

COVER PAGE

Document downloaded by @DAEL

Tue Nov 17 20:17:29 2026
For personal use

When automatic English translation is provided, only the original document is authentic.
The EAA cannot be held responsible of any translation error

Bibliographical reference

Nonlinear Sound Propagation in Trumpets, Pablo L. Rendón, Rodrigo Ezeta and Antonio Pérez-López, Acta Acustica **vol. 99** (Number 4), 2013, pp. 607-614

DOI
https://doi.org/10.3813/AAA.918640

Acta Acustica united with Acustica
Vol. 99 (2013) 607 – 614

DOI 10.3813/AAA.918640

Nonlinear Sound Propagation in Trumpets

Pablo L. Rendón, Rodrigo Ezeta, Antonio Pérez-López
Grupo de Acústica y Vibraciones, Centro de Ciencias Aplicadas y Desarrollo Tecnológico, Universidad Nacional Autónoma de México, Circuito Exterior, Ciudad Universitaria, A.P. 70-186, México D.F. 04510, México.
pablo.rendon@ccadet.unam.mx

Summary
The bright timbre of many brass instruments when played at loud dynamic levels is commonly characterised as "brassy". It is now widely acknowledged that this brassy quality to the sound is mainly due to nonlinear wave propagation along the instrument bore, which in turn results in a transfer of energy to high harmonics. Thus, longer instruments like the trombone might seem better suited to produce brassy sound than shorter ones like the trumpet, and observations of nonlinear propagation effects in trombones have, in fact, been reported. The length of the instrument bore is not, however, the sole factor that determines the scale of nonlinear propagation effects inside the instrument resonator – the shape of the instrument bore and the shape of the time signal at the mouthpiece are also important. Regarding the trumpet, there is some controversy as to the scale of spectral enrichment due to nonlinear wave propagation. With this in mind, we have measured both standing and progressive waves along the bore of a B♭ trumpet. We have observed that for high enough dynamic levels, waveform steepening is quite noticeable in the time domain. By following progressive waves along the bore of the instrument, we have also been able to isolate the effect of nonlinearity from that of attenuation, and identify the transfer of energy to high frequencies that occurs as blowing pressure is increased. Using a live player, shock waves were detected at *fortissimo* levels, confirming that the scale of waveform steepening is sufficient to result in significant spectral enrichment of high frequencies.

PACS no. 43.25, 43.75.Fg

1. Introduction

Wind instruments are typically described as feedback systems consisting of a nonlinear sound source coupled to a linear acoustic resonator, the instrument pipe [1]. The nonlinearity of the source has been confirmed by Backus and Hundley [2] for a trumpet, and by Elliot and Bowsher [3] for a trombone. The validity of the assumption that the resonator essentially behaves as a linear filter, however, has been called into question for brass instruments played at loud dynamic levels. The sound of brass instruments is partly characterised by a gradual enrichment of the upper harmonics that accompanies an increase in blowing pressure. The bright sound produced under these conditions is often referred to as "brassy" sound. Beauchamp [4] was the first to suggest that this effect was perhaps not entirely due to the nonlinearity of the source, and that nonlinear propagation effects in the instrument resonator could play a part as well. He based this observation on measurements he made of the transfer function of a trombone, where he found that the radiated harmonics depended on the amplitude of the initial signal. More conclusively, Hirschberg *et al.* [5] demonstrated through experiments that, for loud enough dynamic levels, nonlinear distortion of standing waves in a trombone pipe can give rise to shock waves. In the frequency-domain, Thompson and Strong [6] developed a model taking into account nonlinear propagation effects and attenuation, and showed that for large-amplitude tones, linear models severely underestimate the amplitudes of the high-frequency harmonics. The transfer of energy towards the high-end of the frequency spectrum is characteristic of nonlinear propagation [7], so these results were taken as further evidence that linear theory alone does not adequately describe wave propagation in the trombone resonator. Subsequently, Rendón *et al.* [8] showed that high-amplitude short pulses moving along the trombone pipe again exhibit this characteristic pumping of energy towards the high harmonics, once a sufficiently large distance has been covered by the pulse. Thus, it seems clear by now that the brassiness associated with loud playing of trombones is due mainly to spectral enrichment caused by nonlinear propagation of sound along the instrument bore. Trombones were first used to study the relationship between brassiness and nonlinear propagation presumably because they are generally much longer than trumpets. The evidence for nonlinear steepening having an effect on the timbre of trumpets at high dynamic playing levels is consequently less extensive than that for the trombone. Backus and Hundley [2] have

Received 30 August 2012,
accepted 21 April 2013.

Document downloaded by @DAEL, 2026-11-17 20:17:32 - personal use only

obtained, through measurements on an artificially blown trumpet, an essentially linear transfer function between the mouthpiece sound pressure and the radiated sound field. On the other hand, both Fletcher and Tarnopolsky [9], using a live player, and Vergez and Rodet [10], using an artificial mouth, observe spectral enrichment of high frequencies measured at the bell of a trumpet as the blowing pressure is increased. Both nonlinear acoustic propagation and the manner in which the shape of the instrument bore may affect nonlinear propagation are discussed in the following section. In this way, we will establish conditions for nonlinear propagation effects in the trumpet bore to be identified and quantified.

