tag{42}$$
$$K_2 = \begin{bmatrix}(\gamma_\infty+1)M_\infty^2\\ M_\infty^2-1\\ M_\infty^2-3+2\gamma_\infty M_\infty^2\\ M_\infty^2+1\end{bmatrix}, \tag{43}$$
$$T_{0x} = \frac{\mathrm{d}T_0}{\mathrm{d}x}. \tag{44}$$

A parameter α_∞ is the high-frequency acoustical gain in the homogeneous gas media. In [7, 12], we showed that this gain is the part of acoustical increment (2) which is proportional to the second viscosity coefficient ξ. Here, both $\xi<0$ and $\alpha_\infty<0$ under the condition

$$\frac{\rho_0}{RT_0}\frac{\partial Q}{\partial\rho}+\frac{1}{C_{V\infty}}\frac{\partial Q}{\partial T}>0. \tag{45}$$

Condition (45) corresponds to the positive feedback between acoustic perturbations and heat release, that is to Rayleigh's criterion of acoustical instability.

The coefficients K_1, K_2 are different for different components of vector $\vec{\varphi}$. Hence, the closed differential equations for acoustical perturbations $\tilde{v}(x,t)$, $\tilde{T}(x,t)$, $\tilde{\rho}(x,t)$, $\tilde{P}(x,t)$ have different forms in inhomogeneous flows.

By $\vec{\psi}$ we denote the vector of relative perturbations

$$\vec{\psi} = \begin{bmatrix} \tilde{v}/c_\infty \\ \tilde{p}/p_0 \\ \tilde{\rho}/\rho_0 \\ \tilde{T}/T_0 \end{bmatrix}, \tag{46}$$

Using equations (34)–(37), we obtain with an accuracy of $\sim \varepsilon^2$ one closed equation for all projections of vector $\vec{\psi}$,

$$\begin{aligned} &\frac{\partial^2\tilde{\psi}}{\partial t^2} + 2v_0\frac{\partial^2\tilde{\psi}}{\partial t\,\partial x} + v_0^2\frac{\partial^2\tilde{\psi}}{\partial x^2} - c_\infty^2\frac{\partial^2\tilde{\psi}}{\partial x^2} \\ &-\frac{1}{\rho_0}b\Big(\frac{\partial}{\partial t} + v_0\frac{\partial}{\partial x}\Big)\frac{\partial^2\tilde{\psi}}{\partial x^2} + 2c_\infty\alpha_\infty\left(\frac{\partial\tilde{\psi}}{\partial t} + v_0\frac{\partial\tilde{\psi}}{\partial x}\right) \\ &+v_0\frac{\gamma_\infty + 1 + M_\infty^2 - \gamma_\infty M_\infty^2}{1-\gamma_\infty M_\infty^2}\frac{T_{0x}}{T_0}\left(\frac{\partial\tilde{\psi}}{\partial t} + v_0\frac{\partial\tilde{\psi}}{\partial x}\right) \\ &+c_\infty^2\frac{2\gamma_\infty M_\infty^2 + M_\infty^2 - 1}{1-\gamma_\infty M_\infty^2}\frac{T_{0x}}{T_0}\frac{\partial\tilde{\psi}}{\partial x} = 0. \end{aligned} \tag{47}$$

Neglecting the inhomogeneity in equation (39) gives

$$\begin{aligned} &\frac{\partial^2\tilde{\varphi}}{\partial t^2} + 2v_0\frac{\partial^2\tilde{\varphi}}{\partial t\,\partial x} + v_0^2\frac{\partial^2\tilde{\varphi}}{\partial x^2} - c_\infty^2\frac{\partial^2\tilde{\varphi}}{\partial x^2} \\ &-\frac{1}{\rho_0}b\left(\frac{\partial}{\partial t} + v_0\frac{\partial}{\partial x}\right)\frac{\partial^2\tilde{\varphi}}{\partial x^2} + 2c_\infty\alpha_\infty\left(\frac{\partial\tilde{\varphi}}{\partial t} + v_0\frac{\partial\tilde{\varphi}}{\partial x}\right) = 0. \end{aligned} \tag{48}$$

Let

$$\tilde{\varphi} \sim \exp\big(\mathrm{i}kx - \mathrm{i}\omega t\big), \tag{49}$$

where $k = \omega/(v_0 \pm c_s) \pm \mathrm{i}\alpha$ for $M_\infty < 1$ (subsonic flow) or $k = \omega/(v_0 \pm c_s) + \mathrm{i}\alpha$ for $M_\infty > 1$ (supersonic flow).

