

The influence of ocean waves in shallow water problems

Kormann J. A.

CSIC, Instituto de Acústica, C/ Serrano 144, 28006 Madrid, jkormann@hotmail.com

C. Ranz Guerra

CSIC, Instituto de Acústica, C/ Serrano 144, 28006 Madrid, iacrg32@ia.cetef.csic.es

A. Fernández Fernández

CSIC, Instituto de Acústica, C/ Serrano 144, 28006 Madrid, iacf319@ia.cetef.csic.es

In this paper we present the influence of ocean waves in shallow water scenarios which have geometric dimensions of about several times the acoustic wavelength. Typically, this concerns costal zones. It is well known that ocean waves are a source of scattering, have dispersive behaviour, and that at low frequencies, are a source of noise. We will base our work on measurements realized in shallow water conditions, and attempt to model ocean waves and include them in existing codes of acoustic propagation using finite difference schemes. Additionally, we will compare the differences between measurements and prediction models and try to discuss the reliability of the models for the Shallow water case.

1 Introduction

In this paper we will present a numerical implementation for calculating normal modes. The influence of the bottom depends on several parameters, as source frequency, water depth, acoustic ducts and ocean wave. Furthermore, in shallow water environment, its influence will be preponderant.

There are some approaches to modelize the ocean bottom interaction, in this paper we choose to use a *continuously stratified elastic layer of finite thickness* resting on rigid basement to represent the ocean bottom. The bottom characteristics will depend on the shear and compressional speed.

To resolve the acoustic problem, we used the normal modes approach, which is one way to solve the *Helmholtz equation*. For this, we will use finite difference schemes, and we could consequently introduce a z dependance for the velocity profiles, in the ocean and in the bottom.

Some problems occur when we have to solve the eigenvalues problems, we need to be accurated in their calculations, because the errors can degrade the accuracy of the normal mode representation as we are more far away from the source plane.

In this paper we will be only working with range independent medium, the medium characteristics will not depend on the "r" direction.

2 Problem formulation

The problem is the following: it consist in a *Pekeris problem* [1] with pressure release and rough ocean surface ($z=0$), and a fluid-elastic interface at $z=D_1$, with z positive in the downward direction. $z=D_2$ correspond to the elastic bottom layer depth. On the second hand, will

introduce the absorption due to the ocean surface roughness.

2.1 Solving the modal problem

We need to solve the modal equation :

$$p''(z) + [\omega^2 / c^2(z) - k_n^2] p(z) = 0 \quad (1)$$

The first step is to linearize (1), using finite difference schemes [1], so we will discretize the downward direction (z), from zero to D_1 , which is the bottom depth. We define ω as the circular frequency of the source, and k_r the horizontal wavenumber.

The second step is to modelize the bottom propagation. Following M.B. Porter and E.L. Reiss approach [2], we lead to solve the following system of four-first order equations :

$$r' = Er \quad (2)$$

where :

$$E(z, k) = \begin{bmatrix} 0 & -1 & 1/(\rho c_s^2) & 0 \\ k^2 \eta & 0 & 0 & 1/(\rho c_p^2) \\ k^2 \zeta - \rho \omega^2 & 0 & 0 & -\eta \\ 0 & -\rho \omega^2 & 0 & 0 \end{bmatrix}$$

$$\eta \equiv \frac{c_p^2 - 2c_s^2}{c_p^2}, \quad \zeta \equiv \rho c_p^2 (1 - \eta^2), \quad ik_r r_1 \equiv u,$$

$r_2 \equiv w$, $ik_r r_3 \equiv \tau_{zx}$, $r_4 \equiv \tau_{zz}$. Here c_p and c_s are the compressional and shear speeds of the elastic bottom.