2. Nonlinear propagation effects and their impact on timbre

Nonlinear propagation of acoustic waves occurs when large-scale wave profile steepening is caused by the cumulative effect over long distances of locally small nonlinearities [7]. The scale of this steepening is determined by both the amplitude and the maximum slope of the compressive phase of the wave profile at a reference position. Although it could be argued that nonlinear propagation effects are present to some degree at even moderate dynamic levels, their presence is normally expected for high-amplitude waves propagating over long distances. Usually, attenuation effects will cause a progressive wave to smoothen and decrease in amplitude over long distances, but for very high-amplitude waves, attenuation may balance with nonlinearity to give rise to very thin regions where the pressure becomes almost discontinuous. These regions are known as shocks, and they have been extensively studied by, among others, Whitham [11]. Any wave profile with a compressive phase may develop shocks, but whether it does so or not depends on the distance over which it travels. An estimate for the shock formation distance may be obtained through the method of characteristics. The form of this estimate given by Pierce [12] for a plane wave moving along a frictionless duct is

$$x_s \approx \frac{2\,\gamma\,p_0\,c_0}{(\gamma+1)(\partial p_m/\partial t)_{max}}, \tag{1}$$

where γ is the ratio of specific heats, p_0 is the atmospheric pressure, c_0 is the local sound speed, and p_m is the mouthpiece pressure. The position, speed and strength of this shock can be estimated within the framework of weak shock theory [13]. It is interesting to note here that it is not the ratio between p_0 and p_m which partly determines the shock formation distance, but rather the maximum slope of the pressure profile. This is especially important in the context of brass instruments because the coupling between the player and the mouthpiece is nonlinear, and the flow pattern produced is not only non-sinusoidal, but results in a marked pressure rise during the opening of the lips. [2, 3] For this reason, signals produced by an actual player will develop shocks more quickly than a sinusoidal signal of the same RMS amplitude generated at the mouthpiece of the instrument.

Table I. Relationship between RMS sound pressure levels inside (position 1) and outside (1 m from the horn exit) the trumpet.

	Interior		Exterior	
	RMS [Pa]	SPL [dB]	RMS [Pa]	SPL [dB]
p	29.716	123.43	0.002	42.01
mf	108.955	134.72	0.009	53.26
ff	3149.384	163.94	0.275	82.78

However, as Myers *et al.* [14] have pointed out, the effective length [15] of the instrument is not the only factor that determines whether nonlinear propagation along the instrument bore can have an effect on the timbre of the instrument, but rather, the shape of the bore itself is also critical. In fact, the shape of the instrument determines the extent to which the bore profile is able to support nonlinear propagation. The optimal shape of a tube in terms of favouring nonlinear distortion is a cylinder, as any outwardly flaring horn will cause a stretching of the x coordinate. For brass instruments that flare outwards, such as the trumpet, the length of the instrument is clearly greater than the length of a cylinder which would permit an equivalent amount of nonlinear steepening. We shall call this length L_c. The brassiness potential parameter, B, is then defined as the ratio of L_c to the equivalent conical length of the instrument, L_{ecl}, which is the length of a cone whose lowest resonance frequency is the same as the nominal fundamental frequency of the instrument. In order to calculate B for actual instruments, an approximation is used,

$$B \approx \sum_{n=1}^{N} \frac{l_n}{L_{ecl}} \left(\frac{2\,D_0}{D_{n+1}+D_n} \right), \tag{2}$$

where the instrument is divided into N sections with arbitrary lengths l_n ($1 \le n \le N$), D_0 is the minimum bore, and D_n is the bore diameter at the beginning of the nth section. Myers *et al* [14] have calculated B for a variety of brass instruments, and have concluded that its value is a good indicator of spectral enrichment due to nonlinear wave propagation in brass instruments.

In terms of sound quality, the main consequence of waveform steepening is a transfer of energy to high harmonics as the wave moves along the instrument bore, due to the fact that as the wave steepens, spectral components interact very efficiently between themselves, as they are essentially in resonance with one another [16]. The scale of this energy transfer is dependent on blowing pressure. This enrichment of the high-end of the frequency spectrum with increasing pressure amplitude furnishes us with a rather straightforward criterion to identify nonlinear propagation, especially for progressive waves. For standing waves, Van Buren [17] and Ilinskii *et al* [18] have shown that steepening of the progressive waves that interact to produce standing wave patterns also results in steepening of the standing waves. Therefore, the waveforms observed at high sound pressure levels in trombones by Hirschberg *et al* [5] and Thompson and Strong [6] are entirely to be

Document downloaded by @DAEL, 2026-11-17 20:17:32 - personal use only

expected, and serve as confirmation of nonlinear steepening in the instrument bore. In the following section, we use both time-domain and frequency-domain arguments to identify nonlinear wave propagation in a trumpet bore.

