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Acoustic Absorption of Multi-Layer Structure Composed of Porous Layers Based on Biot's Model and Transfer Matrix Method

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Summary

This article presents an approach of modelling the surface impedance and absorption coefficient of multi-layer structure composed of porous materials placed against a rigid wall. The Biot's model is used to calculate the complex wave numbers of the two compressional waves propagating in porous materials at normal incidence and the surface impedances of multi-layer structures are predicted by the transfer matrix method (TMM). The resonance effects of structures composed of three layers of different porous materials are studied and the arrangement of each layer and the influence of their thicknesses are also discussed.

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1. Introduction

Acoustic absorption of porous materials has been widely studied. Numerous models have been proposed to describe the sound propagation in porous materials, and many of them provide different methods to calculate the surface impedance. The Delany-Bazley model [1] is established on the measured values of the characteristic impedance and propagation coefficient of a range of fibrous absorbent materials which are normalized as a function of frequency divided by flow-resistance and can be represented by simple power-law functions. Allard [2] reformulated the law of Delany-Bazley by rewriting it as a function of an adimensional parameter that multiplies the density of the fluid by the quotient between frequency and resistivity. The Mechel-Vér model [3] is a more refined adjustment than the Delany-Bazley model. It differentiates two families of absorbing materials and two areas for the normalized adimensional parameter. The Allard-Champoux model [4] assumes that the thermal effects depend on frequency and defines the dynamic density and the compressibility modulus of the media. The equivalent fluid model [2] assumes that the skeleton of the porous material is rigid and offers equivalent formulations for effective density [5] and compressibility [6]. The Biot's model [7, 8] takes into account the stiffness of the frame and demonstrates that two compressional waves and one shear wave can propagate in porous materials. In the case of materials with a heavy frame and a moderate or low flow resistivity, one of these compressional waves, which will be called the airborne

wave, propagates mainly in the air [9], and the other wave called the frame-borne wave is created by frame vibrations of which the amplitude is generally negligible while the material is set in an acoustic field in air. Nevertheless, at the resonant frequency of the frame, the amplitude of the frame-borne wave can be significant and the frame-borne wave can modify the surface impedance. More generally, the acoustic movement of the frame of materials with high flow resistivity may be significant. Allard [10] showed that the $\lambda/4$ resonance of the frame will modify the surface impedance.

Stratified media and structures are widely used in the noise control for their broad absorption frequency band. The sound transmission of the stratified structure composed of elastic porous materials and solid planes was calculated using the Biot theory and Transfer matrix method (TMM) and the results were compared with the experimental data by Bolton, Shiau and Kang [11]. Dunn and Davern [12] provided an analytical analysis for the flat-walled anechoic lining composed of outer-, middle- and inner-layer porous materials. The surface impedance of porous layer was calculated using the expression given by Zwikker and Kosten [13] and the characteristic impedance of the porous materials was based on experiments and the law of Delany-Bazley. It was suggested that the outer-layer porous material with appropriate impedance should encourage the incident sound to enter the composite structure, but the inner-layer porous material should be selected to attenuate the sound energy and prevent the incident sound from recombining. With this instruction, F. C. Lee and W. H. Chen [14] calculated the acoustic absorption coefficients of multi-layer acoustic absorbers composed of three layers of perforated plates with different porosities

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and air layers. It was found that the acoustic resonance frequencies of the multi-layer acoustic absorber are more broadband than those of their individual compartments, the resonance peaks of each perforated plates are retained somehow and the arrangement of the perforated plates also affects the acoustic absorption of the absorber. The TMM [2, 15, 16, 17, 18, 19] is widely used to model the acoustic fields in stratified media composed of fluid, solid, poroelastic, perforated plate or impervious screen. Allard and his fellow employed the transfer matrices obtained from the Biot theory to describe the acoustic properties of homogeneous layers and screens constituting layered materials at normal incidence [17], the three inhomogeneous Biot waves in layered porous media [18] and with impervious screens [19]. The transfer matrix of a layer integrates the quantities which describe the acoustic fields on the front and rear face of the layer, and the coupling matrix integrates the boundary conditions relating to the neighboring layers. The transfer matrices and coupling matrices simplify the stratified media acoustic field problem to a single layer problem. The surface impedance and absorption coefficient of the layer were compared with experiments. Sagartzazu [20] investigated a case of two layers of porous materials adhered to a flexible plate and the surface impedance was calculated using the Biot theory and TMM.

According to published literature, the influence of the resonance effect of the layered porous materials was not studied and it will be the main subject of this work. This article presents a study of the acoustic absorption of multi-layer structure composed of high flow resistivity porous materials, the combination styles of porous materials to increase the sound absorption is also discussed. The Biot theory and TMM are used to model the acoustic fields. The scheme for this study is as follows: in section 2, the Biot's model is presented, which is able to calculate the complex wave numbers of two compressional waves of poroelastic materials. The calculation of the surface impedances of the single layer and multi-layer porous materials using the TMM is presented in section 3 and section 4 is the description of the resonance effect of high flow resistivity porous materials composing the multi-layer structures. The results and discussions of the acoustic properties of multi-layer structures are given in section 5. Finally the conclusion of the work is drawn in section 6.

