Mutually Comparison of Sub-Optimal Passive Sonar Detection Structures

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The two most important sub-optimal detection structures of the passive sonar are considered: the so-called wideband and standard structures. For the both structures is supposed that the useful signal of the vessel noise, on the receiving location, is much weaker then the interference (signal by deep-sea ambient acoustic noise). The sub-optimal structures are synthesized on the basis of optimal detection structure. The idea was to synthesize the sub-optimal structures as optimal structures, but for simplified receiving signal. And so, the wideband sub-optimal structure is a real optimal structure to the receiving signal without narrowband parts. The standard sub-optimal structure is a real optimal structure to the receiving signal that has, besides, the constant power spectral density. The comparison is on the basis of the detection probability values.

1 Introduction

The receiving signal is due either to underwater acoustic deep-sea ambient noise only (null hypothesis H_0) or to additive mixture of underwater acoustic deep-sea ambient noise and degenerate underwater acoustic noise of a vessel (alternative hypothesis H_1). The degeneration of the underwater acoustic vessel noise is the consequence of the sound propagation in the sea mass as acoustic medium. These in homogeneities change in random manner and make the sea mass to be a dispersive stochastic filter [1]. On the other hand, the ambient deep-sea noise is a stochastic process too. So, the receiving sonar signal is an additive mixture of two stochastic processes. We suppose that both stochastic processes have Gaussian distribution. This assumption is very near to real situations for the deep-sea researches [2].

The receiving signal is the underwater acoustic signal picked up by the hydrophones (underwater acoustic sonar sensors) located in the deep-sea.

We suppose that the useful component of the underwater receiving signal is present (hypothesis H_1 is true: vessel is present).

2 Optimal structure

On the output of the sonar receiver we have function $l(y)$, which is the basis to decide that the receiving signal is only deep-sea ambient noise (null hypothesis H_0 : vessel is not present) or that the receiving signal is a sum of deep-sea ambient noise and the useful signal (alternative hypothesis H_1 : vessel is present). We decide on the basis of comparison between the instantaneous value of $l(y)$ and the reduced threshold l_0 of Neyman-Pearson statistical criterion [3]. Now we can optimize detection process and to research the best sub-optimal detection structure.

On the output of the decision-making device we accept the alternative hypothesis H_1 as the true one for

$$l(y) \geq l_0 \tag{1}$$

and we accept the null hypothesis H_0 as true one for

$$l(y) < l_0 \ . \tag{2}$$

The threshold l_0 is a function of an accepted value of the false alarm probability α (the null hypothesis H_0 is true but we decide that the alternative hypothesis H_1 is true). It means that the detection probability D (the alternative hypothesis H_1 is true and we decide that the alternative hypothesis H_1 is true) and the miss probability β (the alternative hypothesis H_1 is true but we decide that the null hypothesis H_0 is true) are functions of the false alarm probability α too.

The receiving signal, for the alternative hypothesis in the time domain, has the following form on the receiving location

$$y(t) = n(t) + s(t) \tag{3}$$

where $n(t)$ is the underwater acoustic deep-sea ambient noise (interference or simple - noise) and $s(t)$ is the degenerate underwater acoustic noise of the vessel (useful signal or simple - signal).

The deep-sea ambient noise $n(t)$ we can approximate as a stationary ergodic Gaussian stochastic process with zero mean and with finite variance $(\sigma_n)^2$. The useful signal $s(t)$ we can approximate as a stationary ergodic Gaussian stochastic process with zero mean and with finite variance $(\sigma_s)^2$. Both processes are mutually statistically independent. Therefore the receiving signal for the alternative hypothesis in the time domain, as their sum, is also stationary ergodic Gaussian stochastic process with zero mean and with finite variance of the following form

$$(\sigma_y)^2 = (\sigma_n)^2 + (\sigma_s)^2 \ . \qquad (4)$$

It means, that testing the hypotheses we really test the value of variance of the receiving signal.

If the signal processing is in digital form then the analog receiving signal has to be transformed in the M discrete samples. If the time discrete samples are mutually statistically independent then the optimal detection structure of the passive sonar has the following form for the alternative hypothesis

$$\sum_{k=1}^{K} y^2(t_k) \geq l_0 \qquad (5)$$

where t_k for k=1,2, … K are discrete times.

But, unfortunately the discrete time samples of the underwater receiving acoustic signal are not mutually statistically independent. This is the consequence of the form of power spectral density of the receiving signal. Its power spectral density is not constant (white process) but has a slope of about -6±1 dB per octave [4].

In order to be mutually statistically independent the discrete samples have to be coefficients of an orthogonal expansion of the receiving signal waveform. The best one is Karhunen-Loéve expansion. Acceptable its approximation is complex Fourier expansion [5]. Its coefficients are in the frequency domain.