3. Experimental results

3.1. Experimental setup

A Silvertone B♭ trumpet, bought new and considered to be a good quality student instrument, was used for all measurements. The trumpet was set up in such a way that all three valves were engaged, thus producing the greatest length possible for the instrument resonator. For the first series of measurements, sound excitation was provided by a Radson U150 S driver, coupled to the mouthpiece-end of the trumpet by means of an 8.4 cm PVC cylindrical tube with an internal diameter of 10.7 mm made especially for this purpose. The whole setup is pictured in Figure 1. Sound pressure measurements were obtained with a Brüel & Kjaer type 4182 probe microphone at two different positions along the trumpet tube, where, in both cases, the probe microphone was inserted and plasticine was used to seal the space around the probe, thus approximating continuous closed bore conditions. For the first position, closest to the driver and labelled from here on as position 1, a 1.5 mm diameter hole was drilled on to the cylindrical fitting part 4 mm from the driver exit. In order to avoid drilling directly on the trumpet, we have taken advantage of a water key positioned 80 cm from the mouthpiece end of the trumpet tube, so that adding the length of the cylindrical fitting piece the total distance to the first position is 88 cm. This orifice corresponds to position 2. The total length of the extended trumpet resonator is then 205.9 cm, as can be appreciated in Figure 2. Using equation (2) we have calculated the brassiness potential parameter, B, by measuring the bore diameter at 38 points along the trumpet bore. We obtained a value of $B = 0.77$, which is high – comparable to the B value of a trombone – due to the large section of cylindrical tubing that forms a part of the trumpet bore, and leads us to expect that the trumpet should be able to sustain nonlinear wave propagation.

The input signals were generated using a Stanford research DS345 function generator and a Yamaha AX-380 power amplifier. The measurements were then acquired by means of a Brüel & Kjaer type 2034 dual-channel analyser, and transferred to a laptop computer through a National Instruments GBIP/IEEE-488.1 interface. Impulse signals were repeated every 125 ms, allowing for significant decay of sound amplitude inside the trumpet between pulses. Each measurement, for both impulse and periodic signals, initially consists of the average of 100 pulse-synchronised sound pressure transients. Further, ten measurements were made, and averaged, for each hole at each dynamic level, taking out and reinserting the probe on each occasion, so that this new average would take into account the statistical effect of minimal changes in positioning of the microphone. All measurements were made

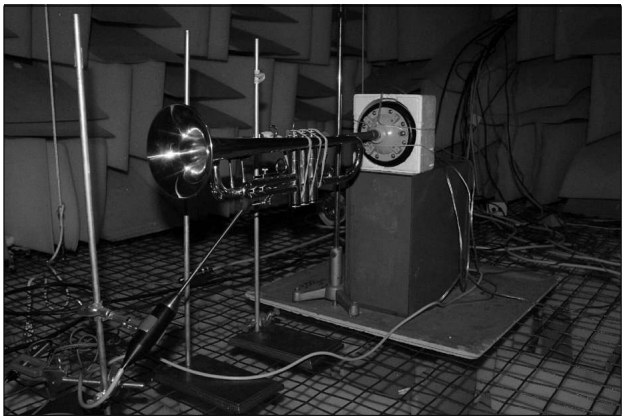


Figure 1. Experimental setup used for sound pressure measurements in a B♭ trumpet.

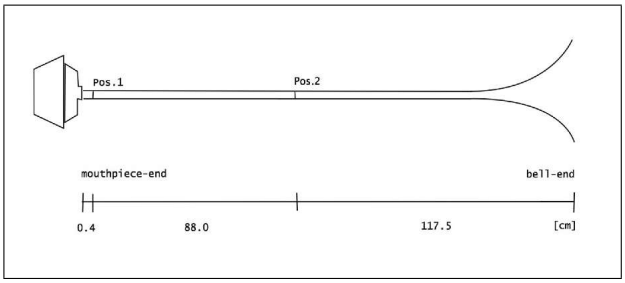


Figure 2. Measurement positions along the body of the trumpet.

in an anechoic chamber at 20°C, with little temperature variation. The peak sound pressure level measured inside the trumpet in the absence of excitation from the driver was 31 dB.

Following Hirschberg *et al.* [5], we compared waveforms produced at three distinct dynamic levels: *piano* (p), *mezzo-forte* (mf), and *fortissimo* (ff). In order to establish reference values for sound pressure levels corresponding to these rather subjective dynamic levels, we asked a professional trumpet player to play a sustained B♭4, with a corresponding frequency of, in modern equal temperament, 466.16 Hz, at each of these dynamic levels in an anechoic chamber. The musician's perception of dynamic levels inside the anechoic chamber may be different than in a regular acoustic since there will be practically no reflections from any walls, but we assume this will not greatly affect the dynamic levels produced, which are, in any case, somewhat subjective. The player was positioned in such a way that the horn exit was located approximately 1 m away from a Brüel & Kjaer type 4165 microphone, and three measurements were obtained for each level in order to obtain average values. The same note, B♭4, was then produced by the function generator, and the amplitude of the signal was adjusted using the amplifier so that the sound pressure level measured 1 m away from the horn exit matched the results produced by the player for each dynamic level. The probe microphone was then used to obtain sound pressure levels at position 1, inside the trumpet, allowing us to establish the relationship between exterior and interior sound pressure levels. RMS levels were cal-

Document downloaded by @DAEL, 2026-11-17 20:17:32 - personal use only

culated in all cases, and the results are shown in Table I. We observe that the sound pressure levels corresponding to *fortissimo* are, roughly, order 10^3 larger than those corresponding to *piano*. Note also that the values obtained inside the pipe are of the same order as those obtained by Hirschberg *et al.* for the trombone, and are such that the RMS pressure [Pa] is $O(1000)$ at *fortissimo*.