2. Biot's model

As is mentioned above, the Biot's model considers that two compressional waves and one shear wave can propagate in a porous medium simultaneously. In the case of normal incidence, which is the case discussed throughout the paper, the shear wave is not excited and the problem can therefore be posed in a one-dimensional diagram. Note that a harmonic time dependence of the $e^{j\omega t}$ type will be assumed and will therefore not appear in the equations given.

The Biot theory gives an approach to calculate the complex wave numbers δ_1 and δ_2 of the two compressional waves

$$\delta_1^2 = \frac{\omega^2}{2(PR - Q^2)} [P\tilde{\rho}_{22} + R\tilde{\rho}_{11} - 2Q\tilde{\rho}_{12} - \sqrt{\Delta}], \quad (1)$$

$$\delta_2^2 = \frac{\omega^2}{2(PR - Q^2)} [P\tilde{\rho}_{22} + R\tilde{\rho}_{11} - 2Q\tilde{\rho}_{12} + \sqrt{\Delta}], \quad (2)$$

where Δ is given by

$$\Delta = [P\tilde{\rho}_{22} + R\tilde{\rho}_{11} - 2Q\tilde{\rho}_{12}]^2 - 4(PR - Q^2)(\tilde{\rho}_{11}\tilde{\rho}_{22} - \tilde{\rho}_{12}^2) \quad (3)$$

and $\tilde{\rho}_{11}$, $\tilde{\rho}_{12}$ and $\tilde{\rho}_{22}$ are the modified Biot's densities,

$$\tilde{\rho}_{11} = \rho_1 + \rho_a - j\sigma\phi^2 \frac{G(\omega)}{\omega}, \quad (4)$$

$$\tilde{\rho}_{12} = -\rho_a + j\sigma\phi^2 \frac{G(\omega)}{\omega}, \quad (5)$$

$$\tilde{\rho}_{22} = \phi\rho_0 + \rho_a - j\sigma\phi^2 \frac{G(\omega)}{\omega}, \quad (6)$$

in which $G(\omega)$ and $\alpha(\omega)$ are defined as

$$G(\omega) = \sqrt{1 + \left(\frac{2\alpha_\infty q_0}{\phi\Lambda}\right)^2 \frac{j\omega}{\bar{\nu}}}, \quad (7)$$

$$\alpha(\omega) = \frac{\bar{\nu}\phi}{j\omega q_0} G(\omega) + \alpha_\infty, \quad (8)$$

where ω is the circular frequency, ϕ is the porosity, ρ_1 and ρ_0 are the densities of the porous skeleton and air respectively, $\alpha(\omega)$ is the dynamic tortuosity, α_∞ is the tortuosity, $\rho_a = \phi\rho_0(\alpha(\omega) - 1)$ is the inertial coupling term, Λ is the viscous characteristic length, σ is the flow resistivity, η is the dynamic viscosity of air, $q_0 = \eta/\sigma$ is the static dynamic permeability, $\bar{\nu}$ is the kinematic viscosity of air.

P , Q and R are the elastic coefficients of the porous material, which can be evaluated from the "Gedanken Experiments" suggested by Biot [21]. In the case that the compressibility modulus of the porous skeleton K_s , is much greater than the compressibility modulus of the porous medium K_b , the simplified expressions of the coefficients are

$$P = \frac{4}{3}N + K_b + \frac{(1 - \phi)^2}{\phi}K_f, \quad (9)$$

$$Q = K_f(1 - \phi), \quad (10)$$

$$R = \phi K_f, \quad (11)$$

where the dynamic elastic moduli are

$$K_b = \frac{2N(\nu + 1)}{3(1 - 2\nu)}, \quad (12)$$

$$K_f = P_0 \left/ \left(1 - \frac{\gamma - 1}{\gamma\alpha'(\omega)}\right)\right., \quad (13)$$

in which $\alpha'(\omega)$ is the function from the Champoux-Allard model for the description of the thermal exchanges,

$$\alpha'(\omega) = \frac{8\bar{\nu}'}{j\omega\Lambda'^2} \sqrt{1 + \left(\frac{\Lambda'}{4}\right)^2 \frac{j\omega}{\bar{\nu}'}} + 1, \quad (14)$$

where N is the shear modulus, ν is the Poisson ratio, Λ' is the characteristic thermal length, $\bar{\nu}' = \bar{\nu}/Pr$, Pr is the Prandtl number, γ is the specific heats ratio of air, P_0 is the equilibrium pressure of air.

The expression of the surface impedance of a single layer of porous material placed against a rigid wall as shown in Figure 1 was worked out by Allard [2],

$$Z = -j \frac{Z_1^s Z_2^f \mu_2 - Z_2^s Z_1^f \mu_1}{D}, \quad (15)$$

where

$$D = (1 - \phi + \phi \mu_2) [Z_1^s - (1 - \phi) Z_1^f \mu_1] \tan(\delta_2 l) + (1 - \phi + \phi \mu_1) [Z_2^f \mu_2 (1 - \phi) - Z_2^s] \tan(\delta_1 l), \quad (16)$$

in which Z_i^s (analogously, Z_i^f) denotes the characteristic impedance of the i th compressional wave in the solid (fluid) part of the porous medium, which can be calculated as

$$Z_i^f = \left(R + \frac{Q}{\mu_i} \right) \frac{\delta_i}{\phi \omega}, \quad Z_i^s = \left(P + Q \mu_i \right) \frac{\delta_i}{\omega}, \quad (17)$$

