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Simulation of the 3D Sound Pressure Level Inside Closed Absorbing Acoustic Rooms Bounded by Non-Parallel Floor and Ceiling Surfaces, and Parallel Sidewalls

A. Tadeu¹⁾, I. Simões¹⁾, J. António¹⁾, L. Sousa²⁾

¹⁾ CICC, Department of Civil Engineering, Faculty of Sciences and Technology, University of Coimbra, Rua Luís Reis Santos - Pólo II da Universidade, 3030-788 Coimbra, Portugal. tadeu@dec.uc.pt

²⁾ ITeCons, Rua Pedro Hispano, Pólo II da Universidade, 3030-289 Coimbra, Portugal

Summary

A model for the simulation of three-dimensional (3D) sound pressure level generated by acoustic sources inside closed rooms bounded by non-parallel floor and ceiling surfaces is presented. The 3D problem is solved as a sequence of two-dimensional (2D) problems with different spatial wavenumbers k_z , after a spatial Fourier transform has been applied along the z direction, which is the width of the room. The problem uses a boundary element method (BEM) formulation in the frequency domain. The use of analytically appropriate Green's functions for wedges, defined by means of an image model technique, allows the required BEM discretization to be limited to the back and front walls of the room and avoids the discretization of the other flat surfaces. The computations are first performed in the frequency domain and inverse Fourier transforms are subsequently applied to obtain time responses. Complex frequencies are used to avoid the time-aliasing phenomenon. This effect is later taken into account by re-scaling the response in the time domain. The model allows frequency dependent absorption coefficients to be defined for the various walls. In the case of the ceiling, floor and lateral room surfaces the absorption is introduced through the direct manipulation of the analytical Green's functions. The absorption of the back and front walls is simulated using impedance boundary conditions. Once the algorithm has been verified against a 3D BEM model, the potential of the proposed procedures is illustrated by simulating the propagation of waves generated by a point source inside an auditorium.

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1. Introduction

The simulation of sound wave propagation inside enclosed spaces has been an area of interest in the field of room acoustics. Analytical solutions to simulate acoustic wave propagation are only known for very simple geometries. Two basically different numerical approaches have been used to handle this problem, these being geometrical and wave-based methods.

The most popular models are based on geometrical acoustics that describe the sound propagation along sound rays, following the principles of ray optics and computer graphics. The ray tracing method [1, 2], the image source method [3, 4], beam tracing [5] and acoustic radiosity [6, 7] are some of these methods. One review of them can be found in [8]. Some of these models are also analyzed in the work of Rindel [9] on the use of computer modeling in room acoustics.

The earlier geometry-based methods have been improved and manipulated mathematically over the years and they have often been combined with one another to generate hybrid methods. An optimized image method for calculating the impulse response of a box-shaped room that requires fewer mathematical operations has been proposed by McGovern [10]. A radiative transfer model has expanded the classical geometrical room acoustic modeling algorithms by incorporating a propagation medium that absorbs and scatters radiation and can thus handle both diffuse and non-diffuse reflections on boundaries and objects in the room [11]. Korany *et al.* [12] has proposed a method to model the impulse response for rooms with partially specularly reflecting walls that combines an extended radiosity method and conventional methods based on geometrical acoustics (ray tracing and image methods). With this algorithm it is possible to handle not only specular-specular and diffuse-diffuse reflections, but also diffuse-specular and specular-diffuse ones. An optimized beam tracing algorithm for finding and updating specular

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reflection paths for a moving listener has been proposed by Laine [5].

Siltanen *et al.* [13] derived an integral equation that generalizes a variety of known geometrical room acoustics modeling algorithms. Based on this equation, an acoustic radiance transfer method has been formulated that can handle both diffuse and non-diffuse reflections.

To overcome the deficiency of the ray tracing method regarding the diffraction effect, Stephenson [14] has proposed an energetic approach that preserves the algorithm but introduces diffraction by means of a sound particle diffraction model. The effect of sound ray detection on the accuracy of the ray-tracing algorithms has been discussed by Z. Xiangyang *et al.* [15]. They proposed a new method to determine the optimal size of the receiver since unsuitable receivers will lead to multiple detection, diminished detection or wrong detection and will affect the accuracy of the calculation of room impulse response and other acoustic parameters.

Wave based methods have been proposed and implemented using the finite element method (FEM), the finite difference method (FDM) and the boundary element method (BEM), to simulate wave related phenomena. However, they are more suitable for handling lower frequencies of excitation due to the need for the discretization of the domains. The full discretization of the domain required by the FEM and FDM leads to the definition of very fine meshes which results in a huge number of equations to solve problems at high frequencies.

Kowalczyk and Walstijn [16] proposed a finite-difference time-domain (FDTD) technique for simulating wave propagation inside the room and the reflections at the boundaries. The simulation of low frequency sound fields in small rooms using the FEM has been performed by Aretz [17]. He used a simple scale model room with well defined boundary conditions to compare the results with measured ones and achieved a good agreement between them. He [18] also investigated the influence of different boundary representations of porous absorbers on the simulated sound field in small rooms by using FEM simulations of the sound field in a scale model room and comparing them with measured results.

The boundary element method (BEM) has long been considered a very promising technique in acoustics, since it allows the simulation of fields in unbounded domains and automatically satisfies the radiation condition at infinity [19]. Many references can be found related to its use in the time and frequency domains to solve acoustic problems (see, for example, [20, 21, 22, 23, 24, 25, 26, 27]). The BEM, is a more efficient method in terms of computer cost as it only requires the discretization of the boundaries [20]. However as the number of elements increases with the frequency, due to the imposition at each frequency of a relation between the wavelength and the length of the boundary elements, the calculation at high frequencies can be unaffordable. A fast solution method for boundary integral equations has been proposed for use in large-scale engineering problems. It combines the effect of sources a

long way away from a collocation point in a far-field term using the multipole expansion, whereas for nearby sources the conventional BEM evaluations are used. The fast multipole boundary element method (FMBEM) based on the improved Burton–Miller formulation was proposed by S. Li [28] for solving large-scale 3D acoustic problems. The method has also been addressed by Sakuma [29, 30] for solving large-scale steady-state sound fields.

More recently, meshless methods have been used to model acoustic problems, including pressure fields inside rooms. António *et al.* developed a method of fundamental solutions (MFS) algorithm for an acoustic half-space in order to model wave propagation inside a room with a dome configuration [31]. But for this method the accuracy of the results depends on the distance between the fictitious sources and boundaries and on the number of nodal points and sources. The appropriate choice of these two parameters ensures reliable results.

The solution for a conventional BEM becomes simpler in two- and-a-half-dimensional spaces (or 2-1/2-D for short), where the medium is two-dimensional (2D), but the dynamic source is three-dimensional (3D), such as a point source. 3D results can be achieved by using a spatial Fourier transform in the direction in which the geometry does not vary, which allows the solution to be obtained as a summation of 2D responses for a continuous variation of spatial wavenumbers [31]. The computational cost can be reduced further if appropriate Greens functions or fundamental solutions are used to satisfy the boundary conditions at some surfaces and so avoid the discretization of those surfaces by boundary elements.

This paper presents an approach to determining the (3D) sound pressure level generated by acoustic sources inside closed rooms that are bounded by non-parallel floor and ceiling surfaces. The BEM is used in the frequency domain. The 3D problem is solved as a sequence of two-dimensional (2D) problems with different spatial wavenumbers k_z , after a spatial Fourier transform has been applied along the z direction, which is the width of the room. When applying an inverse spatial Fourier transform to the 2D results to synthesize the 3D wavefield, the lateral walls are taken into account by using virtual sources placed so as to ensure the required boundary conditions.

