

Comparison of Models to Determine the Radiation Resistance of a Radiating Surface.

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The mechanical radiation resistance is a significant parameter in radiating systems. It is directly related to the performance of the system [1], [2]. This parameter can be determined using different models. In this study, the flat piston model, the resistance matrix model and the Maidanik model used in SEA (Statistical Energy Analysis) [3], [4], are analysed. The experimental measurements of acoustic power in a diffuse field measured with an accelerometer are used as reference values. The velocities in the surface of the panel, obtained using the Near-field Acoustic Holography technique (NAH) [5], are used as input values to determine, by means of numerical integration, the pressure radiated out to any distance from the source in accordance with the methodology used in [1]. These data are used to determine the mechanical radiation impedance, thereby offering a more precise prediction of the power radiated by the speaker in order to reach a comparison between the different models.

1 Introduction

The performance of a DML speaker is based on the principle of distributed modes, which consists of exciting the diaphragm so that it produces the maximum number of flexural resonances. The resulting vibration is so complex that it produces a diffuse field. In this paper, the aim is to predict the acoustic radiation resistance of a DML speaker using three different methods: the flat rectangular piston model, the acoustic radiation resistance matrix model and the Maidanik model. In order to arrive at an objective prediction, velocity input data are used at different points of the panel, by employing NAH. Experimental measurements of acoustic power in a reverberating chamber are carried out in accordance with the proceedings stated in regulation UNE-EN ISO 3741, [6], in order to indirectly calculate the radiation resistance.

2 GENERAL FORMALISM

2.1 Mechanical radiation impedance of a rectangular surface. Flat piston.

The flat piston model begins with the hypothesis that, generally speaking, the radiation produced by the vibration of an extended surface, such as a piston, can be obtained by adding the pressures which would be derived from the association of simple sources. It is assumed that the vibration velocity is perpendicular to the surface and that it is constant at all its points. The

total pressure, radiated by a rectangular surface with harmonic vibration velocity $u_0(x, y)$ results in

$$p = -j\rho_0 f e^{-j\omega t} \int_{x=-Lx/2}^{x=Lx/2} \int_{y=-Ly/2}^{y=Ly/2} \frac{u_0(x, y) e^{jk\sqrt{(x-x_0)^2 + (y-y_0)^2 + z_0^2}}}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + z_0^2}} dx dy \quad (1)$$

By substituting the vibration velocity of a surface in (1), the pressure radiated at a point in the space can be established. This facilitates the prediction of the pressure field generated by the plate.

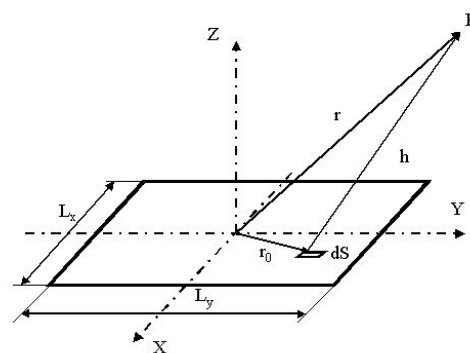


Figure 1- Rectangular vibrating piston

The concept of mechanical radiation impedance is related to the reaction force F_R exerted by the air on the speaker panel, at variance with the movement of the

panel, with regard to the velocity with which the panel moves.

$$Z_{MR} = \frac{F_R}{u_0 e^{-j\omega t}} \quad (2)$$

The reaction force depends on the pressure exerted on the surface of the membrane

$$F_R = -\iint_S p dS' \quad (3)$$

The expression of the radiated pressure is substituted by a rectangular surface (1) in the reaction force, being integrated throughout all the surface elements and by substitution in (3), the result being

$$Z_{MR} = \frac{1}{u_0} j\rho_0 f \int_{x'=-Lx/2}^{x=Lx/2} \int_{y'=-Ly/2}^{y=Ly/2} \int_{x=-Lx/2}^{x=Lx/2} \int_{y=-Ly/2}^{y=Ly/2} \frac{u_0(x,y) e^{jk\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}}}{\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}} dx dy \quad (4)$$

This expression is calculated by applying the Romberg integration method.