$i = 1, 2$, with μ_1 and μ_2 the ratios of the velocity of the air over the velocity of the frame for the two compressional waves,

$$\mu_i = \frac{P \delta_i^2 - \omega^2 \tilde{\rho}_{11}}{\omega^2 \tilde{\rho}_{12} - Q \delta_i^2} = \frac{Q \delta_i^2 - \omega^2 \tilde{\rho}_{12}}{\omega^2 \tilde{\rho}_{22} - R \delta_i^2}, \quad i = 1, 2. \quad (18)$$

The parameters which describe the acoustical properties of the acoustical materials and structures are the reflection coefficient and absorption coefficient and they can be directly obtained from the surface impedance. While a plane wave impinges upon a layer of acoustical medium, the reflection coefficient R_s and absorption coefficient α_s of the layer can be expressed as

$$R_s = \frac{Z_s - Z_c}{Z_s + Z_c}, \quad (19)$$

$$\alpha_s = 1 - |R_s|^2, \quad (20)$$

where Z_c is the characteristic impedance of the medium neighboring to the layer and Z_s is the surface impedance at normal direction of the layer.

3. Transfer matrix method (TMM)

A multi-layer structure composed of n layers of porous materials is shown in Figure 2. The acoustic field in a given layer can be depicted by the particle velocities and stresses of the solid and fluid phase. At the front and rear face of the porous layer 1 it is expressed by the state vectors

$$V(M_1) = [\dot{u}^s(M_1) \dot{u}^f(M_1) \sigma^s(M_1) \sigma^f(M_1)]^T, \quad (21)$$

$$V(M_1) = [\dot{u}^s(M_1) \dot{u}^f(M_1) \sigma^s(M_1) \sigma^f(M_1)]^T, \quad (22)$$

where $\dot{u}^s$ and σ^s (analogously, $\dot{u}^f$ and σ^f) denote the particle velocity and stress of the solid phase (fluid phase,

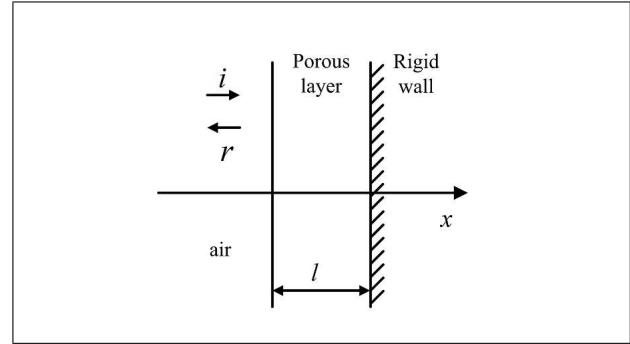


Figure 1. A porous layer placed against the rigid wall.

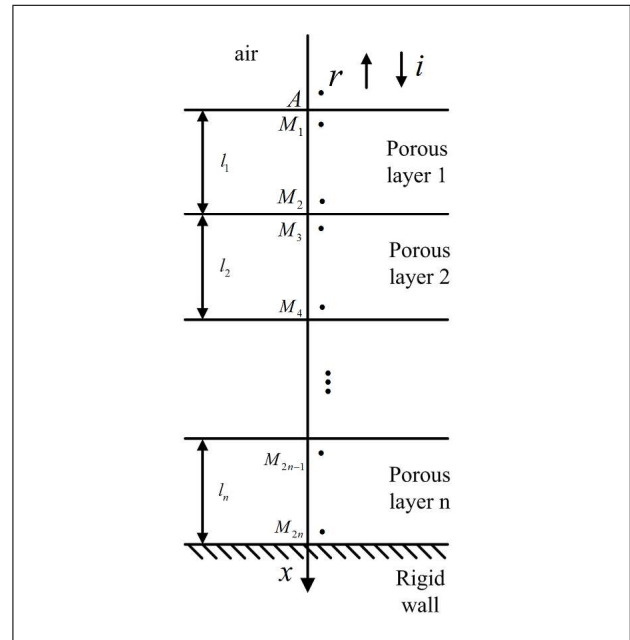


Figure 2. A multi-layer structure composed of n -layer of porous materials backed with rigid wall. Point A is the point of the air near the interface of the air and the structure. Point M_{2i-1} and M_{2i} represent the point of the front face and the rear face of the i th layer while $i = 1, 2, \dots, n$.

respectively). The frame and fluid displacement potentials can be written as

$$\varphi_i^s = (A_i e^{-j\delta_i x} + A'_i e^{j\delta_i x}) \quad i = 1, 2, \quad (23)$$

$$\varphi_i^f = \mu_i (A_i e^{-j\delta_i x} + A'_i e^{j\delta_i x}) \quad i = 1, 2. \quad (24)$$

The particle velocities and stresses are written as

$$\dot{u}^s = j\omega \frac{\partial(\varphi_1^s + \varphi_2^s)}{\partial x}, \quad (25)$$

$$\dot{u}^f = j\omega \frac{\partial(\varphi_1^f + \varphi_2^f)}{\partial x}, \quad (26)$$

$$\sigma^s = P \frac{\partial^2(\varphi_1^s + \varphi_2^s)}{\partial x^2} + Q \frac{\partial^2(\varphi_1^f + \varphi_2^f)}{\partial x^2}, \quad (27)$$

$$\sigma^f = Q \frac{\partial^2(\varphi_1^s + \varphi_2^s)}{\partial x^2} + R \frac{\partial^2(\varphi_1^f + \varphi_2^f)}{\partial x^2}. \quad (28)$$