The use of appropriate Green's functions for wedges, defined by means of an image model technique [24, 32], allows the required BEM discretization to be limited to the back and front walls of the room and to a vertical surface coupling two subdomains, avoiding the discretization of the other flat surfaces. A compact mathematical description of these Green's functions is given in this work. All image sources are placed over a circle and only the visible sources are taken into account.

Frequency-dependent absorption coefficients can be ascribed to the various room surfaces. In the case of the ceiling, floor and lateral room surfaces the absorption is introduced through the direct manipulation of the analytical Green's functions. The absorption of the back and front walls is simulated using impedance boundary conditions.

The frequency domain responses are inverse Fourier transformed in order to obtain time domain pressure field variations. Complex frequencies are used to avoid the time-aliasing phenomenon, with this effect later taken into account by re-scaling the response in the time domain.

The main original contribution of the paper is its use of a compact mathematical description of the Green's functions for wedges to enable the acoustic behaviour of closed rooms bounded by non-parallel floor and ceiling surfaces, such as auditoriums, to be simulated using an efficient BEM model that does not require the discretization of the floor and ceiling surfaces. The proposed model further allows the 3D solution within such 3D closed rooms to be obtained by solving a sequence of two-dimensional (2D) problems with different spatial wavenumbers k_z , which is often called a 2.5D method. The numerical model is formulated in the frequency domain which allows the use of frequency-dependent properties and the simulation of different wavelet source types.

This paper is organized as follows. The problem is defined in section 2, where the boundary element method formulation, the proposed Green's functions and the method used to simulate the sound absorption are presented. The verification of the proposed algorithm is described in section 3. Section 4 defines the procedure for obtaining the time domain responses. Numerical examples demonstrating the applicability and efficiency of the BEM algorithm for the simulation of a three-dimensional pressure wavefield inside a three-dimensional auditorium are presented in section 5. Section 6 states some concluding remarks.

2. Problem definition

Consider a closed room bounded by non-parallel floor and ceiling surfaces (see Figure 1a). A harmonic point pressure source, at position (x_0, y_0, z_0) , oscillating with a frequency ω , illuminates this space, creating the propagation of three-dimensional acoustic waves, that travel with constant velocity c . The back and front walls, the ceiling and the floor are assumed to be parallel to the z axis. The lateral walls are perpendicular to the z axis. All surfaces may exhibit sound absorbent properties.

The resulting incident pressure field at the receiver placed at (x, y, z) is given by

$$\hat{p}_{\text{inc}} = \frac{1}{r_1} e^{i \frac{\omega}{c} (ct - r_1)}, \quad (1)$$

in which the subscript inc denotes the incident field, $r_1 = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$ is the distance from the source and $i = \sqrt{-1}$. In the absence of the lateral walls, which are perpendicular to the z axis, the geometry of the problem is constant along one direction (z), the incident field can be computed as a discrete summation of two-dimensional pressure fields given by

$$p_{\text{inc}}(\omega, x, y, k_z) = \frac{-i}{2} H_0(k_c r_{01}), \quad (2)$$

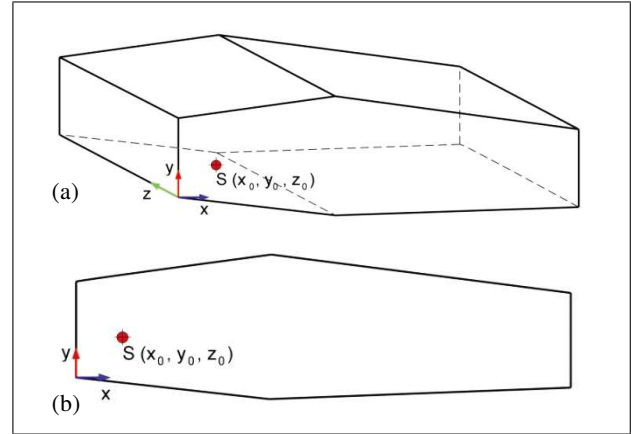


Figure 1. Definition of the problem: a) 3D geometry; b) Longitudinal cross-section.

where $r_{01} = \sqrt{(x - x_0)^2 + (y - y_0)^2}$, $k_c = \sqrt{\omega^2/c^2 - k_z^2}$ with $\Im m\{k_c\} < 0$, $k_z = 2\pi m/L_z$, ($m = 0, -1, 1, -2, 2, \dots, -M, M$) being the spatial wavenumber, obtained after Fourier-transforming equation (1) in the z direction, L_z is the distance between virtual sources placed along the z direction in order to obtain the solution as a discrete summation and $H_n(\cdot)$ are second Hankel functions of order n . The problem is periodic in the z dimension, and the term $e^{-ik_z z}$ is implicit in equation (2).

$\Im m\{k_c\}$ is set to < 0 because the high negative imaginary wavenumber of $k_c = \sqrt{\omega^2/c^2 - k_z^2}$ corresponds to evanescent modes that quickly attenuate with increasing distance from the source. Hence, the summation can safely be truncated above a certain value of $M > (\omega/c)L_z/(2\pi)$, except near the source.

Similarly, the scattered field originated by a point pressure load can be evaluated by solving a sequence of 2D problems, as illustrated in Figure 1b. The lateral walls can be taken into account later by using image sources placed so as to ensure the boundary conditions required (null pressure flows). The first order image sources correspond to the first reflections from the walls, while high order image sources represent multiple reflections on the different walls. The number of virtual sources to be used in the calculations is defined so that the signal responses can be correctly computed in the time frame, which is determined by the frequency increment $2\pi/\Delta\omega$. Thus, the contributions of the image sources placed at a distance from the receiver $c/(2\pi/\Delta\omega)$ are not taken into account.

2.1. Boundary element method formulation

We first describe how the BEM is formulated to obtain the scattered acoustic pressure wave field, i.e. the pressure in the 2D medium generated by a spatially sinusoidal harmonic line load (2.5D model). The use of Green's functions that satisfy the boundary conditions of the wedge allows the discretization to be limited to the vertical interfaces between wedges (s_2), the back (s_3) and front (s_1) walls (see Figure 2), which required far fewer boundary elements.

The classical boundary integral equation can be derived from the Helmholtz equation in the frequency domain by applying the reciprocity theorem. If the acoustic medium is assumed to be confined by rigid front and back walls the following equations are defined:

a) along the interior domain 1,

$$\begin{aligned} bp(x_0, y_0, k_z, \omega) = & \int_{S_2} q(x, y, \mathbf{n}_{n1}, k_z, \omega) G(x, y, x_0, y_0, k_z, \omega) ds_2 \\ & - \int_{S_1} H(x, y, \mathbf{n}_{n1}, x_0, y_0, k_z, \omega) p(x, y, k_z, \omega) ds_1 \\ & - \int_{S_2} H(x, y, \mathbf{n}_{n1}, x_0, y_0, k_z, \omega) p(x, y, k_z, \omega) ds_2 \\ & + p_{\text{inc}}(x_0, y_0, x_s, y_s, k_z, \omega), \end{aligned} \quad (3)$$

assuming that the source is within this domain.

b) along the interior domain 2,

$$\begin{aligned} bp(x_0, y_0, k_z, \omega) = & \int_{S_2} q(x, y, -\mathbf{n}_{n1}, k_z, \omega) G(x, y, x_0, y_0, k_z, \omega) ds_2 \\ & - \int_{S_2} H(x, y, -\mathbf{n}_{n1}, x_0, y_0, k_z, \omega) p(x, y, k_z, \omega) ds_2 \\ & - \int_{S_3} H(x, y, -\mathbf{n}_{n1}, x_0, y_0, k_z, \omega) p(x, y, k_z, \omega) ds_3, \end{aligned} \quad (4)$$

where G and H represent the Green's functions for the pressure (p) and pressure gradient (q) at a point (x, y) on the boundary S ($S = S_1 + S_2 + S_3$) due to a virtual point pressure source at a collocation point (x_0, y_0) . p_{inc} is the incident pressure field at (x_0, y_0) , generated by a source located in domain 1 at (x_s, y_s) . $\mathbf{n}_{n1}$ represents the unit outward normal along the boundary S , at (x, y) , defined by the vector $\mathbf{n}_{n1} = (\cos \theta_{n1}, \sin \theta_{n1})$. The factor b takes the value $\frac{1}{2}$ if $(x_0, y_0) \in S$ and S is smooth.