Setting expression (49) in equation (48), we get the acoustical increment in the homogeneous gas flow

$$\alpha = \frac{b}{2\rho_0}\frac{\omega^2}{|1 \pm M_\infty|^3 c_\infty^3} + \frac{\alpha_\infty}{|1 \pm M_\infty|}, \tag{50}$$

and the condition of the amplification of high-frequency acoustical perturbations

$$\frac{b}{2\rho_0}\frac{\omega^2}{(1 \pm M_\infty)^2 c_\infty^3} + \alpha_\infty < 0. \tag{51}$$

In what follows, we will consider how the inhomogeneity changes increment (50) and condition (51).

3. Acoustical increments in inhomogeneous media. Sub- and supersonic flows

Let the disturbed gas motion in the acoustical wave be described by the functions with the slowly varying amplitudes

$$\tilde{\varphi}(x,t) = \overline{\varphi}(x)\exp\left(\mathrm{i}\int_0^x k\,\mathrm{d}s - \mathrm{i}\omega t\right), \tag{52}$$

where

$$\begin{aligned} &\frac{\mathrm{d}\overline{\varphi}}{\overline{\varphi}k\,\mathrm{d}x} \sim \varepsilon, \\ &k = \frac{\omega}{v_0(x) \pm c_\infty(x)}, \\ &\frac{1}{k}\frac{\partial k}{\partial x} = -\frac{1}{(v_0 \pm c_\infty)}\frac{2M_\infty \pm 1 \mp \gamma_\infty M_\infty^2}{1-\gamma_\infty M_\infty^2}\frac{c_\infty}{2}\frac{T_{0x}}{T_0}. \end{aligned} \tag{53}$$

Setting equation (52) in equation (39) and neglecting the terms $\sim \varepsilon^3$, we reduce equation (39) to

$$\frac{\partial\overline{\varphi}^{(1)}}{\partial x} = \mp G_\infty\overline{\varphi}^{(1)} \quad \text{(subsonic flow)} \tag{54}$$

and

$$\frac{\partial\overline{\varphi}^{(1)}}{\partial x} = -G_\infty\overline{\varphi}^{(1)} \quad \text{(supersonic flow)}, \tag{55}$$

where G_∞ is the high-frequency increment.

In the inhomogeneous flow

$$G_\infty = \frac{\omega^2 b}{2c_\infty^3\rho_0|1 \pm M_\infty|^3} + \frac{\alpha_\infty}{|1 \pm M_\infty|} + G_\infty^i, \tag{56}$$

where

$$\begin{aligned} G_\infty^i = &\frac{\mp\big(-M_\infty^2 + \frac{1}{2} - \frac{1}{2}\gamma_\infty M_\infty^2 + K_2\big)}{2|1 \pm M_\infty|} \\ &\frac{-M_\infty\big(\frac{1}{2} + \frac{1}{2}\gamma_\infty M_\infty^2 - K_1\big)}{2|1 \pm M_\infty|} \\ &\frac{1}{1-\gamma_\infty M_\infty^2}\frac{1}{T_0}\frac{\mathrm{d}T_0}{\mathrm{d}x} \end{aligned} \tag{57}$$

is the increment correction coefficient connected with the longitudinal inhomogeneity of the stationary heat-emitted media.

Solutions of equations (54) and (55) are

$$\overline{\varphi}(x) = \overline{\varphi}(0)\exp\left(\mp\int_0^x G_\infty(s)\,\mathrm{d}s\right) \tag{58}$$

and

$$\overline{\varphi}(x) = \overline{\varphi}(0)\exp\left(-\int_0^x G_\infty(s)\,\mathrm{d}s\right), \tag{59}$$

respectively.

Unlike the homogeneous media, the perturbations of velocity, pressure, density and temperature increase with different increments. Moreover, increments differ in both the magnitude and the sign.