Substitution of equations (23)–(28) into equations (21)–(22) yields

$$V(M_1) = \Gamma(M_1)A, \quad (29)$$

$$V(M_2) = \Gamma(M_2)A, \quad (30)$$

where

$$A = [A_1 + A'_1 \ A_1 - A'_1 \ A_2 + A'_2 \ A_2 - A'_2]^T. \quad (31)$$

Substituting equation (30) into equation (29), the transfer matrix T which relates $V(M_1)$ and $V(M_2)$ can be obtained as

$$V(M_1) = \Gamma(M_1)\Gamma'(M_2)V(M_2) = T V(M_2), \quad (32)$$

where Γ' denotes the inverse of matrix Γ . With the point M_2 taken at the origin of coordinates, the transfer matrix can be written as

$$T = \Gamma(-I_1)\Gamma'(0). \quad (33)$$

It depends only on the thickness of the layer. The elements of the matrices Γ , Γ' and T are given in the Appendix.

Considering the difference in the porosities of two adjacent porous materials, a coupling matrix must be used to connect two successive layers. Its elements are fixed by writing continuity conditions of the type

$$\dot{u}^s(M_2) = \dot{u}^s(M_3), \quad (34)$$

$$\phi_1(\dot{u}^f(M_2) - \dot{u}^s(M_2)) = \phi_2(\dot{u}^f(M_3) - \dot{u}^s(M_3)), \quad (35)$$

$$\sigma^s(M_2) + \sigma^f(M_2) = \sigma^s(M_3) + \sigma^f(M_3), \quad (36)$$

$$\sigma^f(M_2)/\phi_1 = \sigma^f(M_3)/\phi_2, \quad (37)$$

where ϕ_i denotes the porosity of the i th layer. In the case of two layers of porous materials, the global transfer matrix is written as

$$T^g = T_1 I_{12} T_2, \quad (38)$$

where T_1 and T_2 are the transfer matrix of outer and inner layer respectively and I_{12} is the coupling matrix

$$I_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 - \phi_2/\phi_1 & \phi_2/\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 1 - \phi_1/\phi_2 \\ 0 & 0 & 0 & \phi_1/\phi_2 \end{bmatrix}. \quad (39)$$

Suppose that the multi-layer structure is composed of n layers of porous materials and placed against a rigid wall. The relation between the state vectors at the front and the rear face of the structure is

$$V_1 = T^g V_{2n} = T_1 I_{12} T_2 \cdots T_i I_{i,i+1} \cdots T_n V_{2n} \quad (40)$$

and as the n th layer is against the rigid wall, the particle velocities are equal to zero

$$V(M_{2n}) = [0 \ 0 \ \sigma^s(M_{2n}) \ \sigma^f(M_{2n})]^T. \quad (41)$$

Considering the continuity of the velocity and the stresses at the front face of the first layer, we write

$$(1 - \phi)\dot{u}^s(M_1) + \phi\dot{u}^f(M_1) = v_A = p_A/Z_A, \quad (42)$$

$$\sigma^s(M_1) = -(1 - \phi_1)p(A), \quad (43)$$

$$\sigma^f(M_1) = -\phi_1 p(A), \quad (44)$$

where v_A , p_A and Z_A are the acoustic velocity, pressure and impedance at point A . Then using equations (40)–(41), equations (42)–(44) can be written as

$$M \begin{bmatrix} \sigma^s(M_{2n}) \\ \sigma^f(M_{2n}) \\ v_A \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (45)$$

where the coefficient matrix is

$$M = \begin{bmatrix} ((1-\phi_1)T_{13}^g + \phi_1 T_{23}^g) & ((1-\phi_1)T_{14}^g + \phi_1 T_{24}^g) & -1 \\ T_{33}^g & T_{34}^g & (1 - \phi_1)Z_A \\ T_{43}^g & T_{44}^g & \phi_1 Z_A \end{bmatrix}, \quad (46)$$

where T_{ij}^g denotes the j th column of the i th row element of the global matrix T^g in equation (40). A nontrivial solution exists when the determinant of M equals to zero and the surface impedance of the stratified structure Z_A is

$$Z_A = \frac{(T_{33}^g T_{44}^g - T_{34}^g T_{43}^g)}{\phi^2 D_1 + \phi D_2 + D_3}, \quad (47)$$

where

$$D_1 = T_{34}^g T_{23}^g - T_{24}^g T_{33}^g - T_{34}^g T_{13}^g + T_{44}^g T_{23}^g + T_{14}^g T_{33}^g - T_{24}^g T_{43}^g - T_{13}^g T_{44}^g + T_{14}^g T_{43}^g, \quad (48)$$

$$D_2 = T_{34}^g T_{13}^g - T_{23}^g T_{44}^g - T_{33}^g T_{14}^g + T_{24}^g T_{43}^g + 2T_{13}^g T_{44}^g - 2T_{14}^g T_{43}^g, \quad (49)$$

$$D_3 = T_{14}^g T_{43}^g - T_{13}^g T_{44}^g. \quad (50)$$

While there is only one layer of porous material, equation (47) is identical to equation (15) which has been validated by experiments [2].