Finally, we need six more equations to solve the complete problem which are given by the boundary conditions :

$$p(0)=0, \quad \omega^2 r_2(D_1)=p'(D_1), \quad r_3(D_1)=0, \\ r_4(D_1)=-p(D_1), \quad r_1(D_2)=r_2(D_2)=0$$

We will focus only on the water problem (water modes), following Jensen, we introduce the impedance matching condition :

$$g(k_r^2)p'(D_l)+f(k_r^2)p(D_l)=0 \quad (3)$$

We will use equation (2) to compute the f and g functions using explicit second-order integrator for first-order systems [2]. The final step, is to solve the linear system $A(k_r^2)=0$ (4), where A is defined by:

$$A = \begin{bmatrix} a_1 - h^2 k_r^2 & 1 & 0 & \dots & \dots & 0 \\ 1 & a_2 - h^2 k_r^2 & 1 & 0 & \dots & 0 \\ 0 & 1 & a_3 - h^2 k_r^2 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & 1 & a_{N-1} - h^2 k_r^2 & 1 \\ 0 & \dots & \dots & \dots & 0 & 2 & a_N - h^2 k_r^2 - 2hf(k_r^2)/g(k_r^2) \end{bmatrix}$$

and the coefficients a_i are defined by:

$$a_i \equiv -2 + h^2 \omega^2 / c^2(z_i), i=1,2,\dots,N_l$$

Now we need to solve equation (4). For this we need to calculate the characteristic polynomial of A. We will use the Sturm's sequence [3] to determine it:

$$\begin{aligned} S_0 &= 1 \\ S_1 &= a_1 - h^2 k_r^2 \\ S_i &= (a_i - h^2 k_r^2)S_{i-1} - S_{i-2}, i=2,\dots,N_l-1 \\ S_{N_l} &= (a_{N_l} - h^2 k_r^2 - 2hf(k_r^2)/g(k_r^2))S_{N_l-1} - 4S_{N_l-2} \end{aligned} \quad (7)$$

We know that the eigenvalues are the zeros of our characteristic polynomial. To find wich values will null S, we use a *brut force method*. We create a wavenumber vector, whose maximum is the smallest wavenumber of our problem (i.e w/c_{\min} , where c_{\min} is the smallest velocity of our problem) and zero as minimum.

This choices are due to the fact that we will focus on the propagation modes in water. Finally, all sign changes of S define an eigenvalue. The second step is to find on the complex axis the others eigenvalues wich are solutions of our problems. We use the same vectors but with pure imaginary numbers. Once again the zeros of the Sturm's sequence are eigenvalues. The precision of the method is two times the incrementation between each components of the wavenumber vector. Finally, we will calculate the eigenvectors using the inverse iteration method [4].

2.2 Introducing ocean wave effect

Now we will introduce the effect of ocean waves. For this, we will use the Kuperman & Ingenito formulation [5]. The idea is to express the absorption coefficient using the normal mode calculated before. Rather than using a formal solution, Kuperman & Ingenito introduce little perturbations on the boundary conditions and put the perturbed boundary on the modal equation (1). Assuming that the Kirchoff approximation is right [5], we use the following surface spectrum :

$$P(\vec{\eta} - \vec{\zeta}) = 2\pi\delta^2(\vec{\eta} - \vec{\zeta}) \quad (5)$$

where δ is the Dirac function. This leads to *the modal surface absorption coefficient* :

$$\delta_n^s(\text{Kirchoff}) = \rho_l \langle \alpha^2 \rangle (2k_n A_n)^{-1} [v_n'(0)]^2 [k_l^2(0) - k_n^2]^{1/2} \quad (6)$$

where $\alpha(r)$ is the roughness of the ocean surface, ρ_l is the ocean density (as constant), v_n the n^{th} normalized mode [1], k_l is the surface wavenumber, A_n the normalization of the n^{th} mode, and k_n the n^{th} modal wavenumber.