When the back wall is sound absorbing the system can be formulated by prescribing boundary conditions that relate the pressure to the velocity at each collocation point, leading to the following equation that replaces equation (4),

$$\begin{aligned} bp(x_0, y_0, k_z, \omega) = & \int_{S_2} q(x, y, -\mathbf{n}_{n1}, k_z, \omega) G(x, y, x_0, y_0, k_z, \omega) ds_2 \\ & - \int_{S_2} H(x, y, -\mathbf{n}_{n1}, x_0, y_0, k_z, \omega) p(x, y, k_z, \omega) ds_2 \\ & - \int_{S_3} q(x, y, \mathbf{n}_{n1}, k_z, \omega) \left[G(x, y, x_0, y_0, k_z, \omega) \right. \\ & \quad \left. + \frac{Z(\omega)}{i\rho\omega} H(x, y, \mathbf{n}_{n1}, x_0, y_0, k_z, \omega) \right] ds_3. \end{aligned} \quad (5)$$

This can be regarded as a Robin or impedance boundary condition, that is

$$q(x, y, \mathbf{n}_{n1}, k_z, \omega) = -i\omega\rho \frac{1}{Z(\omega)} p(x, y, k_z, \omega). \quad (6)$$

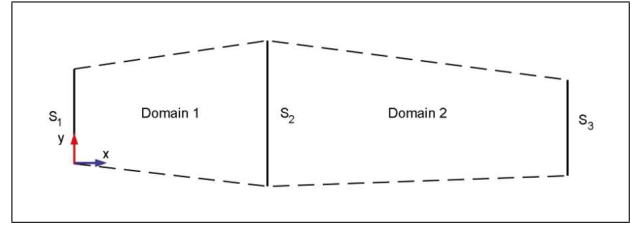


Figure 2. Geometry of the 2.5D BEM model.

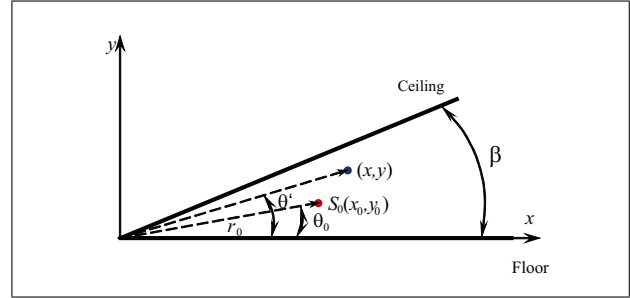


Figure 3. Wedge geometry.

For our purposes the impedance $Z(\omega)$ is expressed by the ratio between the pressure and the velocity and the absorption coefficient α ,

$$Z(\omega) = \frac{p_{\text{inc}}(x_0, y_0, x_s, y_s, k_z, \omega)}{v_{\text{inc}}(x_0, y_0, x_s, y_s, \mathbf{n}_{n1}, k_z, \omega)} \frac{1 + \sqrt{1 - \alpha}}{1 - \sqrt{1 - \alpha}}, \quad (7)$$

when

$$\begin{aligned} p_{\text{inc}}(x_0, y_0, x_s, y_s, k_z, \omega) &= \frac{-i}{2} H_0(k_c r_{00}) e^{-ik_z z}, \\ v_{\text{inc}}(x_0, y_0, x_s, y_s, \mathbf{n}_{n1}, k_z, \omega) &= \frac{i}{2} k_c H_1(k_c r_{00}) e^{-ik_z z} \frac{dr_{00}}{dn_{n1}}, \end{aligned}$$

with $r_{00} = \sqrt{(x_0 - x_s)^2 + (y_0 - y_s)^2}$, are the incident pressure and velocity respectively.

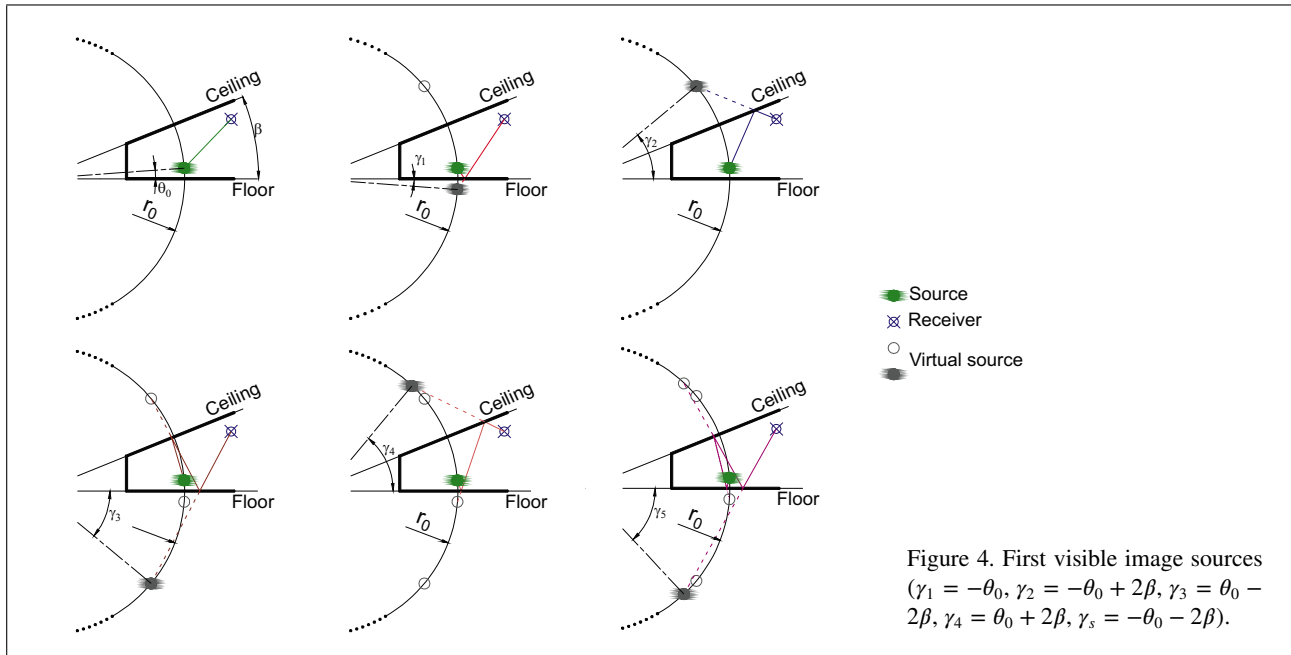
Sound absorption in the front wall is simulated similarly, by modifying equation (3) (not illustrated for the sake of brevity).

The final system of equations is assembled assuring the continuity of pressures and pressure gradients along S_2 . The global solution is found by solving equations (3 and 4) or (3 and 5). This requires the discretization of the interface $S = S_1 + S_2 + S_3$. In this analysis the interface is discretized with N straight boundary elements, with one nodal point in the middle of each element. The unknown nodal pressures and pressure gradients are obtained by solving this system of equations, allowing the pressure field along the domain to be defined.

The required integrations in equations (3–4) are evaluated using a Gaussian quadrature scheme when they are not performed along the loaded element. For the loaded element, the existing singular integrands of the Green's functions are calculated analytically, following the expressions in Tadeu *et al.* [33].

2.2. Green's functions for wedges

To avoid the discretization of the non-parallel rigid floor and ceiling the Green's functions and the incident pres-



sure need to be expressed in a way that satisfy null normal velocities at those boundaries. This can be accomplished by adding the pressure field generated by the real source to that produced by virtual sources (image sources), which are located in such a way that they act as mirrors of the real source in relation to the boundaries.