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So, the form of the power spectral density is the basis of the optimal passive sonar structure. On the other hand, the simplified form of the power spectral density is the basis of the sub-optimal passive sonar structure.

And so, for the very sophisticated underwater acoustic receiving signal $y(t)$ the optimal detection sonar structure for the alternative hypothesis H_1 , in the frequency domain, has the following form

$$\sum_{k=1}^{K} \frac{S(\omega_k)/N^2(\omega_k)}{1+M\,\dfrac{S(\omega_k)}{N(\omega_k)}} \left| \sum_{m=1}^{M} \tilde{y}_{km} \right|^2 \geq C \qquad (6)$$

where:
- $S(\omega_k)$ are the magnitudes of two-sided power spectral densities, for discrete frequencies $f_k > 0$, for the vessel noise component $s(t)$ of the underwater receiving signal $y(t)$.
- $S(\omega_k)$ is the sum of wideband components $S_{wb}(\omega_k)$ and of narrowband components $S_{nb}(\omega_k)$.
- $N(\omega_k)$ are the magnitudes of two-sided power spectral densities, for discrete frequencies $f_k > 0$, for the deep-sea ambient noise component $n(t)$ of the underwater receiving signal $y(t)$.
- $\tilde{y}_{ki}$ are the complex Fourier coefficients, for discrete frequencies $f_k > 0$ and for i-th hydrophone, of the underwater receiving signal $y(t)$.
- K is number of discrete frequencies.
- M is number of hydrophones.
- C is statistical threshold of detection.

To evaluate the optimal passive sonar detection structure we have to compute so called ROC (Receiver Operating Characteristics) diagrams [6]. The ROC diagram is the function of detection probability $D \geq 0.5$ versus false-alarm probability α. In Figure 1 is shown a curve of the ROC diagram of the optimal passive sonar structure and in Table 1 are detection probability values versus several values of false-alarm probability [7].

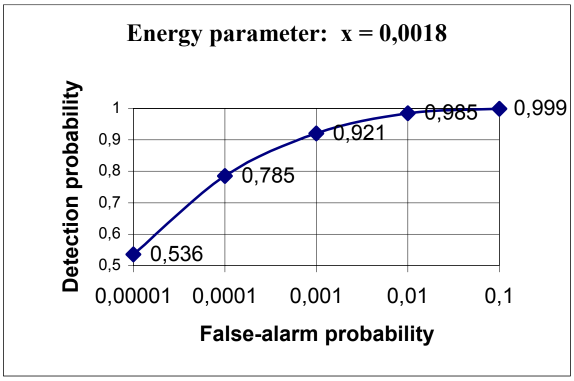

Figure 1: Optimal ROC diagram

Table 1: Detection probability values of the optimal passive sonar structure versus several values of false-alarm probability

D_{opt}	0.536	0.785	0.921	0.985	0.999
α	0.00001	0.0001	0.001	0.01	0.1

The energy parameter x is defined as product of M (number of hydrophones in underwater array) and of ν (ratio of the power spectral densities of the part of useful signal due to cavitation and of deep-sea ambient noise for the equal frequency), or

$$x = M\nu \,. \qquad (7)$$

The values of the chosen parameters are [6]:
- Number hydrophones M =20.
- Processing time of the signal T = 625 s.

- Frequency band B_0 = 10 kHz.
- Frequency band of discrete tone B_q = 2 Hz.
- Number of discrete tones Q = 2.

In Figure 2 is shown another curve (for different energy parameter) of the ROC diagram of the optimal passive sonar structure and in Table 2 are detection probability values versus the same values of false-alarm probability [7].

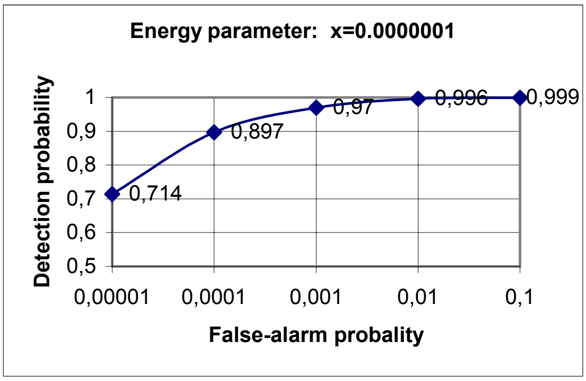

Figure 2: Optimal ROC diagram

Table 2: Detection probability values of the optimal passive sonar structure versus several values of false-alarm probability

D_{opt}	0.714	0.897	0.970	0.996	0.999
α	0.00001	0.0001	0.001	0.01	0.1

In Figure 1 is shown only one curve of optimal ROC diagram. This curve is the worst one for the false-alarm probability from 0.00001 to 0.1, where we have more other curves with energy parameters values from 0.0018 to 0.003. All other curves with the values of energy coefficients more then 0.003 give for the full interval of false-alarm probability the value of detection probability equal $D = 1$.