2.3 Calculating the acoustic pressure

The final step is to compute the acoustic pressure. The exact solution of our problem is an infinite sum of modes, but we can sum over a finite number of them [1]:

$$p(r, z) \approx \frac{i}{4\rho(z_s)} \sum_{n=1}^M v_n(z_s)v_n(z)H_0^1(k_n r)\exp(-\delta_n^s r) \quad (7)$$

where z_s is the source depth, ρ the density at the source depth, H_0^1 the Hankel function of the first kind and p the acoustic pressure. Generally, we represent the Transmission Loss by:

$$TL(r, z) = -20 \log_{10} \left| \frac{p(r, z)}{p(r=1)} \right| \quad (8)$$

3 Two examples

3.1 Deep water case

The first problem we will analyzing is a deep water case [2]. We remind that we are working in a Pekeris problem. The ocean depth is $D_1=5000\text{m}$. The Bottom layer depth is $D_2=6000\text{m}$. As velocity profil, we will use the Munk profil:

$$\begin{aligned} \tilde{z} &= 2(z - 1300)/1300 \\ c(z) &= 1500(1 + 0.00737(\tilde{z} - 1 + \exp(-\tilde{z}))) \end{aligned} \quad (9)$$

We will introduce linear profiles for the compressional and shear velocity :

$$c_p(z) = 4700 + 100(z - D_1)/(D_2 - D_1) \quad (10)$$

$$c_s(z) = 2000 + 100(z - D_1)/(D_2 - D_1) \quad (11)$$

The density is constant for the whole range, and for both the bottom and the ocean $\rho_w=1000 \text{ Kg/m}^3$, $\rho_b=2000 \text{ Kg/m}^3$. The circular frequency is $30\pi/\text{s}$.

We will use the Pierson-Moskowitz spectrum for fully developed wind-generated sea to describe the sea surface roughness [5].

Now we will present the results in the form of transmission loss contour computed using 60 modes, and graphically representing some of the calculated modes.

