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On the Sensitivity of Conjugate Acoustic Wave at Parametric Resonance Conditions to Small Wave Length Variations

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Summary

The magneto-acoustic wave phase conjugation is simulated numerically in a sample of finite length under parametric resonance conditions. The magnetostrictive effect induced in the material by a variable magnetic field is taken into account as a sound velocity modulation under a linear pumping assumption. The present numerical simulations fill up the gaps between the analytical solutions recently discovered for discrete values of the sample length to acoustic wavelength ratio. A high sensitivity of the conjugate wave amplification rate is underlined even for small variations in the above ratio. A technical solution is then proposed in order to limit this effect in practice.

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1. Introduction

The magneto-acoustic wave phase conjugation (WPC) is a nonlinear coupling, inside an active medium sample, between an acoustic or ultrasonic wave and oscillating magnetic field used as an energy source (Figure 1). Energy is transferred from the magnetic field to the acoustic wave through the instantaneous modulation of the elastic properties of the active material by the magnetic field. The mechanism of this transfer is a parametric resonance: an incident acoustic wave of frequency ω is amplified if the frequency of the magnetic field is $\Omega = 2\omega$ and if the energy supplied by the magnetic field exceeds the energy lost by the conjugator by the waves leaving the sample. Over this threshold of energy supply, the amplification becomes exponential until a saturation due to the nonlinearity of the coupling at high energy [1]. This mode, characterized by an energy threshold and an exponential growth rate, is called supercritical. In that case, because of the momentum conservation and since no force is applied to the sample, a time-reversed "conjugate" wave is emitted with the same amplitude and frequency as the incident amplified wave, but traveling in the opposite direction.

A comprehensive presentation of the acoustic wave phase conjugation problem and giant amplification of ultrasound in magnetostrictive materials is given in [2] and more recently in [3]. The applications are numerous in ultrasonic imaging [4, 5, 6, 7], medical diagnostics [8, 9, 10] and fluidics [11, 12]. Specific theoretical approaches address different aspects of the supercritical mode in a one-dimensional approximation of a cylindrical sample [13, 14, 15, 16] or in spherical geometry [17]. The most recent theoretical analysis on the formation of an amplified conjugate acoustic wave by the interaction of an acoustic signal with a finite length sample of magnetostrictive material under parametric resonance conditions (Supercritical WPC) is performed in [18]. In the case of a perfect 1D sample, the authors showed that the amplification rate is very sensitive to the ratio between the sample length and the acoustic wavelength but analytic solutions were obtained only for quarter-integer values of this ratio, the amplification rate being different for the even and odd values. Even if the perfect 1D situation gives a good understanding of the process generating the conjugate wave, this sensitivity to the length of the sample underlines the difficulty of interpretation of the experiments and the necessity to increase fundamental knowledge on WPC. The aim of present paper is to fill the gaps between the above analytic solutions by solving numerically the one-dimensional problem for any real value of the sample length to the wavelength ratio.

The numerical technique used here is of finite-volume type with Riemann solvers adapted to situations where solids and liquids are in contact [19, 20]. This numerical technique had been already successfully used for WPC in one- or multi-dimensional cases [21, 22] and compared with spectral methods [23].

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2. Problem statement

Mathematically, the problem consists in solving the linear wave equation in a space-time domain with a source term caused by variations in the mechanical characteristics of the material. The assumption of linear pumping is adopted below, and the sound speed modulation is given by

$$c^2(t) = c_0^2\big(1 + m\cos(\Omega t)\big),$$

where m is a small parameter called the modulation depth and c_0 the wave velocity without pumping.

In real magneto-strictive conjugators, the modulation depth can be different for longitudinal and shear waves except if the material remains isotropic under pumping conditions as it has been shown in [22]. Here, in a paraxial approximation, only one modulation depth is needed since only longitudinal waves can take place in a 1D conjugator in contact with two 1D fluid domains (Figure 1). The limitation to linear pumping is not a restriction because the nonlinear pumping, treated numerically in [1], does not change the results obtained in the linear case except in the fact that the amplitude of the conjugate wave saturates if the duration of the pumping is long enough. This is due to the electromotive feedback from the elastic field to the magnetic field through the solenoid wrapped around the conjugator in order to produce the modulated magnetic field.