4. Porous materials

The porous materials used in the calculations are the high flow resistivity materials chosen from [2], [20], [22] and their parameters are given in Table I. As mentioned above, the frame motions induced in high flow resistivity materials are apparent and affect the surface impedance very much. The resonance effect of this type of materials may be used to improve the global acoustic absorption and these three materials are chosen to be the representation of porous materials which have the similar flow resistivity level.

Number 1 material is a plastic foam [22] which has lower flow resistivity, number 2 material named Acousti-cell [20] is an absorbing expanded polyurethane foam and has a moderate flow resistivity and number 3 material [2] is the glass wool ŠDomisol Coffrage' manufactured by

Table I. Parameters for the three porous materials.

Material number	1	2	3
Tortuosity	1.5	1.38	1.06
Density of frame	20	26	130
Flow resistivity	10000	22000	40000
Porosity	0.98	0.96	0.94
Shear modulus	$80000(1+j0.1)$	$78050(1+j0.13)$	$2200000(1+j0.1)$
Poisson ratio	0.35	0.23	0
Viscous length	$100 \cdot 10^{-6}$	$17 \cdot 10^{-6}$	$56 \cdot 10^{-6}$
Thermal length	$200 \cdot 10^{-6}$	$40 \cdot 10^{-6}$	$112 \cdot 10^{-6}$

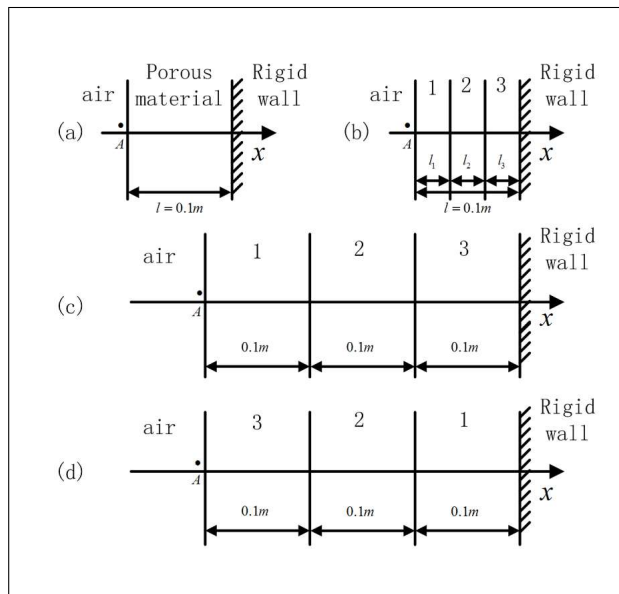


Figure 3. Geometry models calculated. (a) is a layer of porous material which thickness is 0.1 m placed against rigid wall; (b) is a layered structure composed of three different porous materials denoting by the number, the total thickness of this structure is 0.1 m and the thickness of the number one material is and so on; (c) and (d) are structure 1-2-3 and structure 3-2-1 respectively, the thickness of each component is the same and equal to 0.1 m.

St Gobain-Isover (BP19 60290 Rantigny France) with a higher flow resistivity. The materials are anisotropic, but the compressional waves in normal direction are identical in these materials and in an equivalent isotropic material which presents the same stiffness in the case of normal displacements, and whose other acoustical parameters are the same as for the fibrous material in normal direction.

Other constants used in the following calculations are given in Table II. They correspond to air at room temperature.

The normalized surface impedances Z_s/Z_0 of a single layer of porous material placed against a rigid wall and its front face neighbored by free air as shown in Figure 3a can be calculated using the TMM, just making the layer number equal to one in equation (47), as shown in Figure 4. The thickness is 0.1 m and $Z_0 = \rho_0 c_0$ is the characteristic impedance of air.

The corresponding absorption coefficients calculated by equation (20) are given in Figure 5. Frame resonances are

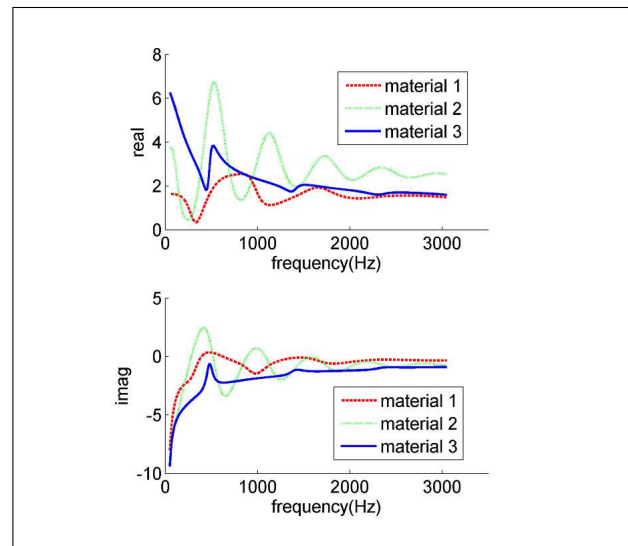


Figure 4. Real and imaginary part of the normalized surface impedances of three materials. Dashed: material 1; Dots: material 2; Solid: material 3.

