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Acoustic Behaviour of Lightly Damped Rooms

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Abstract

The analytical method has been developed to determine the acoustic behaviour of a lightly damped room excited by a sound source. Using the classical modal analysis, a solution of the wave equation was found as a sum of an infinite number of components representing individual responses of acoustic modes to the sound source. For weak sound absorption a general form of this solution was determined by means of the method of variation of parameters. From this solution the equations for the Green's function and the room impulse response were found. By employing the Green's function, the expressions describing a sound build-up and a sound decay for a harmonic excitation were derived.

1. Introduction

Acoustics of rooms has been studied for many years because a room behaviour in steady-state and transient stages needs to be evaluated at the design phase. A prediction of a sound field in rooms and a determination of room acoustic parameters are not simple and several theoretical methods of different complexity have been developed. These include: statistical theory [1], statistical energy analysis [2], diffusion-equation models [3], geometrical methods such as the image source method [4] and the ray-tracing techniques [5], and methods employing a modal expansion approach which are based on the wave theory. Among these methods are: normal mode analysis [6] which is the oldest and simplest type of modal analysis, classical modal analysis [7] developed by using the Green's theorem, asymptotic modal analysis [8] and hybrid modal analysis [9] that combines the free field Green's function and a modal expansion. The methods based on the wave theory are the most appropriate from the physical point of view because they include wave phenomena such as diffraction and interference of sound waves, but their disadvantage is a significant increase in computation times at higher frequencies.

In order to determine the acoustic response of room to any sound source, it is necessary to find a solution of inhomogeneous wave equation for a point source generating an impulse in time. Based on the classical modal analysis, this paper presents a general method for finding this solution when weak sound damping inside a room is assumed. A theory developed provides the Green's function for lightly damped enclosures and also the room impulse response which can be used for a prediction of such significant room parameters as early decay time and reverberation time. The proposed method is a generalization of theoretical models presented by the author in the previous works [10, 11].

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2. General solution

For an arbitrarily shaped enclosure a theoretical description of a room response to acoustic excitation is based on a solution of the inhomogeneous wave equation

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial p}{\partial t} = -Q(t), \tag{1}$$

where $p(\boldsymbol{r}, t)$ is the sound pressure, c is the sound speed, $Q(\boldsymbol{r}, t)$ is the volume source term and $\boldsymbol{r} = (x, y, z)$ is the position vector. This solution must satisfy a causality condition and the following boundary condition on the surface S of room walls [1],

$$\frac{\partial p}{\partial n} = -\frac{\rho}{Z}\frac{\partial p}{\partial t}, \tag{2}$$

where the symbol $\partial/\partial n$ denotes a derivative in a direction outward normal to the boundary, ρ is the air density and Z is the acoustic wall impedance. It is assumed that initial conditions are defined for the time t_0 which means that $p(\boldsymbol{r}, t_0)$ and $p'(\boldsymbol{r}, t_0) = \partial p(\boldsymbol{r}, t)/\partial t|_{t=t_0}$ are given.

In a modal approach, a solution of equation (1) is sought in the form of an infinite sum representing a superposition of individual responses of all acoustic modes excited inside the room by a sound source. Thus, after excluding the trivial solution of equation (1) the sound pressure $p(\boldsymbol{r}, t)$ inside the room can be described as

$$p(\boldsymbol{r}, t) \;=\; \sqrt{V}\sum_{m=1}^{\infty} P_m(t)\,\Phi_m(\boldsymbol{r}), \tag{3}$$

where V is the room volume, the function $P_m(t)$ determines a temporal behaviour of the sound pressure for the mth mode and Φ_m are eigenfunctions. The functions Φ_m are mutually orthogonal and are normalized in the room volume V by the relation

$$\iiint_V \Phi_m\,\Phi_n\,\mathrm{d}v = \delta_{mn}, \tag{4}$$

where δ_{mn} is the Kronecker delta function. Moreover, each eigenfunction Φ_m is related to the corresponding eigenvalue λ_m through the equation