In one-dimensional approximation under a constant modulation depth along the sample, the problem can be considered as the interaction between two phonons ruled by the following equations [18]:

$$\begin{cases} \dfrac{\partial\theta}{\partial t} + c\dfrac{\partial v}{\partial x} = m\dfrac{\Omega\theta}{2}\sin(\Omega t), \\ \dfrac{\partial v}{\partial t} + c\dfrac{\partial\theta}{\partial x} = 0, \end{cases}$$

where $\theta = -\sigma_{xx}/(\rho_0 c_0)$, σ_{xx} is the normal elastic stress along the axis in the solid, v is the particle velocity, ρ_0 is the density of the conjugator material. The first equation is simply the time derivative of the Hooke Law that gives the relation between the particle velocity field and the stresses. The second equation is the 1D unsteady Navier equation. It must be underlined that the modulation of the sound velocity appears in both sides of the system of equation. The source term in right-hand side comes from the time derivative of the Hooke law whereas the wave motion is described by the left-hand side with the actual instantaneous celerity c inside the active zone.

For the external media on the left and on the right from the conjugator, the governing equations are the classical wave equations without pumping ($m = 0$). They correspond to the left-hand side of the previous system, where c is replaced by c_l or c_r the constant sound velocities respectively in the left and right medium. In a fluid σ_{xx} is replaced by $-p$, the acoustic pressure, in the definition of θ. The interface conditions are determined by $\tau_l = \rho_l c_l/(\rho_0 c_0)$ and $\tau_r = \rho_r c_r/(\rho_0 c_0)$ the impedance ratios of the left and right media.

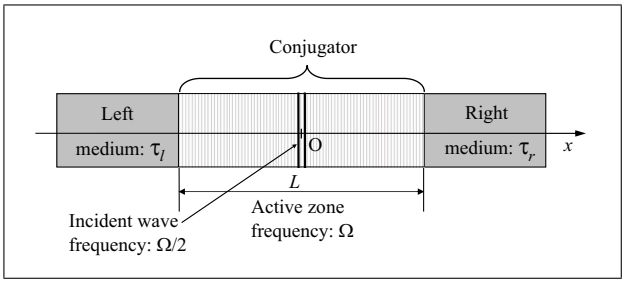


Figure 1. Principle of WPC. Left and right medium are fluids, the conjugator is a sample of magneto-strictive material wrapped in a solenoid circuit that provides the modulated magnetic field. The incident wave is traveling from left to right while the pumping is activated.

The source term and the modulation of c have very little influence since $m \ll 1$ except if the parametric resonance condition is fulfilled. This condition is simply $\Omega = 2\omega$ where ω is the angular frequency of the incident wave [21, 18]. In that case, the system can be solved analytically for two families of pseudo-stationary solutions with exponentially growing amplitudes: one so-called "even mode" in an active medium sample of length L equal to a multiple of a quarter wavelength of the incident wave: $L = 2n\lambda/4$ and one "odd mode" for $L = (2n-1)\lambda/4$ [18]. The analytical solutions gave the threshold values of the modulation depth m, below which there is no signal amplification, as well as the amplification rate Γ of the phonons $w_1 = v+\theta$ (direct wave) and $w_2 = v-\theta$ (conjugate wave), both varying like $A\mathrm{e}^{\Gamma t}$. Under equivalent conditions, the threshold values m_0 of the modulation depth are different for the "even" and "odd" modes, and at the same m, the amplification rates are also different for both modes. Unfortunately, one cannot get analytical solutions for the intermediate values of wavelength between the even and odd modes in the framework of the theory developed in [18]. For that purpose, in present paper, a series of numerical simulations has been performed using the model and methods presented earlier in [19, 20, 22].