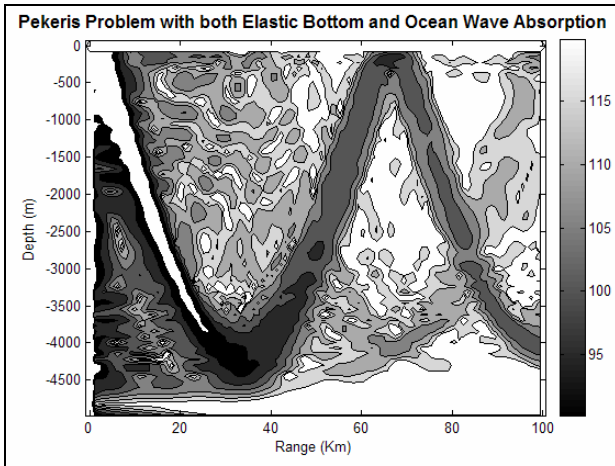


Figure 1

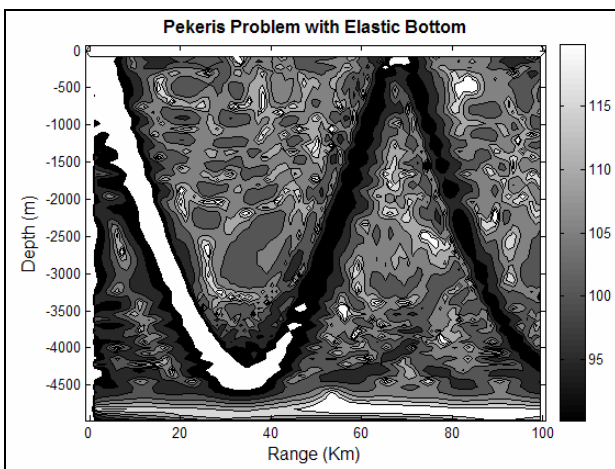


Figure 2

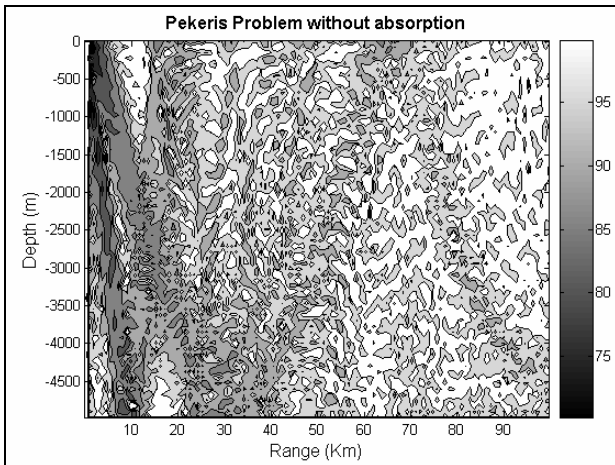


Figure 3

The figures 1-3 represent the transmission loss (dB) for our problem, with three different absorptions.

In Figure 1 we introduce an ocean wave generated by a wind with a 15m/s velocity.

On Figure 2, we only introduce an elastic bottom with the characteristics mentioned earlier.

Figure 3 represents the Pekeris problem without absorption.

We can see from the Figures 1-3 the influence of the absorption of the ocean wave and of the bottom. We can notice that the more absorption is introduced in the medium, the more the TL (Transmission Loss) rises. While the absorption is introduced, the higher modes decay at long range, and finally only the waterborne modes will propagate, since they are less dependent on the boundary condition. This can be easily viewed if we compare Figure 1 and Figure 2: we see that there are more shadow zones on Figure 1, which means that the higher modes decay.

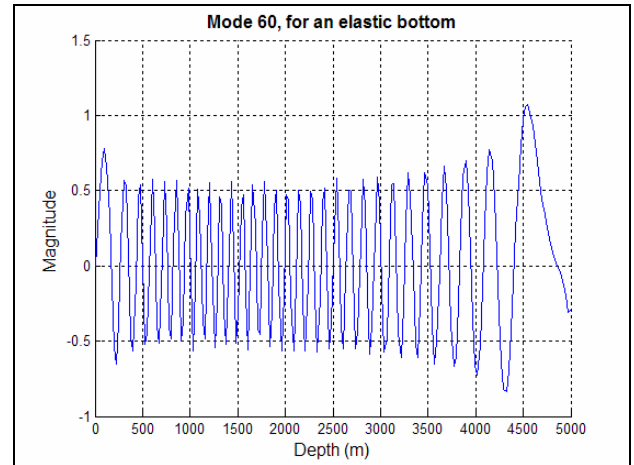


Figure 4

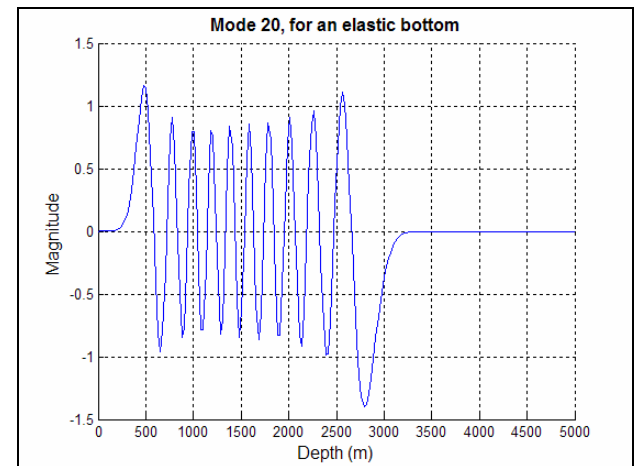


Figure 5

3.2 Shallow Water case

We are still working with the Pekeris problem. The ocean depth is $D_1=100m$. The Bottom layer depth is $D_2=300m$. The ocean velocity is constant and equal to 1500m/s. Densities are constant for the whole range and the same as in the previous case. The circular frequency is $200 \pi/s$. The compressional speed and the shear speed are constant and equal to 1700m/s and 300 m/s respectively. We use the same ocean surface spectrum that for the deep water problem. The wind speed is 6 m/s which corresponds to 21 Km/h.

We will compare the results with the Marsh & Shulkin semi-empirical expressions [6]. This semi-empirical expression are particularly useful, they are based on 100000 measurements in shallow water and are valid for a frequency range of 0.1 to 10 kHz.