Consider a wedge with an opening angle β , in which the two planes of the wedge represent the combination of floor and ceiling of the room (see Figure 3).

For simplicity the origin of the Cartesian coordinate system is placed at its corner. When the source is placed at point (x_0, y_0) the response at the receiver placed at (x, y) is given by

$$G(x, y, x_0, y_0, k_z, \omega) = \quad (8)$$

$$\frac{-i}{4} H_0(k_c r_{01}) - \frac{i}{4} \sum_{n=1}^{NS} \left[\sum_{j=1}^4 H_0(k_c r_j) \right],$$

$$H(x, y, \mathbf{n}_{n1}, x_0, y_0, k_z, \omega) = \quad (9)$$

$$\frac{i}{4} k_c H_1(k_c r_{01}) \frac{\partial r_{01}}{\partial n_{n1}} + \frac{i}{4} k_c \sum_{n=1}^{NS} \left[\sum_{j=1}^4 H_1(k_c r_j) \frac{\partial r_j}{\partial n_{n1}} \right],$$

where $r_j = \sqrt{(x - r_0 \cos \gamma_j)^2 + (y - r_0 \sin \gamma_j)^2}$, with $\gamma_1 = -\theta_0 - 2\beta(n-1)$, $\gamma_2 = -\theta_0 + 2\beta n$, $\gamma_3 = \theta_0 - 2\beta n$, $\gamma_4 = \theta_0 + 2\beta n$, θ_0 is the azimuth distance of the source.

NS is associated with the number of sources generating the responses that reach the receivers within the time interval fixed by the frequency increment in the calculation. The number of image sources considered is finite because only the observable images or audible image sources are taken into account [8]. Those surfaces can also be simulated as absorbent ones. This is accomplished by multiplying the reflected pressure generated by the mirrored image source at those surfaces by a reflection coefficient $\sqrt{1 - \alpha}$, calculated from the sound absorption coefficient α .

Figure 4 illustrates how the virtual sources are defined. The two planes of the wedge represent the floor and ceiling of the room. All image sources are placed over a circle of radius r_0 . Figure 4 shows the visible image sources. The non-visible virtual sources are not plotted. They would represent image sources whose mirror is the wedge surface but whose line connecting it to the receiver does not cross that mirror surface.

Note that when the wedge opening tends to zero, equations (9-10) become the Green's function for a waveguide with $r_j = \sqrt{(x - x_0)^2 + (y - y_0)^2}$, where, $y_1 = -y_0 - 2h(n-1)$, $y_2 = y_0 - 2hn$, $y_3 = -y_0 + 2hn$, $y_4 = y_0 + 2hn$ and h is the thickness of the waveguide.

2.2.1. Verification of the Green's functions

The proposed Green's functions have been verified by comparing the results with those provided by a BEM model that requires the discretization of both wedge surfaces. Equation (3) was simplified so as to simulate rigid boundaries,

$$bp(x_0, y_0, k_z, \omega) = \quad (10)$$

$$- \int_S H(x, y, \mathbf{n}_{n1}, x_0, y_0, k_z, \omega) p(x, y, k_z, \omega) ds$$

$$+ p_{inc}(x_0, y_0, x_s, y_s, k_z, \omega).$$

In this case the Green's functions used were those for an unbounded medium,

$$H(x, y, \mathbf{n}_{n1}, x_0, y_0, k_z, \omega) = \frac{i}{4} k_c H_1(k_c r_{01}) \frac{\partial r_{01}}{\partial n_{n1}}. \quad (11)$$

Each of these surfaces was modeled with a very large number of boundary elements (1500), along a length long enough to prevent the contamination of the results from the far field radiation (150 m). The boundary elements vary in length so that shorter elements are located in the vicinity

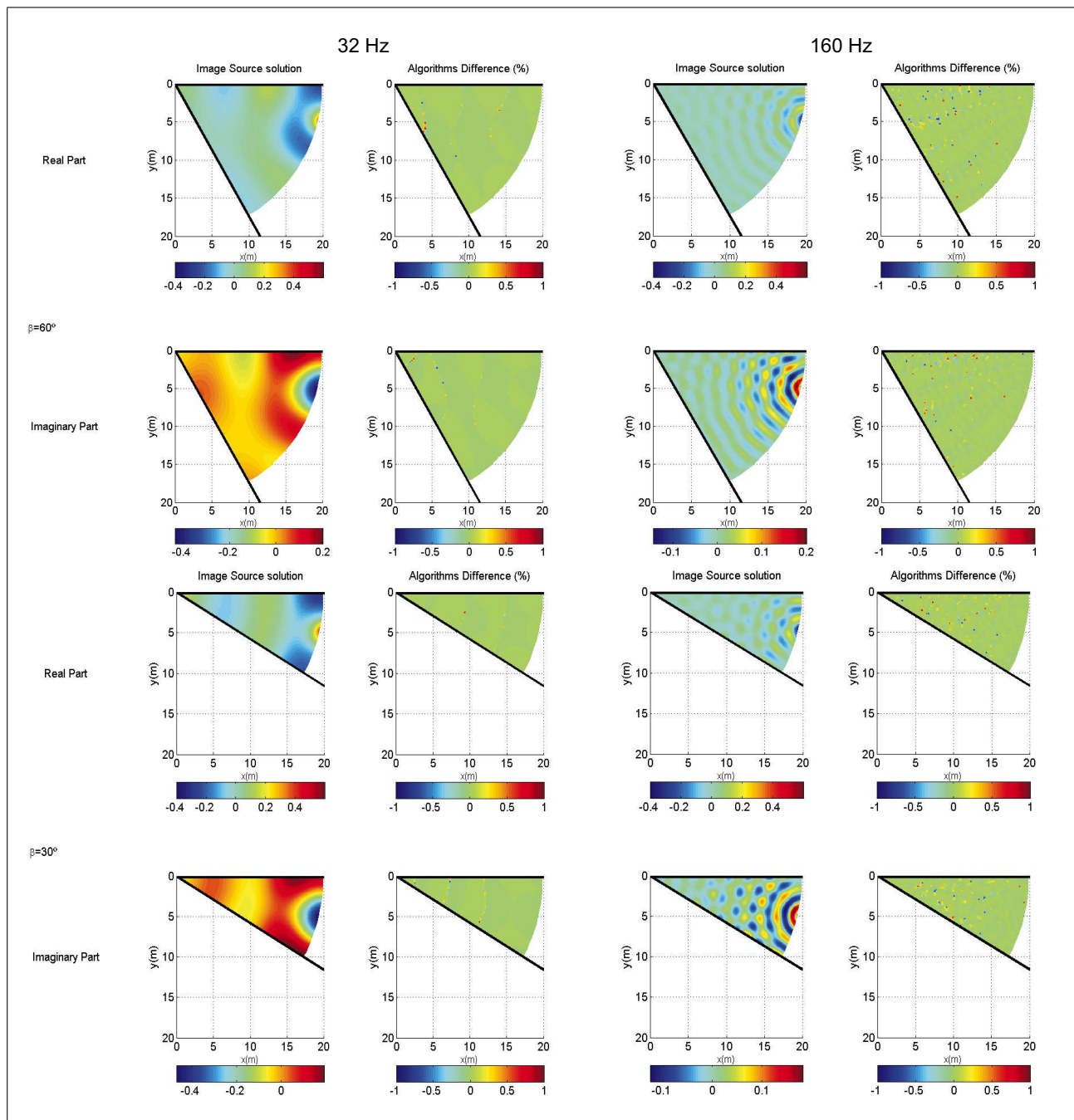


Figure 5. Verification of the Green's functions.

of the wedge corner. To further attenuate the far field radiation emanating from the nodal points placed at the extremities of the wedge, the responses were computed for complex frequencies $\omega_c = \omega - i\eta$ (with $\eta = 0.7\Delta\omega$, and $\Delta\omega = 8.0$ Hz).