The numerical model of elastic continua ensures a detailed resolution of wave processes in space and in time and is based on the momentum equations and generalized Hook's law, both written in a form of conservation laws for elementary control volumes of a computational grid. The amplification rate Γ is evaluated automatically by runtime post-processing of the numerical data for the propagating waves in the simulated elastic medium. The initial data simulated an incident acoustic wave and the active medium pumping was represented in the parametric resonance conditions. In Figures 2 and 3, the amplification rate Γ is presented in dimensionless units normalized by its theoretical value for the nearest even mode of the analytical solution.

3. Results

The modulation in the mechanical properties of the elastic medium appears in the source terms of the model equations as well as in the flux terms. Equivalent results can be

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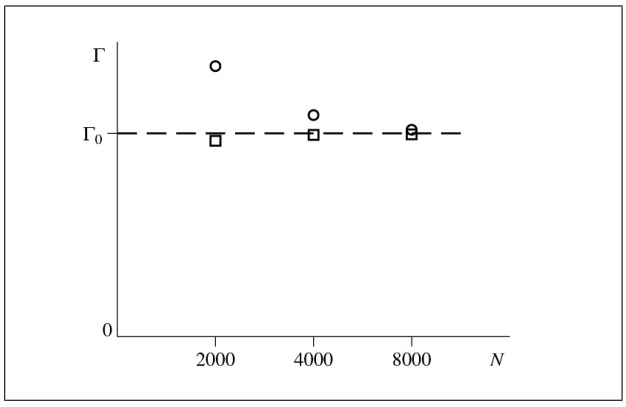


Figure 2. Convergence of the numerical solution for the amplification rate Γ with the multiplication of the grid cells number. Γ_0 analytical solution (even mode), circles complete model with $m = 0.04$, squares reduced model.

obtained for Γ by a reduced model with a constant sound velocity c_0 instead of c in the flux terms. Nevertheless, as shown in [18], for a given geometry, solutions with the same Γ can exhibit small differences in the phase of the phonons due to variations in the initial conditions. The same happens by switching from the complete model to the reduced one. A series of computations has been performed to test the convergence of the numerical solution following the grid refinement (multiplication of the grid cells) and its sensitivity to these variations. A representative example of those results for the value of the amplification rate is presented in Figure 2. Apart from confirming convergence, it has been noted that the complete numerical model always overestimates the exact (analytical) value of the amplification rate, while the reduced model converges to the same solution but always underestimates it. This feature of the numerical model allows for a simple estimate of the solution accuracy by comparing results for identical conditions computed by the complete model, which provides the upper limit of the error bar, and by the reduced model providing the lower limit of the error bar.

The computed Γ values for various L/λ are exemplified in Figure 3. The computations were performed on identical grids using the complete and reduced models at various modulation depths. The sample length to wavelength ratio varied in the range $20 \leq L/\lambda \leq 20.5$. A good agreement with the analytical solutions for the even and odd modes can be observed. In all cases of supercritical values of m, gradual variations of the amplification rate Γ were obtained for intermediate values of L/λ. In Figure 3a the computed data are presented for $m = 0.04$, which is supercritical for the even mode and subcritical for the odd mode. At moderate supercritical values of the modulation depth, the maximum amplification rate is achieved at the conditions corresponding to the even mode of the analytical solution, and the minimum at the odd conditions (see Figure 3b). For higher m, the difference between the maximum and minimum amplification rate reduces, while the position of minimum Γ deviates from the odd mode conditions (Figure 3c,d), the minimum values of the amplification rate becoming essentially lower (Figure 3d) compared to the analytics for the odd mode.