2.2 Acoustic radiation resistance matrix

The acoustic radiation resistance matrix R_{ik} corresponds to the transference function which connects the normal velocities of a vibrating structure with the sound pressures at its surface. This amount is independent from the vibration, being only dependent upon the geometry of the structure. The dimension of R_{ik} is defined by the number of virtual subdivisions within the structure. The local specific radiation impedance on surface S can be defined as the complex ratio between the sound pressure amplitude p_i at point i due to the specific source placed at point k, and to velocity V_{n_k} .

$$Z_{S_{ik}} = \left(\frac{p_i}{V_{n_k}} \right) \quad (5)$$

Assuming that the characteristic length of surface elements is small in comparison with the typical acoustic wavelength, then pressure and velocity can be considered constant for each element and can be represented using a mean value. The acoustic impedance being

$$Z_{ik} = \left(\frac{p_i}{u_k} \right) \quad (6)$$

The acoustic radiation resistance matrix R_{ik} is obtained taking the real part of Z_{ik} . The radiated sound power of a vibrating structure is defined as the mean time ratio in which the sound energy flows along a complete surface surrounding the structure. When considering that the structure is divided into N small elements, the radiated power is

$$W_{rad} = \frac{1}{2} \sum_{i=1}^N R_{ii} |u_i|^2 + \frac{1}{2} \sum_{i=1}^N \sum_{\substack{k=1 \\ k \neq i}}^N R_{ik} |u_i| |u_k| \cos(\phi_k - \phi_i) \quad (7)$$

where the first part of equation (7) corresponds to the normal resistance and the second part to the crossed resistance, which offers a measurement of the acoustic coupling between the *i*-th and *k*-th surface elements. This is numerically assessed using the Hashimoto method (*discrete calculation method*), in which the panel is divided into virtual small elements. Each rectangular element is treated as a circular piston with an area similar to that of the corresponding element. Using this approach, the normal and crossed resistance result in

$$R_{ii} = \rho_0 c S_i \left[1 - \frac{J_1(2ka_i)}{ka_i} \right] \quad (8)$$

$$R_{ik} = \frac{2\rho_0 c k^2 S_i S_k}{\pi} \left[\frac{J_1(2ka_i)}{ka_i} \frac{J_1(2ka_k)}{ka_k} \right] \frac{\text{sinc} kr_{ik}}{kr_{ik}} \quad (9)$$

Respectively, where c is the sound velocity, S_i y

S_k are the surfaces of the equivalent pistons,

$a_i = \sqrt{S_i / \pi}$ and $a_k = \sqrt{S_k / \pi}$ are the radii of the pistons and J_1 is the Bessel function of the first type.

2.3 Radiation resistance of a panel derived by Maidanik

The radiation resistance is the reaction exerted by the environment to the displacement of the diaphragm or of a plate for a DML speaker. The acoustic power is defined as the product of this resistance by the square of the velocity of the plate.

$$W = R_{rad} \cdot u^2 \quad (10)$$

R_{rad} : radiation resistance (kg/s), W acoustic power of the source (Watt), u : displacement velocity (m/s)

The first mathematical expression to model the behaviour of radiation resistance was formulated by Maidanik [7], and was subsequently corrected [8] [9]:

$$R_{rad} = 2 \cdot A \rho_a c \left\{ 2 \left(\frac{\lambda_c \lambda_a}{A} \right) \frac{f}{f_c} g_1 + \frac{p \lambda_c}{A} g_2 \right\} \quad \text{donde !} \quad (11.1)$$

fora $f < f_c$ y $k \cdot a > 2\pi$

where $g_1 = \frac{4}{\pi^4} \left\{ \frac{1 - 2\psi^2}{\psi(1 - \psi^2)^{1/2}} \right\}$ para! $f/f_c < 0,5$
 $g_1 = 0$ para! $f/f_c \geq 0,5$

$$g_2 = \frac{1}{4\pi^2} \left\{ (1 - \psi^2) \ln \left[\frac{1 + \psi}{1 - \psi} \right] + 2\psi \right\} \left\{ \frac{1}{(1 - \psi^2)^{3/2}} \right\}$$

where $\psi = \left(\frac{f}{f_c} \right)^{1/2}$ $p = 2a + 2b$ (11.2)

for $f < f_c$ y $k \cdot a < 2\pi$

$$R_{rad} = 2 \cdot A \rho_a c \frac{4}{\pi^4} \left(\frac{p \lambda_c}{A} \right) \left(\frac{f}{f_c} \right)^{1/2} \quad (11.3)$$

for $f = f_c$

$$R_{rad} = 2 \cdot A \rho_a c \left\{ \left(\frac{a}{\lambda_c} \right)^{1/2} + \left(\frac{b}{\lambda_c} \right)^{1/2} \right\} \quad (11.4)$$