Note, that $\eta = 0.7\Delta\omega$ will lead to an attenuation of about $1/8$ at the end of the time window given by $2\pi/\Delta\omega$. During this time window the waves will travel a distance of $340/8 = 42.5$ m. The discretization of the wedge surfaces for a length of 150 m ensures that the contribution of the far field radiation, coming from the farthest nodal sources, is very weak. If η takes a smaller value the wedge surfaces are discretized over a greater length.

Two wedges with opening angles of $\beta = 60^\circ$ and $\beta = 30^\circ$ were modeled. 5 and 11 virtual sources were required to define the Green's function for the two wedges with $\beta = 60^\circ$ and $\beta = 30^\circ$, respectively. The pressure wave velocity is that of the air $c = 340.0$ m/s (density $\rho = 1.22$ kg/m³). Both systems are excited by a harmonic cylindrical pressure line load ($p_{inc} = -i/4 H_0(k_c r_{01})$ with $k_z = 0.0$ rad/m), applied to point O (20.0 m, 5.0 m). The resulting total pressure field is obtained over a two-dimensional grid of receivers arranged along the x and y directions at equal intervals of 0.2 m. Two frequencies were modeled (32.0 Hz and 160.0 Hz). Figure 5 displays the real and the imaginary part of the responses obtained

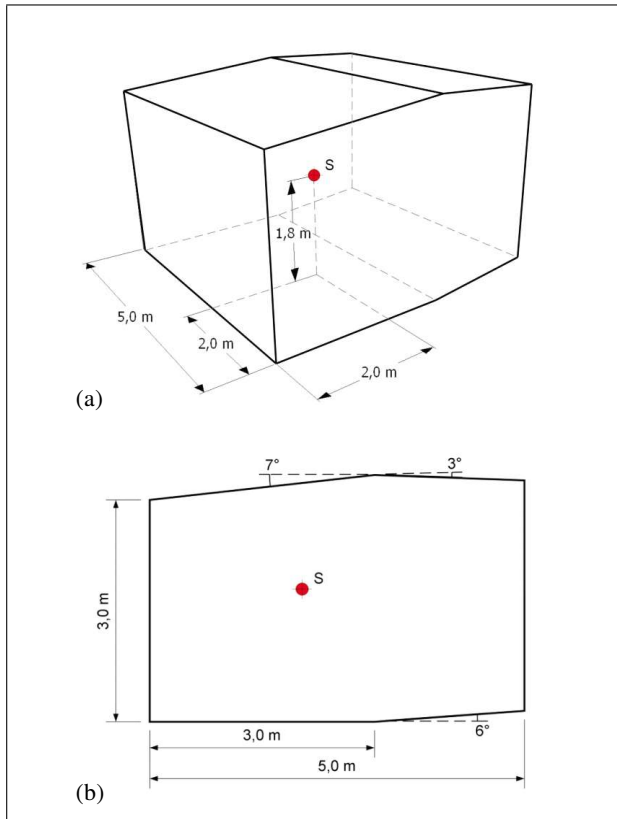


Figure 6. Geometry of the 3D room used to verify the BEM formulation: a) 3D geometry; b) Longitudinal cross-section.

using the proposed Green's functions. This Figure also presents the difference between these responses and those obtained by using the BEM model. These differences are plotted, normalized in relation to the value given by the Green's functions, at each point. Given the type of normalization, the higher values are associated with receivers that register small response amplitudes. Analysis of the plots shows that the two responses agree, exhibiting very small differences of less than 1.0%. Additional verifications for different frequencies and wedge openings were performed and it was found that the proposed Green's functions are appropriate for modeling wedges.

Note, fewer virtual sources are required to define the Green's function as the opening angle increases. For $\beta = 90^\circ$ only 3 virtual sources are required.

2.3. Simulation of the lateral walls

The lateral walls, perpendicular to the z axis, are simulated using the image source technique. In their absence, the geometry of the stated problem does not vary along the longitudinal z direction. Thus, the 3D incident field can be formulated as a summation of 2D solutions after applying a Fourier transformation along that direction. Therefore, the solutions are first obtained for the 2D cross-section for varying axial wavenumbers, as explained above. Then the lateral walls are simulated by applying an image source technique and the 3D solution is calculated by applying an

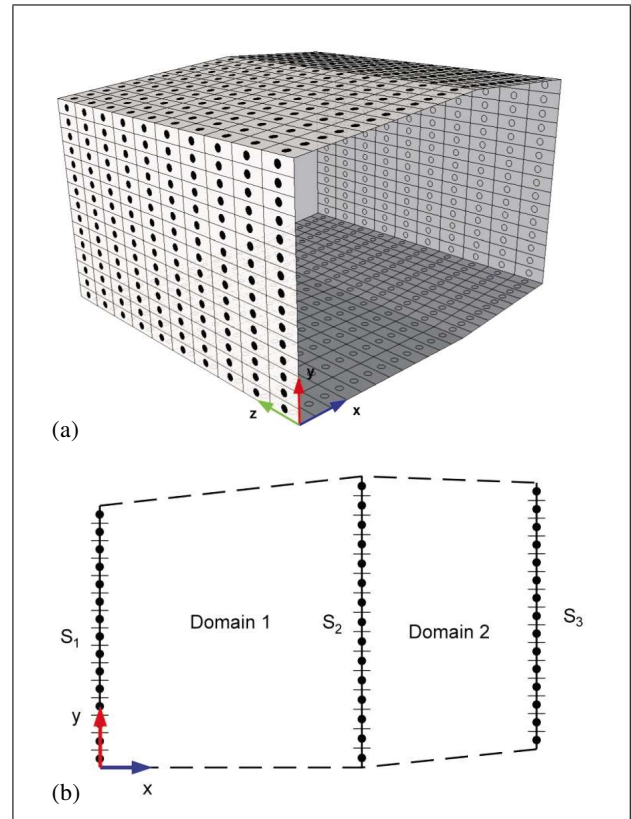


Figure 7. Boundary element discretization: a) 3D BEM; b) 2.5D BEM.

inverse spatial Fourier Transform (equation 12),

$$p(x, y, z, \omega) = \frac{2\pi}{L_z} \sum_{m=-M}^{m=+M} p(x, y, k_{zm}, \omega) e^{-ik_{zm}(z-z_s)} \quad (12)$$

$$+ \frac{2\pi}{L_z} \sum_{m=-M}^{m=+M} \sum_{n=1}^{NS} \left[\sum_{j=1}^4 p(x, y, k_{zm}, \omega) e^{-ik_{zm}(z-z_j)} \right],$$

with $z_1 = -z_s - 2d(n-1)$, $z_2 = z_s - 2dn$, $z_3 = -z_s + 2dn$, $z_4 = z_0 + 2dn$, d is the width of the room, and $k_{zm} = 2\pi m/L_z$.