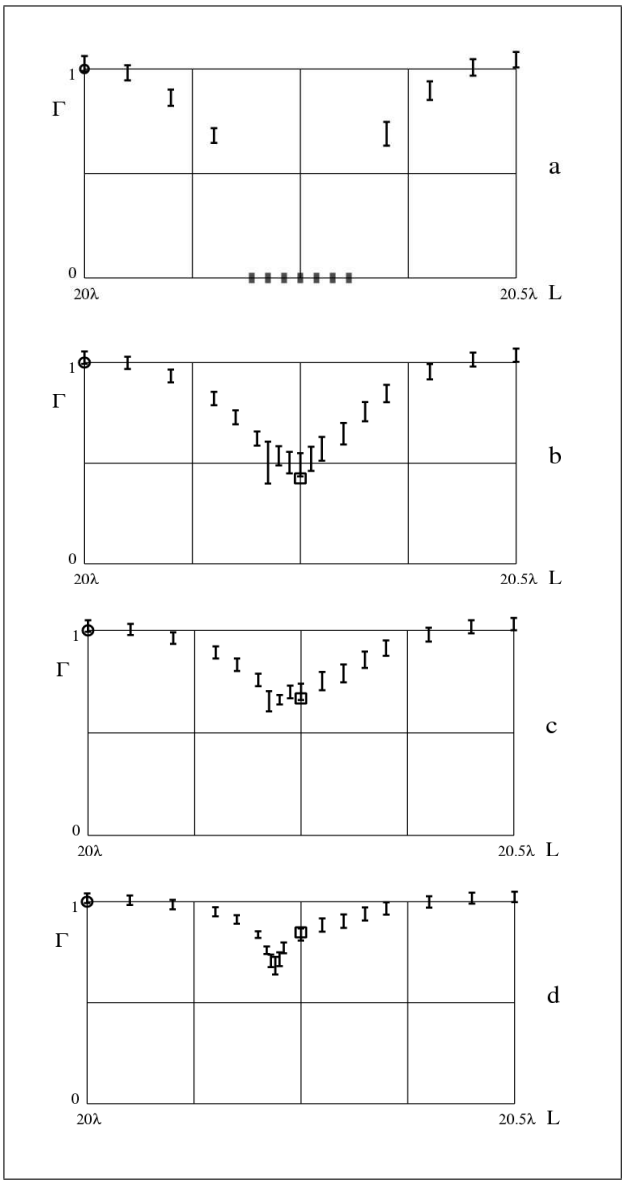


Figure 3. Amplification rate Γ for various L/λ. Circles: analytical solution for even mode. Squares: same for odd mode. (a) $m = 0.04$; (b) $m = 0.05$; (c) $m = 0.06$; (d) $m = 0.08$.

4. Conclusion

From the above results one can conclude that at least both even and odd threshold values must be taken into account to understand the behavior of a conjugator. In the present example, the value of the modulation depth predicted by the odd solution should be used in practice, as choosing an m exceeding only the even threshold may cause no amplification for large parts of the L/λ range . At low supercritical values of the modulation depth, unstable experimental results can be expected, as small variations in ω would lead to essential changes in amplification, even switching from super to subcritical mode. In 3D, this effect can also

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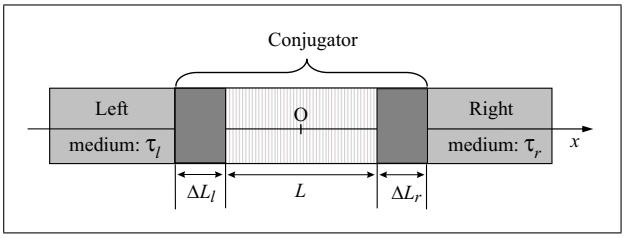


Figure 4. Conjugator partially wrapped in a solenoid circuit. The active zone is smaller than the sample.

affect the focusing of the conjugate signal because the effective distance between the reflecting surfaces of the active medium sample for a non-planar wave front would depend not only on the sample length L, but also on the path of each ray within the sample. As a consequence, variations in the amplification rate for different directions within the active medium may deteriorate the quality of retro-focusing. In order to minimize such problems, it is possible to use an extended sample (Fig. 4)

According to [18], the difference between "even" and "odd" solutions dicreases when ΔL_l and ΔL_r increase. In the limiting case of an homogenous material with a finite active zone, the difference vanishes. According to present simulations, the general solution for Γ is most of the time situated between the odd and the even value. It is therefore possible to reduce the distortion due to the sensitivity of Γ with L/λ by enlarging ΔL_i and ΔL_r. Nevertheless, according to [18] a total elimination of the distortion would need $\Delta L_i = \Delta L_r = 2.5L$ which increases considerably the length of the conjugator and introduces other drawbacks like dissipation, time delay and technical difficulty in realizing the conjugator. A reasonable value would be $\Delta L = L$.

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