$$\nabla^2\Phi_m = \lambda_m\,\Phi_m. \tag{5}$$

The use of equations (1) and (2)–(5) together with the Green's theorem [12],

$$\iiint_V \left(p\nabla^2\Phi_m - \Phi_m\nabla^2 p\right)\mathrm{d}v = \iint_S \left(p\frac{\partial\Phi_m}{\partial n} - \Phi_m\frac{\partial p}{\partial n}\right)\mathrm{d}s, \tag{6}$$

leads to the system of coupled differential equations

$$\frac{\partial^2 P_m}{\partial t^2} + 2\sum_{n=0}^{\infty} C_{mn}\frac{\partial P_n}{\partial t} - c^2\lambda_m P_m + c^2\iint_S p\frac{\partial\Phi_m}{\partial n}\,\mathrm{d}s \;=\; f_m(t), \tag{7}$$

where C_{mn} are cross-modal coefficients given by

$$C_{mn} = \frac{\rho c^2}{2}\iint_S \frac{\Phi_m(\boldsymbol{r})\Phi_n(\boldsymbol{r})}{Z}\,\mathrm{d}s \tag{8}$$

and $f_m(t)$ is the modal source function

$$f_m(t) \;=\; \frac{c^2}{\sqrt{V}}\iiint_V Q(\boldsymbol{r}, t)\Phi(\boldsymbol{r})\,\mathrm{d}v. \tag{9}$$

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When a weak sound damping is assumed, series components in equation (7) containing the coefficients C_{mn} for $m \neq n$ can be omitted and it is possible to approximate the functions Φ_m by eigenfunctions for rigid room walls [7]. In such a case, the boundary condition $\partial\Phi_m/\partial n = 0$ is satisfied and $\lambda_m = -(\omega_m/c)^2$, where ω_m is the mth natural eigenfrequency. Thus, for lightly damped rooms equation (7) simplifies to uncoupled differential equations

$$\frac{\partial^2 P_m}{\partial t^2} + 2r_m \frac{\partial P_m}{\partial t} + \omega_m^2 P_m = f_m(t), \tag{10}$$

where the quantity r_m represents modal damping coefficient and is given by

$$r_m = \frac{\rho c^2}{2} \iint_S \frac{\Phi_m^2(\boldsymbol{r})}{Z}\,\mathrm{d}s. \tag{11}$$

To solve equation (10) the method of variation of parameters was applied [13]. Using this method the general solution of equation (10) was found as

$$\begin{aligned} P_m(t) \;=\; & \frac{[x_m(t)y'(t_0) - y_m(t)x'(t_0)]P_m(t_0)}{W_m(t_0)} \\ & - \frac{[x_m(t)y_m(t_0) - y_m(t)x_m(t_0)]P'_m(t_0)}{W_m(t_0)} \\ & - x_m(t)\int_{t_0}^{t} \frac{y_m(\tau)f_m(\tau)}{W_m(\tau)}\,\mathrm{d}\tau \\ & + y_m(t)\int_{t_0}^{t} \frac{x_m(\tau)f_m(\tau)}{W_m(\tau)}\,\mathrm{d}\tau, \end{aligned} \tag{12}$$

where the functions $x_m(t)$ and $y_m(t)$ given by

$$\begin{aligned} x_m(t) \;&=\; \exp\left[(-r_m - \mathrm{j}\Omega_m)t\right], \\ y_m(t) \;&=\; \exp\left[(-r_m + \mathrm{j}\Omega_m)t\right] \end{aligned} \tag{13}$$

are fundamental solutions of the homogeneous equations

$$\frac{\partial^2 P_m}{\partial t^2} + 2r_m \frac{\partial P_m}{\partial t} + \omega_m^2 P_m = 0, \tag{14}$$

the function $W_m(t) = x_m y'_m - x'_m y_m$ is the Wronskian of x_m and y_m, $P_m(t_0)$ and $P'_m(t_0)$ are initial conditions for the mth mode and the quantity

$$\Omega_m = \sqrt{\omega_m^2 - r_m^2} \tag{15}$$

is the eigenfrequency for oscillations with damping, also called the damped natural eigenfrequency.