3. Algorithm Verification

This section describes the corroboration of the proposed BEM algorithm results with those given by a 3D BEM, using Green's functions for an unbounded medium. Given the computation time and memory storage required, a smaller 3D closed room, bounded by non-parallel floor and ceiling surfaces and parallel sidewalls, is used to verify the use of the proposed 2.5D BEM formulation. The lateral sidewalls in both models are simulated using the image source technique described above. All room surfaces are assumed to be rigid. The proposed boundary element formulation is verified by comparing its numerical results with a 3D BEM solutions that incorporate Green's functions for an unbounded medium, which requires the full discretization of the boundary. Consider the 3D closed

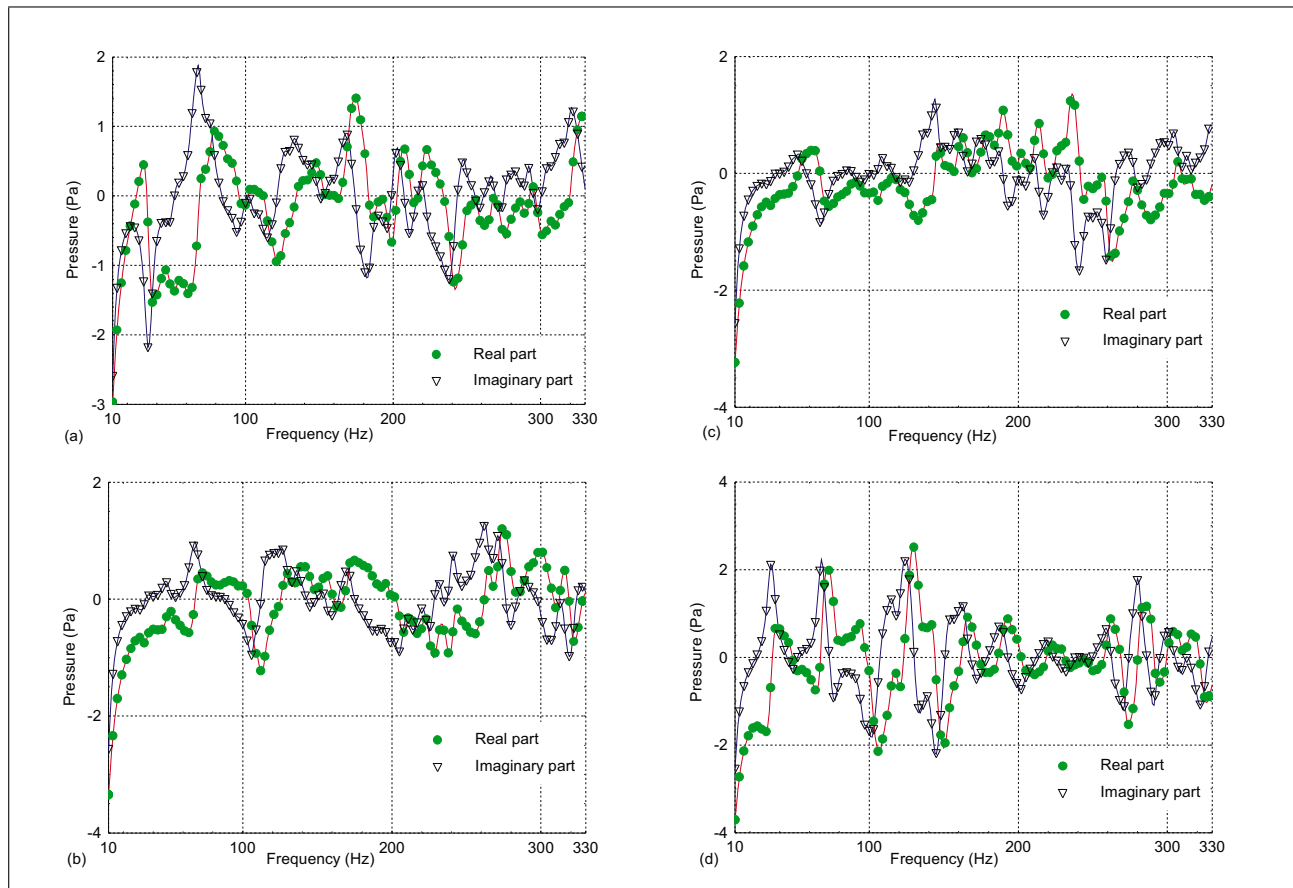


Figure 8. 2.5D BEM algorithm validation: a) response at Receiver 1; b) response at Receiver 2; c) response at Receiver 3; d) response at Receiver 4.

room with the geometry shown in Figure 6. This room is subjected to a harmonic point pressure source S at (2.0 m, 1.8 m, 2.0 m). The pressure wave field generated is evaluated in four receivers placed at (1.0 m, 1.0 m, 0.5 m), (4.0 m, 2.0 m, 1.0 m), (3.5 m, 1.0 m, 1.5 m) and (4.5 m, 1.5 m, 4.5 m). The host medium is air, with the same properties used above.

Computations were performed in the frequency range from [10.0 Hz, 330 Hz]. The responses were computed for complex frequencies $\omega_c = \omega - i \cdot 0.7 \cdot 5.0 \cdot 2\pi$. L_z is assumed to be 800.0 m. Given that $\eta = 0.7 \cdot 5.0 \cdot 2\pi$ the number of virtual sources in the z direction used in both models has been defined so as to guarantee that the most distant source is at least at $2c/\Delta f = 136.0$ m in order to make a weak contribution to the response. In the present example this distance has been set to 165 m, which required the use of 64 virtual sources. A wider room would require fewer virtual sources.

In each BEM formulation, the number of boundary elements is defined by the relation between the wavelength λ and the length of the boundary elements L , which is set to 10. Only the vertical interfaces between wedges, the back and front walls, need to be discretized using the proposed BEM formulation, which requires far fewer boundary elements than the 3D BEM that uses Green's functions for a full 3D space, $G(x, y, z, x_0, y_0, z_0, \omega) = e^{-i\omega r_1/c} / (4\pi r_1)$ [34].

Figure 7 illustrates the discretization of the boundary surface used to simulate the lower frequencies: the minimum number of boundary elements was 740 for the 3D BEM and 45 for the 2.5D BEM. 49 virtual sources were used to define the 2.5D BEM Green's functions for the left (domain 1) and 39 for right (domain 2) wedges.

The pressure obtained by the two BEM formulations are compared in Figures 8a to 8d. Both the real and imaginary parts of the responses can be seen, with the proposed BEM solution represented by solid lines, while the marked points correspond to the 3D BEM model. The two results are in very good agreement.

The computational efficiency of the proposed formulation was evaluated by determining the CPU time taken to compute the above solution for the 2.5D BEM and the 3D BEM. All solutions were computed on a cluster of DELL computers with Intel(R) Xeon(R) CPU E5620 @ 2.4GHz processors. Figure 9a shows the number of boundary elements required by each BEM model. We can see that far more boundary elements are needed by the 3D model than by the 2.5D model, which only requires the discretization of three lines. Figure 9b gives the CPU time for each formulation, for the full range of frequencies computed. The CPU time is displayed on a logarithmic scale for easier interpretation of the results. The results show that the proposed 2.5D BEM requires much less CPU time. This difference would be enhanced for wider or/and longer rooms.

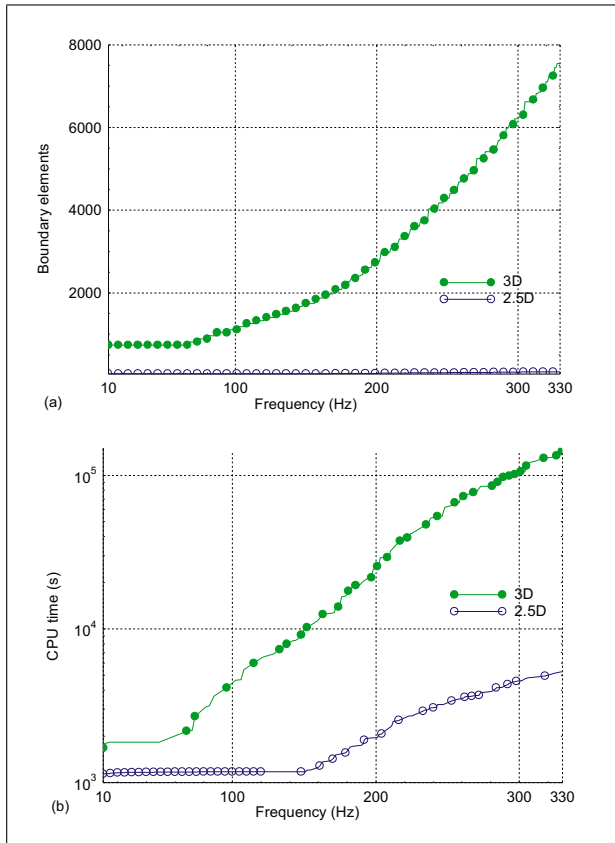


Figure 9. Number of boundary elements (a) and CPU time (b) for the 3D BEM and 2.5D BEM models.