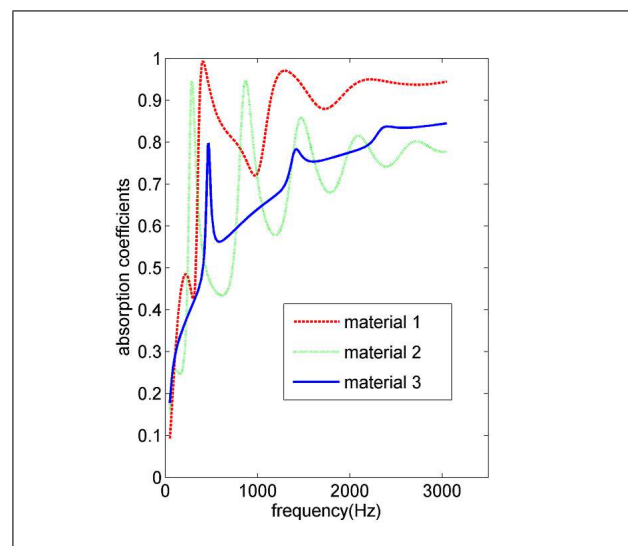


Figure 5. Absorption coefficients of three materials. Dashed: material 1; Dots: material 2; Solid: material 3.

clearly shown in Figure 4 and Figure 5 while the first resonance is the $\lambda/4$ resonance and the resonance frequency

Table II. Constants of air used during the calculations followed.

kinematic viscosity $\eta = 1.82 \cdot 10^{-5}$	Specific heats ratio $\gamma = 1.4$	Atmospheric pressure $P_0 = 1.0132 \cdot 10^5 \text{ Pa}$	Prandtl number $Pr = 0.713$	Density $\rho_0 = 1.205 \text{ kg/m}^3$	Sound speed $c_0 = 340 \text{ m/s}$
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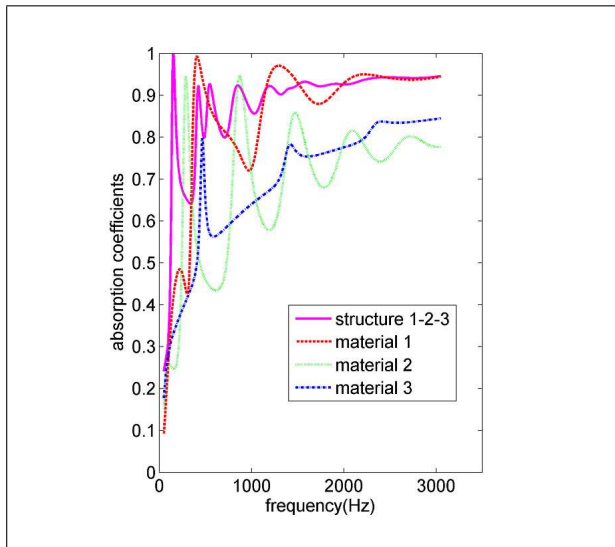


Figure 6. variations of the acoustic resonance frequencies of the three-layer structure and its components. Solid: structure 1-2-3 corresponding to Figure 3c; Dashed: material 1; Dots: material 2; Dashed-dotted: material 3.

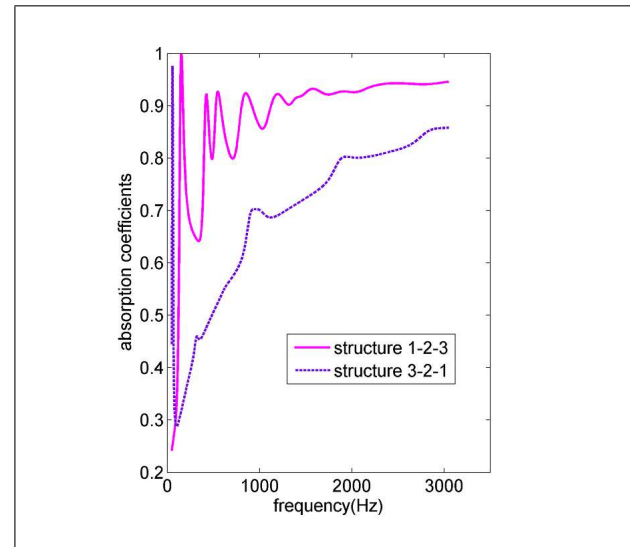


Figure 7. Comparison of the acoustic absorption coefficients for different arrangements of materials. Solid: structure 1-2-3 corresponding to Figure 3c; Dashed: structure 3-2-1 corresponding to Figure 3d.

has an approximate expression [8]

$$f_r \approx \frac{1}{4l} \sqrt{\frac{E(1-\nu)}{\rho_1(1+\nu)(1-2\nu)}} = \frac{1}{4l} \sqrt{\frac{2N(1-\nu)}{\rho_1(1-2\nu)}}, \quad (51)$$

where E is the young's modulus and l is the thickness of the layer. The surface impedance and absorption coefficient of the material 3 (Solid in Figure 4 and Figure 5) have been validated by experiments [10].