3.2. Results: standing waves

We first seek to identify nonlinear propagation effects inside the trumpet resonator by producing standing waves of different amplitudes in the resonator, and then measuring the variation with time of the sound pressure level at a fixed location. This is, essentially, the same approach taken by Hirschberg *et al.*. We may choose an arbitrary location for measurement in the resonator, such as position 2, but care must be taken to avoid the presence of a pressure node at or near this position. As we have discussed before, feeding a continuous, time-periodic signal into the trumpet will result in an approximately stationary wave being formed in the trumpet resonator, as long as the frequency of the input signal does not exceed 1600 Hz, the cut-off frequency of the instrument as obtained by Benade [19]. The shape and frequency of the input signal will then, in turn, determine the shape and wavelength of this wave. Thus, in order to obtain clear and intelligible differences between waves of different amplitudes, we seek not only to avoid proximity to a pressure node, but also to maximise the range of variation of these amplitudes by trying to locate position 2 as close to a pressure antinode as possible. This is achieved by sweeping the frequencies of the input signal, starting with B♭3 (233.08 Hz) and going up to C6 (1046.50 Hz), and looking for local maxima and minima of pressure amplitudes at position 2. Two extrema were found within this range, varying frequencies by intervals of 1 Hz. Since high frequencies favour wave-steepening, as we discuss below, we have chosen the frequency closest to the top-end of the range of a B♭ trumpet, at 720 Hz, corresponding roughly to the note F♯5, which is well within the playing range of a B♭ trumpet.

If we feed the driver at the mouthpiece with a sinusoidal signal of the form $p_m(t) = A \sin \omega t$, it is clear that $(\partial p_m/\partial t)_{max} = A\omega$, and then the estimation of shock formation distance given by equation (1) can be written as

$$x_s \approx \frac{2\gamma\, p_0\, c_0}{A\,\omega\,(\gamma+1)}. \tag{3}$$

Thus, larger frequencies and amplitudes lead to shorter shock formation distances, and consequently to faster rates of waveform steepening, if it does occur. Calculating the value of x_s for a *fortissimo* sine wave with frequency 720 Hz, using $\gamma = 1.4$, $p_0 = 101325$ Pa, $c_0 = 343$ m/s, $\omega = 720$ Hz, and $A = 3150$ Pa, we obtain $x_s \approx 17.88$ m. Clearly, we do not expect shock waves to form inside the trumpet in this case, but we may still be able to observe waveform steepening, which would eventually lead to shock formation given a long enough tube. We should here point out, however, that a live player is normally able to control the value of $(\partial p_m/\partial t)_{max}$, and in this way shorten the shock formation distance considerably.

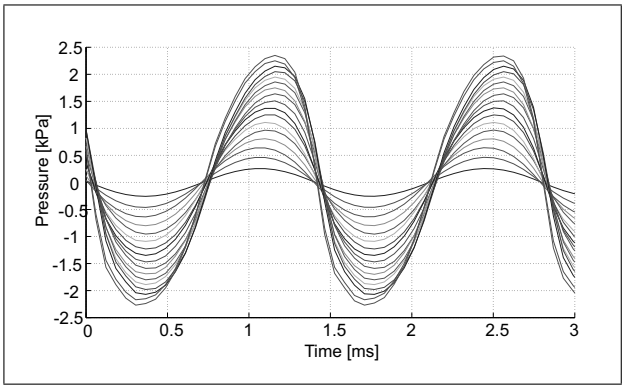


Figure 3. Sine wave pressure signals measured at position 2 for different amplitudes covering the range between mf and ff.

We proceeded to vary the amplitude of pressure sine waves with the same frequency, 720 Hz, produced by the DS345 function generator, so as to determine whether waveform steepening takes place as the amplitude increases. In total, 17 measurements of sound pressure were done at position 2, corresponding to RMS input levels between 183.35 Pa (mf) and 1744.32 Pa (ff). Each successive step in input pressure amplitude was produced by adjusting the amplifier so as to increase by steps of 1 V the potential measured between the driver and the amplifier output. The results are shown in Figure 3, where a slight steepening of the successive wave profiles is, indeed, apparent. In order to illustrate this steepening, we calculated the maximum slope of the compressive phase of the wave for each wave profile. So as to establish that the observed nonlinearity is effectively due to propagation effects and not to nonlinearities associated with the driver, we also calculate the maximum magnitude o f the gradient during the decompressive phase. As we can see from Figure 4, the relationship between input pressure amplitude and maximum magnitude of the slope is perfectly linear for both phases at moderate amplitudes, but these relationships eventually become nonlinear as the amplitude increases, and waveform steepening is clearly discernible at this point. Eventually, since the rate of change of the maximum gradient of the compressive phase decreases very slightly, and the rate of change of the maximum magnitude of the gradient of the decompressive phase increases, we can rule out nonlinear distortion due to the driver as the cause of the observed steepening. Thus, even though shocks do not form, as expected, waveform steepening does occur, and consequently energy is transferred to the high-end of the frequency spectrum.