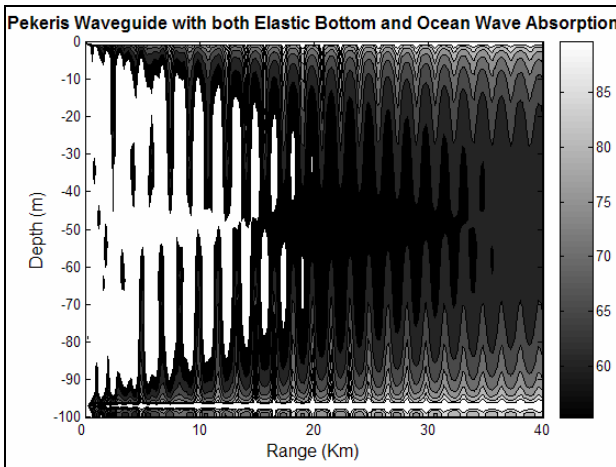


Figure 6

First we note the effect of the ocean wave: we can see that behind a range of 10 km we have a cylindrical spreading. The higher mode are attenuated due to both effects of the bottom and the ocean wave.

Now we will compare with the results obtained with the M. & S. expressions. At 20 km, we have a mean pressure of 69 dB, and at 40 km we have a mean pressure of 71 dB. The M. & S. expression gives us 70 dB at 20 km and 79 dB at 40 km with for both a 4-5 dB errors. Thus the two approaches are concurring.

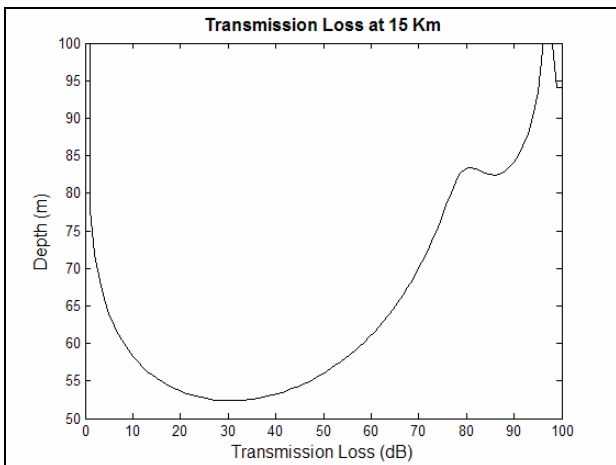


Figure 7

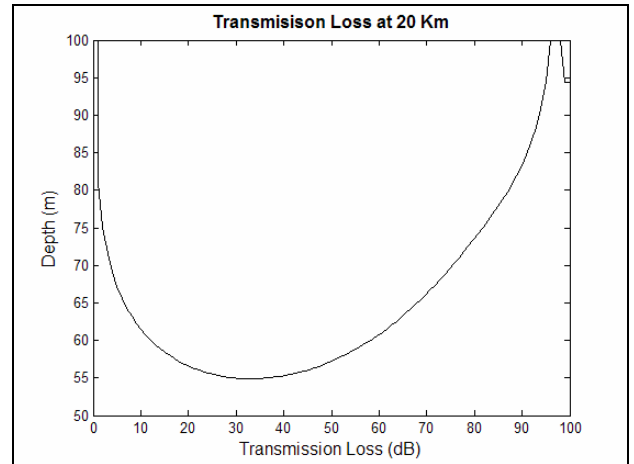


Figure 8

The Figures 8-9 confirm that we are losing modes with spreading loss, and higher is the ocean wave, higher will be the transmission loss.

We will now present the wavenumbers founded and those presented by Jensen & al. [7]:

Table 1: Comparison between SACLANT results and CSIC results (for the horizontal wavenumber)

CSIC Model (k_r)	SNAP Model (k_r)
0,41762	0.41728
0,41381	0.41329
0,40741	0.40734
0,39829	0.40037

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