5. Results and discussions

Absorption coefficients of the structure 1-2-3 and single layer of each material are shown in Figure 6. As shown in Figure 3c, the structure 1-2-3 is a three-layer structure composed of three porous materials while each layer's thickness is 0.1 m. The material 1 and material 3 are bonded on the front and rear face of the material 2, respectively. The material 3 is backed with a rigid wall. Figure 6 reveals that the acoustic resonance frequencies of the three-layer structure are more broadband than those of their individual components. The resonance peaks of each layer are retained and have some frequency shifts and amplitude variations. The global sound absorption becomes better owing to the arrangement of the structure. When the incident frequency is equal to the acoustic resonance frequency of the structure, a large absorption coefficient nearly equals to one can be got and the porous materials existing in the structure will vibrate with the incident

sound wave and the incident sound energy will be largely absorbed. Except for the resonant mechanism to absorb the energy, the porous materials attenuate noise by dissipating energy and converting it into heat due to friction at high frequencies and heat exchange at low frequencies and that is why the porous materials give a good absorption performance in the whole frequency domain, especially at high frequencies. Nevertheless the low frequency absorption performance is improved by the arrangement of the structure.

Figure 7 shows the absorption coefficients of the structure 1-2-3 and structure 3-2-1. The difference between the arrangements of porous materials of these two structures is that the position of the material 1 and material 3 are interchanged, as shown in Figure 3d. It can be seen that the acoustic absorption of the structure 1-2-3 is generally better than the structure 3-2-1 except for a resonant frequency domain. This may be attributed to the difference of the materials' flow resistivity. The flow resistivity of the material 1 is the lowest and the material 3 the highest. The incident sound wave can gradually enter the structure 1-2-3 and be absorbed. However, for the structure 3-2-1, the incident sound energy is almost reflected from the outermost porous material except at the acoustic resonance frequency band; therefore, the acoustic absorption is largely reduced at the frequency bands away from the acoustic resonance frequencies. Other resonance peaks at high frequency domain are not notable as for the good sound attenuation of porous materials. The characteristic of the porous materi-

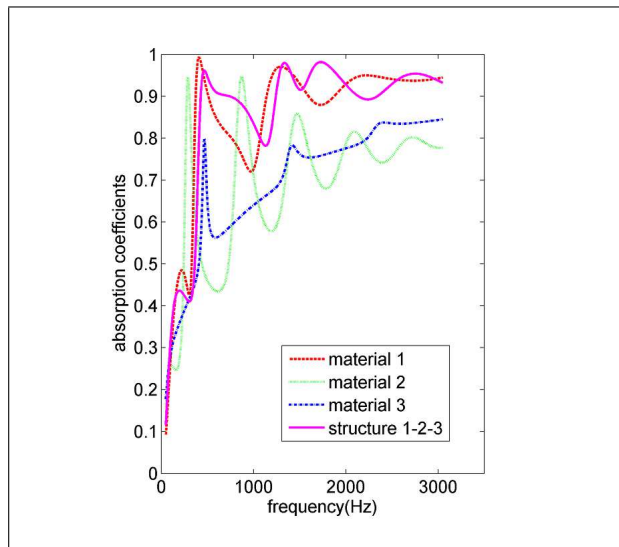


Figure 8. Comparison of the acoustic absorption coefficients for the layered structure and each single layer of material. Solid: layered structure corresponding to Figure 3b; Dashed: material 1; Dots: material 2; Dashed-dotted: material 3.

als' arrangement is familiar to that of the perforated plates discussed in [14].

The thickness of the structure 1-2-3 employed above is three times of each single porous layer for the structure is just the composition of three single layers; the great improvement of the sound absorption performance is therefore consequent. The sound absorption of the structure composed of three layers of porous materials with a comparable thickness to that of each single layer mentioned above is discussed and shown in Figure 8. The thicknesses of material 1, material 2, material 3 and the layered structure shown in Figure 3b are the same, equal to 0.1 m. The components' thickness of the layered structure is the same and equals to 0.033 m. The arrangement of this layered structure is the same as that of the structure 1-2-3: the material 1 is the outermost layer neighbored by free air and the material 3 is the innermost layer backed with a rigid wall. It can be seen from Figure 8 that the global sound absorption of this layered structure is not good as that of the structure 1-2-3 shown in Figure 6, especially the low frequency domain, while the disparity at the high frequency band is small. It is well known that the high frequency absorption of porous materials is not improved apparently with the increase of thickness while the low frequency performance related to thickness tightly, and the performance of layered structure is expectable. Nevertheless, the arrangement of different materials improves the absorption performance at some frequency domains comparing with individual layer, such as 500 Hz to 1000 Hz for material 1.

It can be seen from the equation (51) that the resonance frequency of the porous materials is about negatively correlated to the thickness. The global absorption coefficients of the layered structure may be improved by distributing

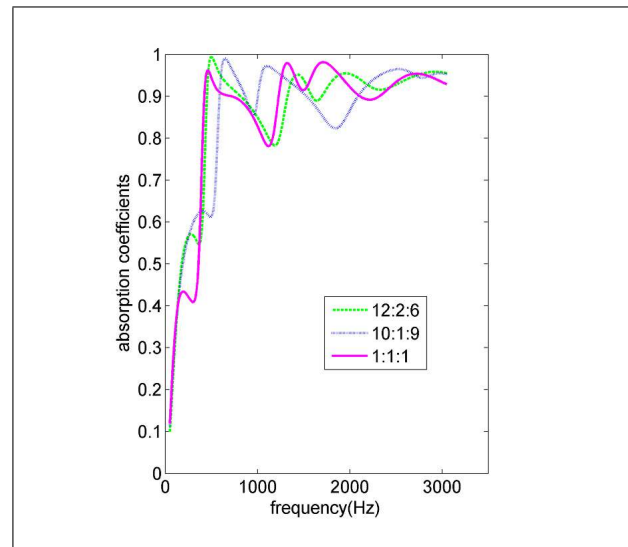


Figure 9. Absorption coefficients of layered structures with different components' thickness ratio. Solid: 1:1:1; Dashed: 12:2:6; Dots: 10:1:9.

the thickness of each material and separating the resonance frequencies. The optimizations are shown in Figure 9.