3.3. Results: progressive waves

Rendón *et al.* [8] have pointed out that progressive waves afford what is in many ways a more natural method for the detection of nonlinear propagation effects than standing waves. As we have already discussed, the cumulative effects of nonlinearity on sound propagation are exhib-

Document downloaded by @DAEL, 2026-11-17 20:17:32 - personal use only

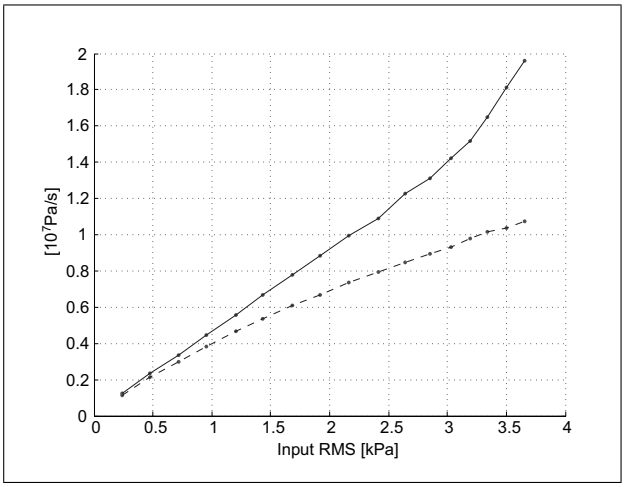


Figure 4. Maximum magnitude of the slope of the compressive (dotted line) and decompressive (solid line) phases of pressure signals measured at position 2 plotted against increasing input pressure amplitudes.

ited through waveform steepening of progressive waves. Waveform distortion of standing waves is a consequence of the steepening of the progressive waves which interact to give rise to the standing wave itself, but the nature of this interaction is, as yet, unclear. Two-microphone methods, such as that used by Rendón *et al.* [8], can be used to track progressive waves through the use of two microphones placed a few millimetres apart, but here have chosen instead to follow a single, narrow pulse between positions 1 and 2. We need to be mindful, though, that the pulse be narrow enough to prevent the reflection from the bell-end from arriving back at position 2 before the actual incoming pulse has completely passed. Although the pulse will be slightly broadened as it passes from the driver to the trumpet pipe, a duration of 1 ms was found to be small enough to guarantee that the reflected pulse would not interfere with measurements of the incoming pulse at position 2 – the calculated length of a rectangular pulse of duration 1 ms, for $c_0 = 343$ m/s, is 34.3 cm. There is clearly a trade-off between the duration of the pulse and the maximum pressure amplitude achievable, due to constraints posed mainly by the driver unit, but it was still possible to choose references for *piano* and *fortissimo* according to Table I. Thus, a pulse with a measured RMS sound pressure level at position 1 of 123 dB corresponds to *piano*, whereas a level of 163 dB corresponds to *fortissimo*. The time-domain signals corresponding to both *piano* and *fortissimo* are shown in Figure 5, as measured over the initial 120 ms at positions 1 and 2.

It is clear that waveform steepening cannot be observed directly from Figure 5, and simply examining the spectra of the different pulses would not allow for observation of spectral enrichment either, since the effect of attenuation is already evident in the same figure. Attenuation gradually reduces the amplitude of the pressure pulses, making the slopes less pronounced as well. It also makes direct comparison between the spectra at positions 1 and 2 basi-

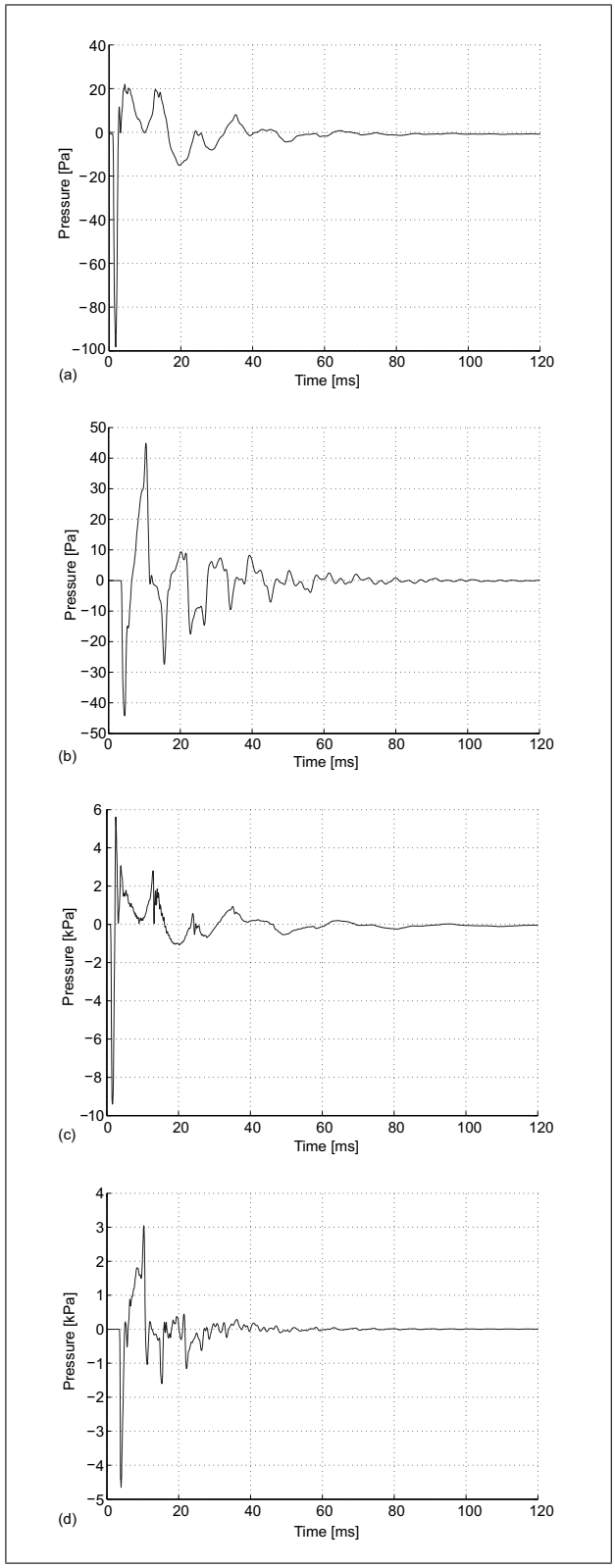


Figure 5. Pressure pulses measured over 120 ms: (a) *piano* pulse at position 1, (b) *piano* pulse at position 2, (c) *fortissimo* pulse at position 1, (d) *fortissimo* pulse at position 2.