On the other hand, in Figure 2 also is shown only one curve of optimal ROC diagram. This curve is the worst one for the false-alarm probability from 0.00001 to 0.1, but for completely different values of energy coefficients compare to the situation in the Figure 1. Now, in that interval of the false-alarm probability, we have more other curves with the energy parameters that are about 18000 times less then that in the Figure 1. This is very interesting fact for the research the optimal detection structure of the passive sonar

3 Sub-optimal structures

The underwater acoustic receiving signal for the passive sonar structure has a very complicated form in the time domain and consequently also in the frequency domain. Therefore the optimal detection process for the passive sonar structure (6) is very sophisticated too. So, the practical and in the same time rational realization is not an easy work.

Because of that, we try to simplify detection process and, in the same time, to simplify the practical realization. To simplify the detection process means to simplify the complicated and not enough known receiving signal. The most disturbing parts in the power spectral density of the receiving signal are narrowband processes due to the vessel harmonic oscillations generated by the operating machinery, mechanisms and propeller(s). So, we suppose that the receiving signal has not the narrowband processes.

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3.1 Wideband structure

It means that the power spectral density of the receiving signal becomes only wideband and much simpler. The power spectral density of the useful signal, due to vessel cavitation only, now completely resembles to the power spectral density of the underwater deep-sea ambient acoustic noise and has the following form

$$S(\omega_k) = \nu N(\omega_k). \tag{8}$$

Besides, we supposed that the useful part of the underwater receiving signal is extremely weak compare to the deep-sea acoustic noise and we may to cancel the second member of the denominator in (6), or

$$M \frac{S(\omega_k)}{N(\omega_k)} << 1 \ . \tag{9}$$

On the basis of (8) and (9) we can write the approximate relation for (6). This approximation is the wideband sub-optimal detection structure.

Consequently, the wideband sub-optimal detection structure of the passive sonar has the form

$$\sum_{k=1}^{K} \frac{\nu}{N(\omega_k)} \left| \sum_{m=1}^{M} \tilde{y}_{km} \right|^2 \geq C \ . \tag{10}$$

To evaluate the wideband sub-optimal passive sonar detection structure we have to compute the Receiver Operating Characteristics diagram. The ROC diagram is the function of detection probability $D \geq 0.5$ versus false-alarm probability α. In Figure 3 is shown a curve of the ROC diagram of the sub-optimal wideband passive sonar structure and in Table 3 are detection probability values versus several values of false-alarm probability [7].

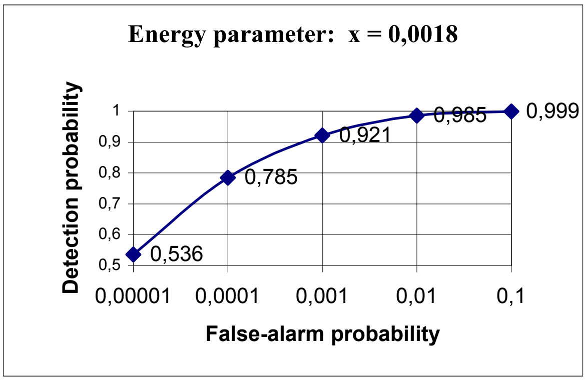

Figure 3: Sub-optimal wideband ROC diagram

Table 3: Detection probability values of the sub-optimal wideband passive sonar structure versus several values of false-alarm probability

D_{sow}	0.536	0.785	0.921	0.985	0.999
α	0.00001	0.0001	0.001	0.01	0.1

In Figure 3 is shown only one curve of sub-optimal ROC diagram. This curve is the worst one for the false-alarm probability from 0.00001 to 0.1, where we have more other curves with energy parameter of the values from 0.0018 to 0.003. All other curves with the values of energy coefficients more then 0.003 give for the full interval of false-alarm probability the value of detection probability equal $D = 1$. It is evident that sub-optimal ROC diagram in Figure 3 is completely the same as the optimal ROC diagram in Figure 1. It means that sub-optimal wideband passive sonar structure is an excellent substitution to the optimal passive sonar structure but only for the energy parameter $x > 0.0018$. Sufficient value of energy parameter is 0.003.

On the other hand, the sub-optimal wideband passive sonar structure has not the sub-optimal ROC diagram similar to one as in Figure 2 for optimal passive sonar structure with very small values of energy parameter $x<10^{-7}$.