The memory storage and CPU time required by the 3D BEM model would limit its applicability to small rooms and very low frequencies.

The proposed 2.5D BEM model would be more efficient for rooms with larger wedge opening angles since fewer virtual sources would be needed to define the Green's functions than for rooms with smaller wedge opening angles. But it is only applicable to rooms whose sidewalls are perpendicular to the z axis and whose geometries do not change in that direction, that is, they have a 2D geometry.

4. Transient responses

The pressure responses in the time domain are obtained by means of an inverse fast Fourier transform in the frequency domain. In order to prevent the aliasing phenomena, complex frequencies with a small imaginary part of the form $\omega_c = \omega - i\eta$ (with $\eta = 0.7\Delta\omega$, and $\Delta\omega$ being the frequency step) are used in the computation procedure. The constant η cannot be made arbitrarily large, since this leads either to a severe loss of numerical accuracy, or to underflows and overflows in the evaluation of the exponential windows. This procedure is later taken into account by rescaling the responses in the time domain with an exponential factor $e^{\eta t}$.

A Ricker pulse is used as the time variation of the excitation source. The temporal variation of the Ricker pulse is given by

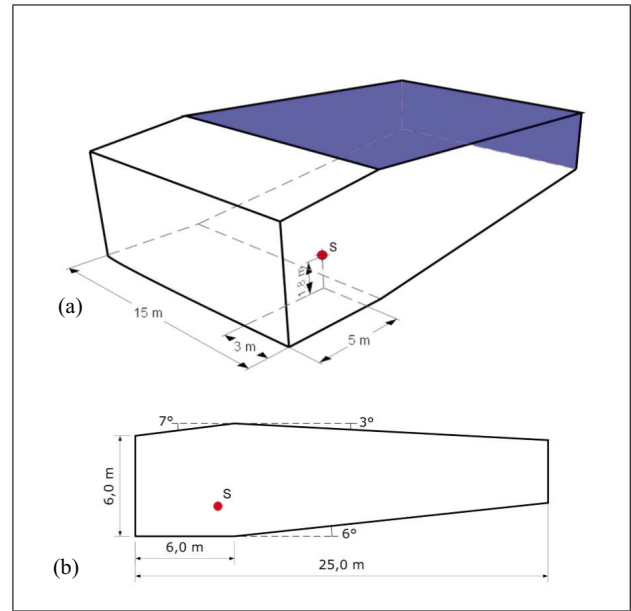


Figure 10. Geometry of the auditorium: a) three-dimensional view; b) longitudinal cross-section.

$$u(\tau) = A(1 - 2\tau^2)e^{-\tau^2}, \quad (13)$$

where A represents the amplitude; $\tau = (t - t_s)/t_0$, t corresponds to the time, t_s when the wavelet attains its highest value, and πt_0 to the characteristic (dominant) period of the Ricker wavelet. Applying a Fourier transform to equation (13), we get

$$U(\omega) = A[2t_0\sqrt{\pi}e^{-i\omega t_s}]\Omega^2 e^{-\Omega^2}, \quad (14)$$

where $\Omega = \omega t_0/2$.

5. Case study

The pressure wave propagation in a three-dimensional auditorium is simulated by means of the boundary element formulation proposed and verified above. The geometry is shown in Figure 10. The room has a width of 15.0 m. This host medium is excited by a 3D harmonic point load located at point S (5.0 m, 1.8 m, 30 m).

As above, only the vertical interfaces between wedges, the back and front walls, need to be discretized to obtain the two-dimensional solution. As above, the number of boundary elements is defined by the relation between the wavelength (λ) and the length of the boundary elements (L), which is set to 10. The minimum number of boundary elements used for discretizing each surface was 30, at an initial frequency.

Frequency results were computed in the range 8.0 Hz to 1024.0 Hz with time responses obtained by applying an inverse (fast) Fourier transform in ω , using a Ricker pulse with a characteristic frequency of 350.0 Hz for the source time evolution. A frequency increment of 8.0 Hz specifies a total time window of 0.125 s.

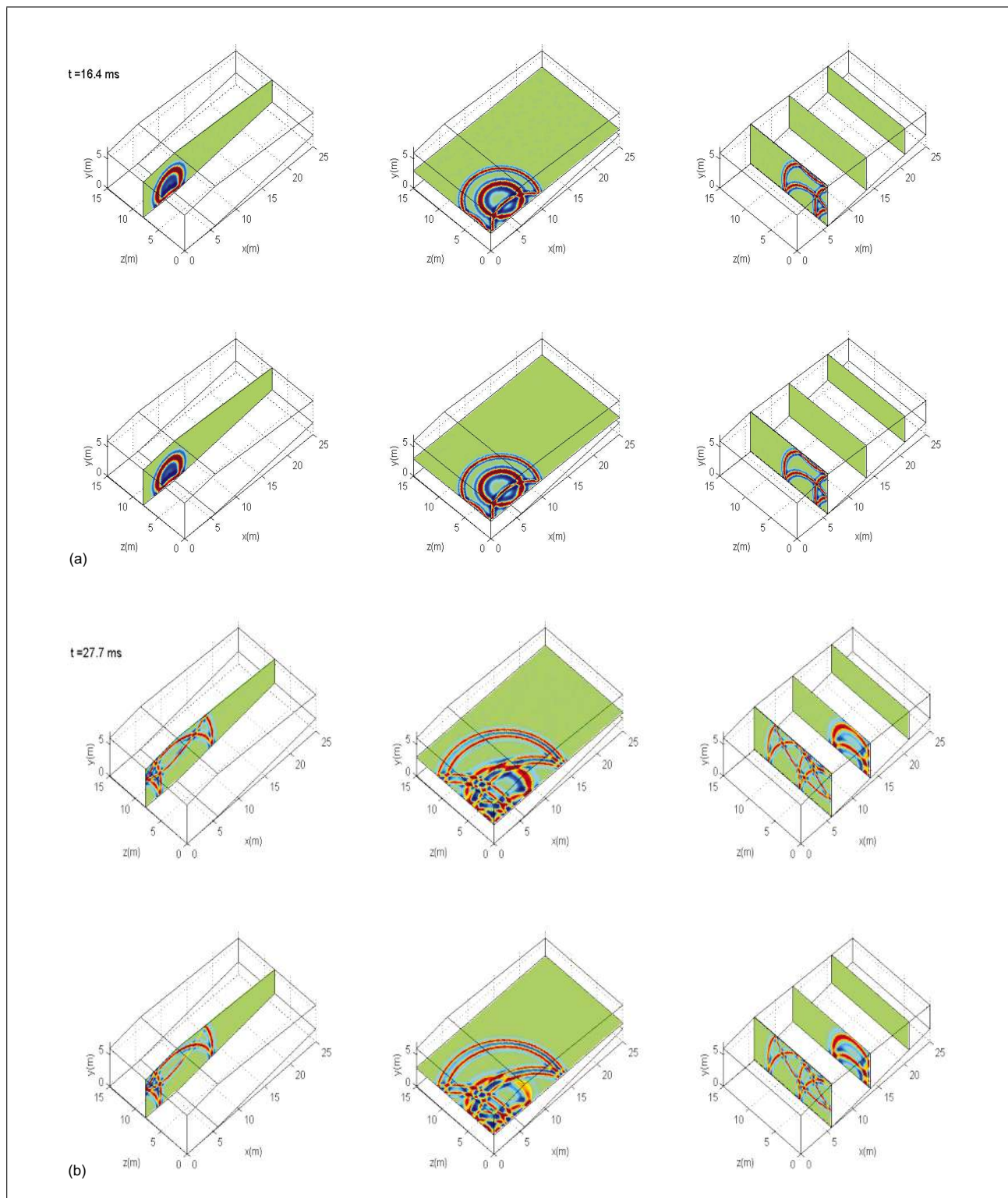


Figure 11. Snapshots of responses at $t = 16.4$ ms (a) and $t = 27.7$ ms (b).