cally meaningless, as the total energy content of the signal at position 2 is much smaller than at position 1. A way around this problem has been suggested by Rendón *et al.*

Document downloaded by @DAEL, 2026-11-17 20:17:32 - personal use only

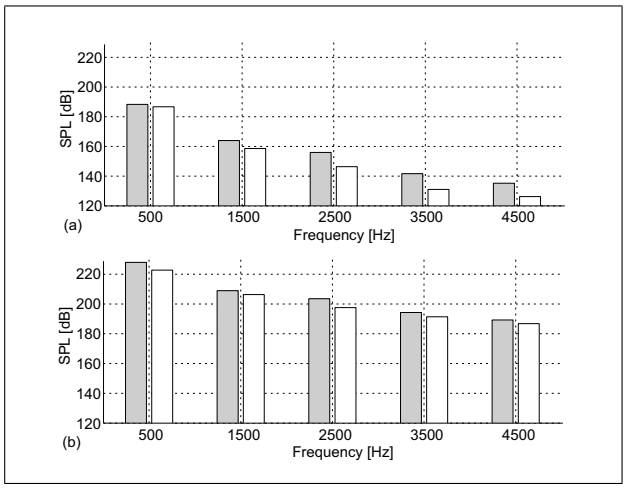


Figure 6. ESDs calculated, in frequency bands of width 1000 Hz, for (a) a *piano* pulse, and (b) a *fortissimo* pulse, at positions 1 (grey) and 2 (white).

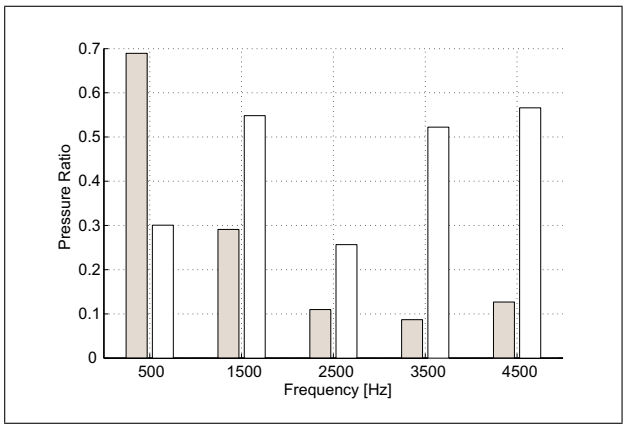


Figure 7. Ratio between ESDs at positions 2 and 1, for frequency bands of width 1000 Hz, and for a *piano* pulse (grey) and a *fortissimo* pulse (white).

[8], and it consists of calculating the energy spectral densities (ESDs) of contiguous frequency bands of equal width. The width of these bands is chosen according to the resolution afforded by measurements, and so that the energy cascade may be appreciated over a wide enough spectrum of frequencies. In this manner, we may compare the transfer of energy to high frequencies for p and ff pulses as they propagate along the trumpet bore. To this effect, we divide the spectrum in 5 bands of equal width (1000 Hz), centred at 500 Hz, 1500 Hz, 2500 Hz, 3500 Hz and 4500 Hz. The results are shown in Figure 6, in a decibel scale, for clarity. We immediately notice that the effect of attenuation is very marked in both cases. Since the effect of attenuation is more marked for higher frequencies [12], we would expect the ESDs for the higher frequencies to be more depleted than those for lower frequencies, and this is indeed the case for the *piano* pulse. For the *fortissimo* pulse, however, this is clearly not the case, and for the higher-frequency bands, the ESDs at position 2 are only slightly smaller than at position 1. In order to compare the rate at which energy has been transferred to the higher-frequency bands, we plot in Figure 7 the ratio between the ESDs at positions 2 and 1, for each frequency band and for each dynamic level. It is immediately clear from this figure that the transfer of energy to the high harmonics is much more significant for the *fortissimo* pulse than for the *piano* pulse. This result is clearly consistent with the assumption that nonlinear wave steepening is at the root of the energy cascade towards the high-end of the frequency spectrum, especially because, as we have mentioned previously, the effect of attenuation is also expected to be larger at this end of the spectrum.