3.2 Standard structure

Another configuration of the sub-optimal structure is so called standard structure. The power spectral densities of the useful signal and of the ambient deep-sea noise are now supposed to be both wideband and constant. That is the simplest form of power spectral density (white random process). Therefore, now we have

$$2S(\omega_k) = S_0 \quad (11)$$

and

$$2N(\omega_k) = N_0 \quad (12)$$

and finally (8) becomes

$$S_0 = \nu N_0 . \quad (13)$$

Besides, we have supposed again that the useful part of the receiving signal is extremely weak. So, we may cancel the second member of the denominator in (6), and (9) now becomes

$$M \frac{S_0}{N_0} << 1 . \quad (14)$$

On the basis of (13) and (14) we can write the approximate relation for (6). This approximation is the sub-optimal standard detection structure.

Consequently, the sub-optimal standard detection structure of the passive sonar has the following form

$$\frac{2\nu}{N_0} \sum_{k=1}^{K} \left| \sum_{m=1}^{M} \tilde{y}_{km} \right|^2 \geq C . \quad (15)$$

To evaluate the sub-optimal standard passive sonar detection structure (15) we have to compute the ROC diagram. In Figure 4 is shown a curve of the ROC diagram of the sub-optimal standard passive sonar structure and in Table 4 are detection probability values versus several values of false-alarm probability [7].

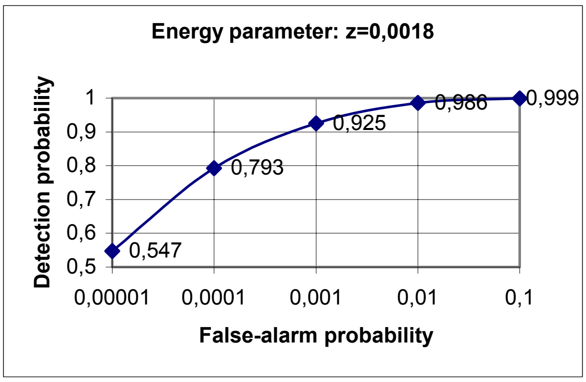

Figure 4: Sub-optimal standard ROC diagram

Table 4: Detection probability values of the sub-optimal standard passive sonar structure versus several values of false-alarm probability

D_{sos}	0.547	0.793	0.925	0.986	0.999
α	0.00001	0.0001	0.001	0.01	0.1

In Figure 4 is shown only one curve of sub-optimal standard ROC diagram. This curve is the worst one for the false-alarm probability from 0.00001 to 0.1, where we have more other curves with energy parameter of the values from 0.0018 to 0.003. All other curves with the values of energy parameter more then 0.003 give for the full interval of false-alarm probability the value of detection probability equal $D = 1$. It is evident that sub-optimal standard ROC diagram in Fig. 4 is almost the same as the optimal ROC diagram in Figure 1. It means that sub-optimal standard passive sonar structure is an excellent substitution to the optimal passive sonar structure but only for the energy parameter $x > 0.0018$. Sufficient value of energy parameter is 0.003.

On the other hand, the sub-optimal standard passive sonar structure has not the sub-optimal ROC diagram similar to one as in Figure 2 for optimal passive sonar structure with very small values of energy parameter $x<10^{-7}$.

4 Conclusions

The sub-optimal structures, wideband and standard, have been analyzed through the knowledge of the optimal passive sonar structure. The fundamental idea was simplification of the passive sonar detection structure.

The first basic simplification was to suppose that the receiving underwater acoustic signal has not very sophisticated narrowband Gaussian processes due to the vessel operating machinery, mechanisms and propeller(s). So, we have supposed that the useful part of the underwater acoustic receiving signal was only the wideband Gaussian process due to the vessel cavitation phenomenon. Its power spectral density resembles to the power spectral density of the deep-sea acoustic ambient noise (random color process with slope about -6±1 dB per octave and with frequency band of 10 kHz), but is ν-times weaker. So, we have got the so-called wideband sub-optimal passive detection structure.

The second basic simplification was to suppose again that the receiving underwater acoustic signal has not very sophisticated narrowband Gaussian processes due to the vessel operating machinery, mechanisms and propeller. But now we have supposed one thing more: that both parts of the receiving signal, the useful signal and the deep-sea ambient noise, are the wideband white Gaussian process with the constant power spectral densities.

Finally, we have concluded that sub-optimal passive sonar detection structures, wideband and standard, are excellent substitution to the optimal passive sonar detection structure, but only for the energy parameter $x > 0.0018$. Here, we have to emphasize, that the sub-optimal standard passive sonar detection structure has the trivial advantage over a wideband sub-optimal passive sonar detection structure: a little greater detection probability values for the same false-alarm probability values and a bit simpler detection relation.

On the other hand, neither of sub-optimal structures has the ROC diagram with the values of detection probability $D \geq 0.5$ for the very small values of the energy parameter $x<10^{-7}$. Therefore, for these values of energy parameter, the sub-optimal wideband and standard structures are completely useless.

Acknowledgement

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