Using the similar procedure for vector $\tilde{\psi}$ of relative perturbations (46), we obtain the common increment correction coefficient for all perturbations

$$\begin{aligned} G_{r\infty}^i = &\frac{\mp\big(-\frac{1}{2} + \frac{3}{2}\gamma_\infty M_\infty^2\big)}{2|1 \pm M_\infty|} \\ &\frac{-M_\infty\big(-\frac{1}{2} + \frac{3}{2}\gamma_\infty M_\infty^2 - \gamma_\infty - M_\infty^2\big)}{2|1 \pm M_\infty|} \\ &\frac{1}{1-\gamma_\infty M_\infty^2}\frac{1}{T_0}\frac{\mathrm{d}T_0}{\mathrm{d}x}. \end{aligned} \tag{60}$$

Let us consider the limiting cases of subsonic ($M_\infty \ll 1$) and supersonic ($M_\infty \gg 1$) flows.

a) For $M_\infty \ll 1$, equations (56)–(57) are reduced to

$$G_\infty = R\frac{\mp\left(1+2\lim_{M_\infty\to 0} K_2\right)}{4C_{p\infty}c_\infty M_\infty \tau_Q} + \frac{\omega^2 b}{2c_\infty^3\rho_0} + \alpha_\infty, \tag{61}$$

$$\lim_{M_\infty\to 0} K_2 = \begin{bmatrix} 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}. \tag{62}$$

Earlier obtained increments [7] follow from equation (61).

It should be highlighted that the contribution of correction coefficient G_∞^i into increments becomes very significant when the Mach number of flow is low. This coefficient dominates if the inequality

$$M_\infty < \frac{R}{2C_{p\infty}|\tau_Q|}\frac{c_\infty^2\rho_0}{|\omega^2 b + 2c_\infty^3\rho_0\alpha_\infty|} \tag{63}$$

is valid. Here, we have the increase of velocity and temperature perturbations propagating along the flow and the increase of density and pressure perturbations propagating in the opposite direction. For the endothermic media, when $Q < 0$, the situation will be opposite.

For $M_\infty \ll 1$, equation (60) is reduced to equation (61) with

$$\lim_{M_\infty\to 0} K_2 = -1. \tag{64}$$

b) For $M_\infty \gg 1$, we obtain

$$G_\infty = \frac{\omega^2 b}{2c_\infty^3\rho_0 M_\infty^3} + \frac{\alpha_\infty}{M_\infty} + R\frac{\gamma_\infty - 2\lim_{M_\infty\to\infty}\left(K_1/M_\infty^2\right)}{4C_{p\infty}c_\infty M_\infty\tau_Q}, \tag{65}$$

$$\lim_{M_\infty\to\infty}\frac{K_1}{M_\infty^2} = \begin{bmatrix} 1 \\ \gamma_\infty + 1 \\ 1-\gamma_\infty \\ \gamma_\infty + 1 \end{bmatrix}. \tag{66}$$

In supersonic flows, all small amplitude waves will propagate in the same direction but with different phase speeds. The modes are stable or unstable simultaneously.

Under the condition

$$\frac{4C_{p\infty}c_\infty T_0}{\gamma_\infty - 2}|\alpha_\infty| < |Q|, \tag{67}$$

the correction coefficient G_∞^i dominates. For $Q > 0$, velocity, pressure and temperature perturbations increase and density perturbation decreases.

For $M_\infty \gg 1$, equation (60) is reduced to

$$G_{r\infty}^i = R\frac{3\gamma_\infty - 2}{4C_{p\infty}c_\infty M_\infty\tau_Q}. \tag{68}$$

Since the linear analysis is not applicable to hypersonic flows ($M >\sim 5$) , the limiting cases (65), (68) of general formulae (56), (57), (60) gives only qualitative notion about the tendency of change of acoustical instability increments in the inhomogeneous supersonic flow as compared with the subsonic flow.

4. Conclusion

In this paper, we obtained linear acoustical equations describing propagation of the acoustical perturbations in inhomogeneous heat-emitted gas flows. These equations have different forms for the perturbations of velocity, pressure, density and temperature. We considered the conditions of acoustical instability of heat-emitted gas flow in dependence on the Mach number. The acoustical instability takes place due to Rayleigh's mechanism and is connected with the positive feedback between acoustic perturbations and heat release. The inhomogeneity in the flow changes the acoustic increment significantly. Inhomogeneous correction coefficients are different for the perturbations of velocity, pressure, density and temperature. For subsonic flows, they depend on the acoustical mode direction. Correction coefficients are more significant for the low-Mach flow. For the heat emitted gas flow with $M_\infty \ll 1$ and condition (63), the velocity and temperature perturbations grow along the flow and the pressure and density perturbations grow in the opposite direction. For the heat emitted gas flow with $M_\infty \gg 1$ and condition (67), the velocity, pressure and temperature perturbations increase and the density perturbation decreases. Relative perturbations $\tilde{v}/c_\infty$, $\tilde{p}/p_0$, $\tilde{\rho}/\rho_0$, $\tilde{T}/T_0$ are described by the single equation and have the common increment.

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