3. Room response

Equation (12) makes it possible to determine the function $P_m(t)$ which may be regarded as a modal response to a sound source described by the function $f_m(t)$. An insertion of $P_m(t)$ into equation (3) enables a prediction of a room response to the acoustic excitation whose a space distribution and temporal changes are characterized by the source term $Q(\boldsymbol{r}, t)$. For example, when the room is excited by the harmonic source, the source term can be expressed as: $Q(\boldsymbol{r}, t) = q(\boldsymbol{r})\cos(\omega t)$, where $q(\boldsymbol{r})$ is the volume source distribution and ω is the source frequency. It indicates that the source function has the form: $f_m(t) = Q_m \cos(\omega t)$, where Q_m is the factor determining source strength for the mth mode

$$Q_m \;=\; \frac{c^2}{\sqrt{V}} \iiint_V q(\boldsymbol{r})\Phi_m(\boldsymbol{r})\,\mathrm{d}v. \tag{16}$$

Now, if it is assumed for simplicity that the source starts to radiate sound at the moment $t_0 = 0$ then imposing the initial conditions $P_m(0) = P'_m(0) = 0$ in equation (12) and inserting the obtained function $P_m(t)$ into equation (3), the formula describing a sound build-up inside a room can be found as

$$\begin{aligned} p(\boldsymbol{r}, t) \;=\; & -\sqrt{V}\sum_{m=1}^{\infty} Q_m \mathrm{e}^{-r_m t}\left[\Omega_m(\omega_m^2 - \omega^2)\cos(\Omega_m t)\right. \\ & \left. + r_m(\omega_m^2 + \omega^2)\sin(\Omega_m t)\right]\Phi_m(\boldsymbol{r}) \\ & \cdot \left\{\Omega_m\left[(\omega_m^2 - \omega^2)^2 + 4r_m^2\omega^2\right]\right\}^{-1} \\ & - Q_m\left[(\omega_m^2 - \omega^2)\cos(\omega t) + 2r_m\omega\sin(\omega t)\right]\Phi_m(\boldsymbol{r}) \\ & \cdot \left[(\omega_m^2 - \omega^2)^2 + 4r_m^2\omega^2\right]^{-1}, \end{aligned} \tag{17}$$

where the first component in the series constitutes transient term which decays exponentially with the time while the second one represents a steady-state harmonic term.

An alternative and more general way of finding a room response relies on the use of the Green's function. According to [12], this function determines room response when the acoustic source located at the point $\boldsymbol{r} = \boldsymbol{r}_0$ generates an impulse at the time $t = t_0$. Equation (9) shows that in this special case the modal source function $f_m(t)$ is given by

$$f_m(t) = \frac{c^2\Phi_m(\boldsymbol{r}_0)\delta(t - t_0)}{\sqrt{V}}. \tag{18}$$

An insertion of equation (18) into equation (12) enables to determine the function $P_m(t)$ for the impulse source. Next, substituting $P_m(t)$ in equation (3) the Green's function can be found. Since integrals in equation (12) have the lower limit corresponding exactly to the peak of the delta function $\delta(t - t_0)$, the integration should be carried out from $t_0^- = t_0 - \epsilon$, where $\epsilon > 0$ is arbitrarily small. In this case the interval of integration includes the value at which the delta function peaks and $P_m(t_0^-)$ and $P'_m(t_0^-)$ are equal to zero, because of a causality condition. This leads to the following formula for the Green's function

$$G(\boldsymbol{r}, t|\boldsymbol{r}_0, t_0) = c^2\sum_{m=1}^{\infty} \frac{\mathrm{e}^{-r_m(t-t_0)}\sin\left[\Omega_m(t-t_0)\right]\Phi_m(\boldsymbol{r}_0)\Phi_m(\boldsymbol{r})}{\Omega_m}. \tag{19}$$

Additionally, it is assumed that G and $\partial G/\partial t$ are zero for $t < t_0$ because if an impulse occurs at t_0, no effects of the impulse should be present at the earlier time (a causality condition). Since the right-hand side of equation (19) is a symmetric function of the sound source and the observation points coordinates, the function G satisfies the reciprocity principle.