3.4. Results: live player

Whereas in the previous sections sound was produced by means of an electromechanical driver to allow for repeatable measurements, in this section we use a live player with the principal aim of detecting, if possible, the emergence of shock waves. As we have discussed before, a live player can reduce the shock formation distance through modification of the input signal generated at the mouthpiece. The setup in this case is very similar to that described in section 3.1, with two main differences: first, we used a regular trumpet mouthpiece instead of the fitting cylinder used earlier, and second, we measured the time signals at different positions than those used before. In order to avoid exceeding the saturation level for the microphone at the position closest to the mouthpiece, we now place the first measurement position, referred to as position A from now on, at a distance of 44.5 cm from the mouthpiece, again taking advantage of a water key. To allow for a reasonably long distance to the second position, a 1.5 mm diameter hole was drilled at a distance of 44 cm from the open end of the instrument, near the end of the cylindrical pipe segment. This position will be labelled position B, and is at a distance of 117 cm from position A, and at a a total distance of 161.5 cm from the mouthpiece. The distances and amplitudes involved in the present setup are comparable to those used by Gilbert and Petiot [20] with a trombone, and which did permit them to observe the formation of shocks. A live player was asked to sustain a note at two different dynamic levels, *piano* and *fortissimo*. The frequency of oscillation recorded with all three valves engaged, so as to maximise the length of the instrument, was 505 Hz, which corresponds to a slightly sharp B4. Waveform steepening for the *fortissimo* dynamic can be appreciated in Figure 8, and the beginning of a shock is clearly seen in Figure 8b.

4. Conclusions

Nonlinear wave propagation has been identified by many authors as the principal mechanism whereby brass instruments are able to produce the typically bright timbre associated with brassy sound. As effective length of an instrument is an important factor in determining whether nonlinear propagation will occur, the trumpet seems, on this account, to be less likely to produce brassy sound than a trombone. Calculation of the brassiness potential param-

Document downloaded by @DAEL, 2026-11-17 20:17:32 - personal use only

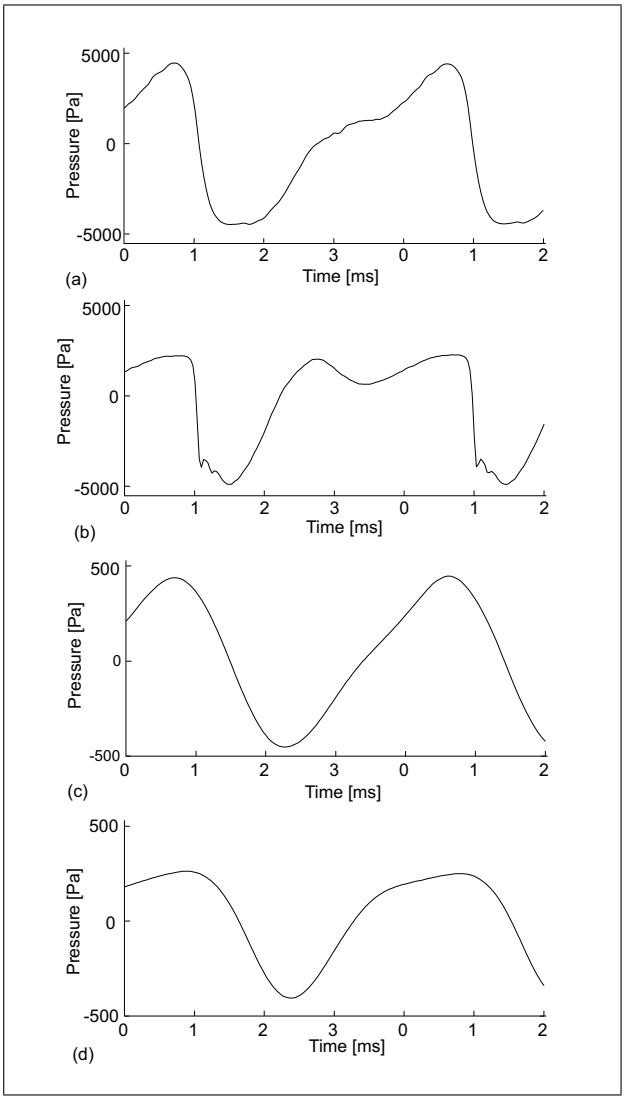


Figure 8. Measured pressure waveforms for a frequency of approximately 505 Hz. The pressure was measured in the trumpet when played (a) *fortissimo* at position A, (b) *fortissimo* at position B, (c) *piano* at position A, and (d) *piano* at position B.

eter for the trumpet, however, indicates that this instrument is actually among the most likely of the brass instruments to sustain nonlinear sound propagation. With regard to the initial shape of the waveform, Norman *et al.* [21] have shown that some degree of control over brassiness is achievable at moderate dynamic levels through slight changes of embochure. Initial waveforms with pronounced slopes will give rise to significant nonlinear distortion faster than waveforms with the same energy content, but less pronounced maximum slopes. We have successfully detected nonlinear waveform distortion in a trumpet bore in the time-domain by producing a stationary wave which visibly steepens as the dynamic level increases beyond a certain point. As the signal produced by the driver was sinusoidal, slopes were not very pronounced, and shocks were never observed. It is difficult to imagine that shocks would appear under these conditions - Hirschberg *et al.* managed to obtain louder dynamic levels in a trombone because they used a live player, and the stationary waves thus produced for low frequencies are pulse-like, as discussed by Luce and Clark [22], and consequently contain more pronounced slopes. Thus, it is natural to use a live trumpet player in order to try and replicate the results obtained, in similar conditions, by Gilbert and Petiot [20]. Although the length of an extended trombone is roughly double that of a trumpet, we nevertheless managed, using a live player, to observe an incipient shock in the trumpet at *fortissimo* levels.