As shown in Figure 9, the configuration of equal thickness is not the best. The configurations with thickness ratio $l_1 : l_2 : l_3$ equals to 12:2:6 and 10:1:9 give better absorption coefficients. Different objective functions have different optimizations while the optimization is not the object of this work.

6. Conclusions

This article described the Biot's model for calculating the complex wave numbers of the compressional waves propagating in porous materials, and the surface impedances and acoustic absorption coefficients of multi-layer structures composed of different porous materials are predicted by the TMM. The acoustic absorption of the combination of the materials and their arrangements are also discussed. It is shown that the acoustic absorption of the multi-layer structure is better than that of each single layer while the performance of the low frequency band is improved and the resonance peaks of each layer retained. The arrangement of the materials influences the performance heavily and the arrangement of decreased flow resistivity from the outermost layer to the innermost layer has better acoustic absorption coefficient while the sound waves enter the materials easily and be absorbed inside. The influence of the multi-layer structure's thickness is removed by making it equal to that of single layer and the comparisons show that some of the resonance peaks exist and improve the absorption coefficient. Separating the resonance frequencies by distributing the thickness of each material to improve the absorption coefficient is also done and is available.

Appendix

The acoustic field at the point x can be expressed as

$$V(x) = \Gamma(x) A. \quad (A1)$$

The elements of the matrix $\Gamma(x)$ are

$$\begin{aligned} \Gamma_{11}(x) &= -j\omega\delta_1 \sin(\delta_1 x), \\ \Gamma_{12}(x) &= \omega\delta_1 \cos(\delta_1 x), \\ \Gamma_{13}(x) &= -j\omega\delta_2 \sin(\delta_2 x), \\ \Gamma_{14}(x) &= \omega\delta_2 \cos(\delta_2 x), \\ \Gamma_{21}(x) &= -j\omega\mu_1\delta_1 \sin(\delta_1 x), \\ \Gamma_{22}(x) &= \omega\mu_1\delta_1 \cos(\delta_1 x), \\ \Gamma_{23}(x) &= -j\omega\mu_2\delta_2 \sin(\delta_2 x), \\ \Gamma_{24}(x) &= \omega\mu_2\delta_2 \cos(\delta_2 x), \\ \Gamma_{31}(x) &= -(P + \mu_1 Q)\delta_1^2 \cos(\delta_1 x), \\ \Gamma_{32}(x) &= j(P + \mu_1 Q)\delta_1^2 \sin(\delta_1 x), \\ \Gamma_{33}(x) &= -(P + \mu_2 Q)\delta_2^2 \cos(\delta_2 x), \\ \Gamma_{34}(x) &= j(P + \mu_2 Q)\delta_2^2 \sin(\delta_2 x), \\ \Gamma_{41}(x) &= -(\mu_1 R + Q)\delta_1^2 \cos(\delta_1 x), \\ \Gamma_{42}(x) &= j(\mu_1 R + Q)\delta_1^2 \sin(\delta_1 x), \\ \Gamma_{43}(x) &= -(\mu_2 R + Q)\delta_2^2 \cos(\delta_2 x), \\ \Gamma_{44}(x) &= j(\mu_2 R + Q)\delta_2^2 \sin(\delta_2 x). \end{aligned}$$

The nonzero elements of the inverse of matrix $\Gamma(0)$ are

$$\begin{aligned} \Gamma'_{13}(0) &= \frac{Q + R\mu_2}{\delta_1^2(PR - Q^2)(\mu_1 - \mu_2)}, \\ \Gamma'_{14}(0) &= -\frac{P + Q\mu_2}{\delta_1^2(PR - Q^2)(\mu_1 - \mu_2)}, \\ \Gamma'_{21}(0) &= \frac{-\mu_2}{\delta_1\omega(\mu_1 - \mu_2)}, \\ \Gamma'_{22}(0) &= \frac{1}{\delta_1\omega(\mu_1 - \mu_2)}, \\ \Gamma'_{33}(0) &= -\frac{Q + R\mu_1}{\delta_2^2(PR - Q^2)(\mu_1 - \mu_2)}, \\ \Gamma'_{34}(0) &= \frac{P + Q\mu_1}{\delta_2^2(PR - Q^2)(\mu_1 - \mu_2)}, \\ \Gamma'_{41}(0) &= \frac{\mu_1}{\delta_2\omega(\mu_1 - \mu_2)}, \\ \Gamma'_{42}(0) &= \frac{-1}{\delta_2\omega(\mu_1 - \mu_2)}. \end{aligned}$$

The transfer matrix of a porous layer which thickness is d is

$$T = \Gamma(-d) \Gamma'(0). \quad (A2)$$

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