The Green's function G enables a simple determination of a room response because following on the Green's function theory [12], the acoustic pressure $p(\boldsymbol{r}, t)$ generated inside a room by a sound source described by the function $Q(\boldsymbol{r}, t)$ is expressed as

$$p(\boldsymbol{r}, t) \;=\; \iiint_V \int_{-\infty}^{t} \left[Q(\boldsymbol{r}_0, t_0)G(\boldsymbol{r}, t|\boldsymbol{r}_0, t_0)\right]\mathrm{d}t_0\,\mathrm{d}v_0. \tag{20}$$

For example, the result such as in equation (17) is obtained after inserting $Q(\boldsymbol{r}_0, t_0) = q(\boldsymbol{r}_0)\cos(\omega t_0)$ in equation (20) and integrating over the room volume V and the time t_0, where a lower limit of t_0 is set to zero. On the other hand, the reverberant response of a room for the harmonic source which is stopped at the moment $t_0 = 0$, is obtained by performing a time integration

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from $-\infty$ to zero. It yields the formula

$$p(\boldsymbol{r},t) = \sqrt{V}\sum_{m=1}^{\infty} Q_m \mathrm{e}^{-r_m t}\left[\Omega_m(\omega_m^2-\omega^2)\cos(\Omega_m t) + r_m(\omega_m^2+\omega^2)\sin(\Omega_m t)\right]\Phi_m(\boldsymbol{r}) \cdot\left\{\Omega_m\left[(\omega_m^2-\omega^2)^2+4r_m^2\omega^2\right]\right\}^{-1}, \quad (21)$$

which shows that the function describing the reverberant response and the transient terms in equation (17) taken with opposite sign are identical. An inspection of equation (20) clearly shows that an alternative expression for pressure $p(\boldsymbol{r},t)$ can be written as

$$p(\boldsymbol{r},t) = \iiint_V \left[Q(\boldsymbol{r}_0,t) * H(\boldsymbol{r},\boldsymbol{r}_0,t)\right]\mathrm{d}v_0, \quad (22)$$

where the symbol * denotes a convolution operation and the function

$$H(\boldsymbol{r},\boldsymbol{r}_0,t) = c^2\sum_{m=1}^{\infty}\frac{\mathrm{e}^{-r_m t}\sin(\Omega_m t)\Phi_m(\boldsymbol{r}_0)\Phi_m(\boldsymbol{r})}{\Omega_m}, \quad (23)$$

which corresponds to the Green's function G for the time t_0 equal to zero, can be interpreted as the room impulse response between a sound source situated at the position $\boldsymbol{r}_0$ and a receiver located at the point $\boldsymbol{r}$. It should be emphasised that the formula (23) is consistent with a general expression for the room impulse response proposed by Kuttruff [1] and employed later by Dragonetti *et al.* [14, 15] to compute the noise-free synthesized room impulse response. From equation (23) it follows that a modal response is inversely proportional to the damped natural eigenfrequency Ω_m. This allows us to draw the important conclusion that for lightly damped rooms higher-order modes weakly contribute to the room impulse response.

4. Summary and conclusions

In the paper, the analytical method has been used to determine the acoustic behaviour of a lightly damped room subjected to a volume sound source located inside a room space. A solution of the inhomogeneous wave equation has been sought in the form of an infinite sum representing a superposition of individual responses of acoustic modes to the sound excitation. For weak sound damping a general form of the solution was found from uncoupled differential equations by means of the method of variation of parameters. In this solution a contribution of absorptive room walls to the room response manifests itself through an exponential decay of modal amplitudes with the time and a decrease in natural modal frequencies. From this general solution the equations for the Green's function and the room impulse response were found. These formulas allow to determine the room response to any arbitrary sound source and in this study, the expressions describing a sound build-up and a sound decay for a harmonic excitation were derived, for example. A knowledge of the room impulse response enables the evaluation of reverberation time by using the Schroeder's backward integration method and provides information about other important parameters used in assessment of the acoustic quality of a room such as early decay time (EDT), clarity (C), definition (D), centre time (CT), strength (S), and lateral energy fraction (LEF).

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