for $f > f_c$

$$R_{rad} = 4 \cdot A \rho_a c \left\{ 1 - \left(\frac{f_c}{f} \right) \right\}^{1/2}$$

where a, b are: dimensions of the plate (m), p: perimeter of the plate (m), f_c : critical frequency of the plate (Hz), A: surface of the plate (m^2), ρ_a : air density (kg/m^3), c: velocity of sound in air (m/s), λ_a : wavelength in air (m) : $\lambda_a = c/f$, λ_c : wavelength at the critical frequency (m) : $\lambda_c = c/f_c$, k: wavenumber in air at frequency f : $k = 2\pi/\lambda_a$.

2.4 NAH

The NAH technique is used to reconstruct the sound field and the vibration velocity of an object or object

source or sound source from pressures measured in an array of microphones, on a parallel plane placed near the source. This technique includes the recording of the amplitude and the phase of the sound pressure using a flat array of microphones, in order to calculate the structure of the sound source, the object of study, using digital reconstruction techniques.

The NAH retropropagation equation is based on the spatial Fourier transform [10]. This allows the normal component of velocity to be determined as follows:

$$\hat{V}(k_x, k_y, d) = (1 / \omega \rho) \hat{P}(k_x, k_y, h) (k_z e^{-ik_z(h-d)}) \quad (12)$$

3 Experimental procedure

A framed DML 59x57 cm2 speaker was analysed. An electromechanic transducer consisting of a magnet and reel was located at the back of the upper part of the panel. The speaker was placed in a semi-anechoic chamber in order to measure the velocity in the surface of the panel using NAH. A linear array of '1/4'' microphones with loop transmission power, a professional sound card, and a 16 channel amplifier were used. A robot was employed for measurements on a parallel plane placed at 6 cm. from the speaker. Measurements were taken every 5 cm on the x-axis and on the y-axis. A multilayer audio recorder was used to record the signals. The signal emitted for the study was a white noise at a sampling frequency of 44100 Hz. The acoustic power measurements were carried out in the reverberating chamber of the Higher Polytechnic School of Gandia in accordance with regulation UNE-EN ISO 3741. The acceleration on the panel was measured with an accelerometer and integration was carried out to obtain the values for the velocity.

4 Results

The prediction of the normalised radiation resistance values using the rectangular piston model and the resistance matrix model can be seen in Figure 2, assuming a mean constant velocity calculated from the measurements with NAH for a DML speaker of 60x60 cm². In turn, the SEA radiation resistance, calculated at the critical frequency of 4000 Hz [11], and the curve obtained from the experimental measurement, are represented. It can be observed that from 100 Hz to 4 kHz, the radiation resistance increases., Maximum resistance is present at 4 kHz and from this frequency it is maintained at more or less a constant value. The piston and resistance matrix models display a similar behaviour, gradually increasing with frequency up to 4kHz. For the case of SEA and the experimental

measurements, an increase of up to 4 kHz presents a steeper slope than in the previous case and an abrupt decrease can be observed in the maximum radiation resistance value, which tends to stabilise.

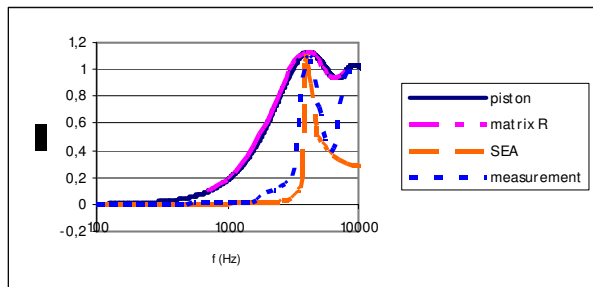


Figure 2- Normalised radiation resistance for different methods: flat piston, radiation resistance matrix, SEA and experimental measurements of a 45x60cm panel.

5 Summary

The flat piston method and the resistance matrix method offer similar results in the prediction of the acoustic radiation resistance of a DML speaker.

Maidanik's radiation resistance method and the experimental measurements present similar behaviour.

The difference between these two behaviours can be accounted for by the fact that for the flat piston method and the resistance matrix method, the mechanical part of the panel is not considered.

The four methods confirm that the maximum value of the radiation resistance of a flat speaker is presented at around 4 kHz.

6 Acknowledgements

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