Another manner of examining whether nonlinear sound propagation in a trumpet bore is enough to account for a noticeable change in the timbre of sound is through the observation of progressive waves. Pulses corresponding to *piano* and *fortissimo* dynamic levels were followed from the mouthpiece to a location at roughly mid-pipe. We then devised a scheme involving the ratio of energy spectral densities of adjacent frequency bands which permits us to isolate the effect of nonlinearity and quantify the transfer of energy to high frequencies. For the ff pulse, a very noticeable transfer of energy to high frequencies is observed. Although the maximum slopes associated with the pulses are more pronounced than those associated with the standing waves, shocks are not formed in this case either. Even so, we have observed waveform steepening for standing waves, and the energy cascade that accompanies this steepening for short progressive pulses. It is precisely the steepening, and not necessarily the shock formation itself, that results in the spectral enrichment of high frequencies which we associate with brassy sound. The scale at which these effects are observed permits us to suggest that nonlinear wave propagation should be taken into account as an important mechanism that affects the timbre of the trumpet, especially for large blowing pressures.

Acknowledgement

The authors acknowledge the financial support of DGAPA, UNAM, for this project through grant PAPIIT IN110411. They also wish to thank Miguel Rosales González and David Sánchez García for their trumpet playing and advice.

References

[1] N. H. Fletcher, T. D. Rossing: The physics of musical instruments. Springer-Verlag, New York, 1998.

[2] J. Backus, T. C. Hundley: Harmonic generation in the trumpet. J. Acoust. Soc. Am. **49** (1971) 509–519.

[3] S. J. Elliott, J. M. Bowsher: Regeneration in brass wind instruments. J. Sound Vib. **83** (1982) 181–217.

[4] J. W. Beauchamp: Analysis of simultaneous mouthpiece and output waveforms of wind instruments. Audio Engineering Society preprint no. 1626, Tech. Rep., 1980.

[5] A. Hirschberg, J. Gilbert, R. Msallam, A. Wijnands: Shock waves in trombones. J. Acoust. Soc. Am. **99** (1996) 1754–1758.

[6] M. Thompson, W. Strong: Inclusion of wave steepening in a frequency-domain model of trombone sound production. J. Acoust. Soc. Am. **110** (2001) 556–562.

Document downloaded by @DAEL, 2026-11-17 20:17:32 - personal use only

[7] D. G. Crighton: Nonlinear acoustics. – In: Modern Methods in Analytical Acoustics. Springer-Verlag, New York, 1992, 648–670.

[8] P. L. Rendón, F. Ordu na-Bustamante, D. Narezo, A. Pérez-López: Nonlinear progressive waves in a slide trombone resonator. J. Acoust. Soc. Am. **127** (2010) 1096–1103.

[9] N. Fletcher, A. Tarnopolsky: Blowing pressure, power and spectrum in trumpet playing. J. Acoust. Soc. Am. **105** (1999) 874–881.

[10] C. Vergez, X. Rodet: Trumpet and trumpet player: model and simulation in a musical context. Proceedings of the International Computer Music Conference, La Habana, Cuba, 2001.

[11] G. Whitham: Linear and nonlinear waves. John Wiley and Sons, New York, 1999.

[12] A. D. Pierce: Acoustics - An introduction to its physical principles and applications. Acoustical Society of America, Melville, New York, 1989.

[13] D. T. Blackstock, M. F. Hamilton, A. D. Pierce: Progressive waves in lossless and lossy fluids. – In: Nonlinear Acoustics. M. F. Hamilton, D. T. Blackstock (eds.). Academic Press, London, 1998, 65–150.

[14] A. Myers, J. R. W. Pyle, J. Gilbert, D. M. Campbell, J. P. Chick, S. Logie: Effects of nonlinear sound propagation on the characteristic timbres of brass instruments. J. Acoust. Soc. Am. **131** (2012) 678–688.

[15] R. W. Pyle: Effective length of horns. J. Acoust. Soc. Am. **57** (1975) 1309–1317.

[16] S. N. Gurbatov, O. V. Rudenko: Statistical phenomena. – In: Nonlinear Acoustics. M. F. Hamilton, D. T. Blackstock (eds.). Academic Press, London, 1998, 377–398.

[17] A. L. Van Buren: Mathematical model for non-linear standing waves in a tube. J. Sound Vib. **42** (1975) 273–280.

[18] Y. A. Ilinskii, B. Lipkens, T. S. Lucas, T. W. Van Doren, E. A. Zabolotskaya: Nonlinear standing waves in an acoustical resonator. J. Acoust. Soc. Am. **104** (1998) 2664–2674.

[19] A. H. Benade: The physics of brasses. – In: The Physics of Music. W. H. Freeman and Company, San Francisco, 1978, 44–55.

[20] J. Gilbert, J. F. Petiot: Brass instruments, some theoretical and experimental results. Proceedings of the Institute of Acoustics **19** (1997) 391–400.

[21] L. Norman, J. P. Chick, D. M. Campbell, A. Myers, J. Gilbert: Player control of 'brassiness' at intermediate dynamic levels in brass instruments. Acta Acustica united with Acustica **96** (2010) 614–621.

[22] D. Luce, M. Clark: Physical correlates of brass-instrument tones. J. Acoust. Soc. Am. **42** (1967) 1232–1243.

Document downloaded by @DAEL, 2026-11-17 20:17:32 - personal use only