Time results in terms of pressures were determined in fine grids of receivers placed over five planes corresponding to $x = 5.925$ m, $x = 13.425$ m, $x = 20.925$ m, $y = 2.925$ m and $z = 8.0$ m (Figure 11). The receivers were spaced at equal intervals of 0.15 m along the x , y and z directions.

The effect of the absorption in the acoustic wavefield is illustrated by assuming that the back wall and ceiling of the room have a constant sound absorption coefficient at all frequencies equal to $\alpha = 0.8$ (although the model allows the absorption coefficients to vary with the frequency) while the remaining flat boundaries are rigid.

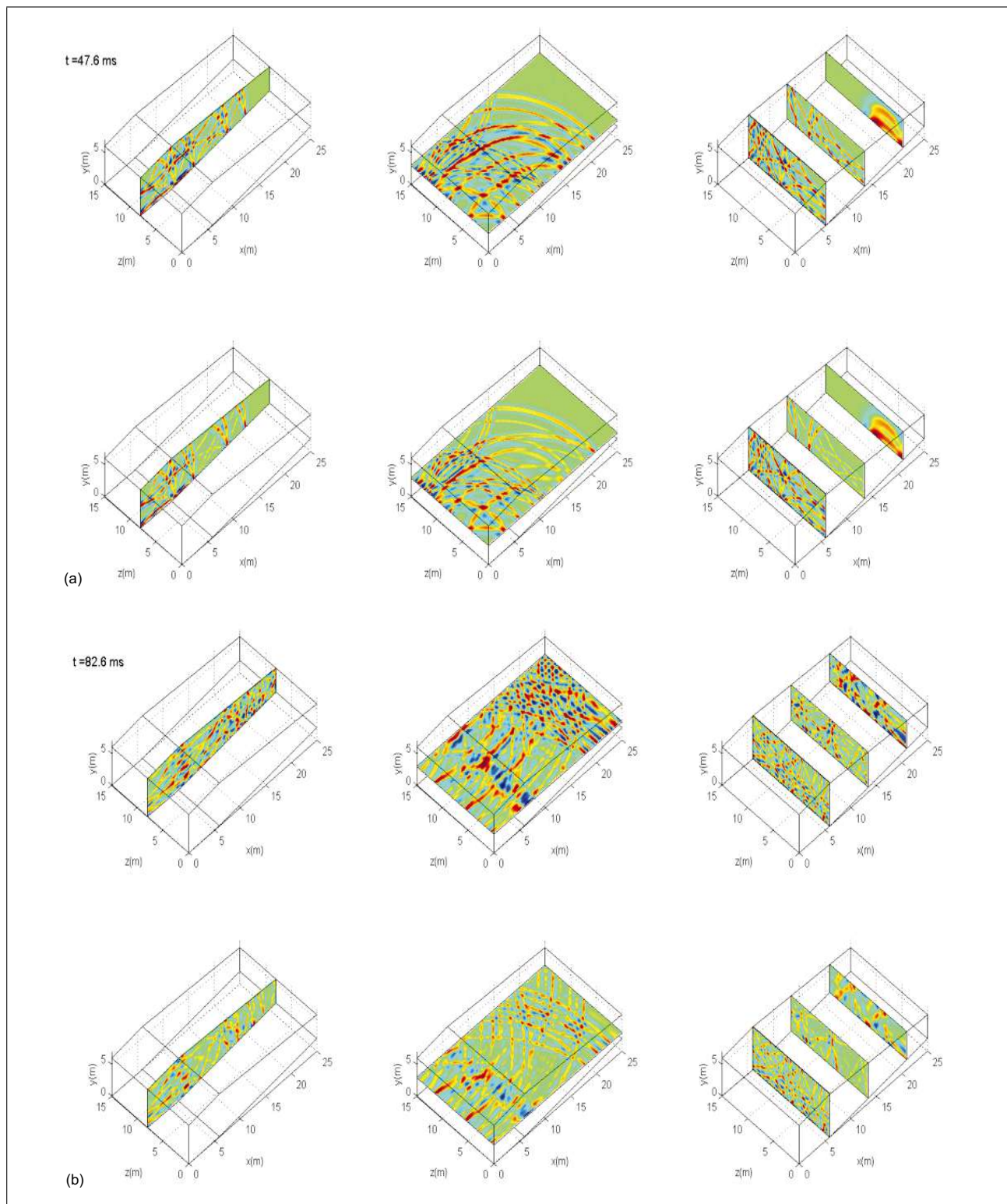


Figure 12. Snapshots of responses at $t = 47.6$ ms (a) and $t = 82.6$ ms (b).

The results are compared with those obtained when all the boundaries are perfectly reflective.

The results are illustrated by a set of 3D snapshots captured at different time instants from computer animations. In these 3D snapshots of pressure, a colour scale is used in which the red and blue shades correspond respectively to higher and lower values of pressure amplitude.

The first row of each plot illustrates the result obtained in the presence of rigid boundaries, while the second row gives the time signatures computed in the presence of an absorbing ceiling and back wall.

The propagation of the incident pulses, diverging away from the source and spreading wave energy, can be observed at the initial time instants. In Figure 11a), at $t =$

16.4 ms the first reflections are clearly evident, coming from the floor, from the right wall and from the ceiling. However, the influence on the sound pressure level of the absorption at the ceiling surface is not yet perceptible, since the absorbent material is only placed from $x = 6.0$ m to the end of the ceiling. When $t = 27.7$ ms the amplitude of the pulse reflected by the ceiling is lower for the absorbing ceiling (see left plots at Figure 11b). Reflections coming from the left wall are evident in Figure 12a ($t = 47.6$ ms). At this instant the incident wave has suffered multiple reflections between the lateral walls, the floor and ceiling. The wave front propagating towards the rear wall is now closer to it. The plots of the second row of Figure 12a ($\alpha = 0.8$) reveal pulses of weak amplitude as a result of the partial absorption of sound energy.

As time elapses the overlapping of multiple reflections from different room surfaces produces a more complex pressure field in the room, in both cases ($\alpha = 0.0$ and $\alpha = 0.8$). Although, the sound amplitudes have decreased when the rear wall and ceiling are sound absorbing (see Figures 11 and 12).

6. Conclusions

A model for simulating the 3D acoustic sound wave propagation inside rooms with non-parallel floor and ceiling surfaces and parallel sidewalls has been presented. A BEM formulation was used to solve the problem for as a sequence of two-dimensional (2D) problems for different spatial wavenumbers across the width of the room. Appropriate Green's functions for wedges, defined by means of an image model technique, were incorporated in this formulation in order to reduce the number of boundary elements needed to discretize the geometry of the problem. Thus the computational cost of modeling the problem is substantially lower. The model allows different frequency-dependent sound absorption coefficients to be ascribed to the boundary surfaces of the room.

The results obtained by the simulations described the physical phenomena involved well and revealed the influence of the sound absorption on the pressure wavefield inside the room. The proposed formulation was found to be a reliable method to predict the acoustics for rooms with similar geometric characteristics, meaning that the wave propagation simulation can be performed at frequencies higher than those allowed by a conventional BEM formulation that incorporates Green's functions for an unbounded medium.

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