An array beampattern synthesis using partial constrained adaptive optimization

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With the underwater acoustic systems, signals and information about a target can be retrieved through a beamforming method. The most important aspect in beamforming is finding the best way to optimize the mainlobe beamwidth and the sidelobe level to a desired value. One of the prominent studies conducted on the beamforming method is the Philip's weighting function method. This method adaptively adjusts its weights of array to meet the desired mainlobe beamwidth and sidelobe level. It is very similar to the design method in adaptive filter. Unfortunately, with Philip's method, it is difficult to meet the sidelobe level due to a complementary relation between mainlobe beamwidth and sidelobe level.

Therefore, in this paper, we propose a new algorithm using partial constrained adaptation which allows us to circumvent the above problem and easily meet the design specification.

1 Introduction

Array processing has played an important role in many diverse applications. For example, most modern radar and sonar system rely on antenna arrays or hydrophone arrays as it is an essential component of the system. [1]

The design of arrays to achieve certain performance criteria involves trade-offs among the array geometry, the number of sensors, signal-to-noise, and signal-to-interference ratios, as well as a number of other factors, etc.

Philip presented a new pattern synthesis algorithm for arbitrary arrays based on the Adaptive Array Theory. [2] The main objective of Philip's method is to find the optimal array weight vector that can minimize the weighted power of the difference between the synthesized pattern and the desired pattern. This algorithm iterates the values of the weighting function in order to minimize the exceedable the desired sidelobe levels and to minimize the absolute difference between desired and achieved patterns in the mainlobe region. This algorithm is very effective in maintaining sidelobe level at a regular state. But it takes up a considerably long computation time to adjust the desired sidelobe level.

In order to resolve this defect, this paper will propose a new optimum beamforming algorithm with which the desired sidelobe level can be adjusted by applying Tseng's method [3], which is effective in controlling sidelobe level. With this newly proposed algorithm, the shape of sidelobe is kept very stable and the computational complexity is greatly reduced compared with pre-method.

2 Proposed Algorithm

In brief, the most important step of the proposed algorithm is defining the initial weight vector. In the proposed algorithm, all of the iteration procedure of the algorithm follows Tseng's method[2], but the definition of initial weight vector shall be calculated using Philip's method [3].

The weight used to obtain the desired sidelobe level can be calculated by adding a residual weight vector $\Delta\mathbf{w}$ on initial weight vector $\mathbf{w}$. This weight vector $\Delta\mathbf{w}$ is a detail adjustment for a error between desired and synthesized pattern.

$$\mathbf{w} \leftarrow \mathbf{w} + \Delta\mathbf{w} \tag{1}$$

Here, $\mathbf{w}$ is calculated by Philip's method as follows.

$$\mathbf{w} = \mathbf{R}_s^{-1}\mathbf{R}_d \tag{2}$$

$$\mathbf{R}_s = \sigma^2\mathbf{I} + \sum_{n=1}^{N} f(\theta)\mathbf{v}_s(\theta_n)\mathbf{v}_s^{'}(\theta_n) \tag{3}$$

$$\mathbf{R}_d = \sum_{n} f(\theta)P_r(\theta_n)\mathbf{v}_s^{'}(\theta_n) \quad \theta_n \text{ in mainlobe} \tag{4}$$

And then, $\Delta\mathbf{w}$ is calculated by Tseng's method again using linear constrained minimum variance(LCMV). That is,

$$\min_{\Delta\mathbf{w}} \Delta\mathbf{w}^H\mathbf{R}_s\Delta\mathbf{w} \tag{5}$$

subject to

$$\mathbf{v}_s^H\Delta\mathbf{w} = 0$$

$$\mathrm{Re}\{\mathbf{v}_d^H \Delta\mathbf{w}\} = 0 \qquad (6)$$

$$\mathbf{v}_i^H \Delta\mathbf{w} = f_i \qquad \text{for } i=1,2,\ldots,n$$

The step-by-step flow chart of the proposed algorithm is shown below.

Table 1: Summary of Proposed algorithm

Initialize weight vector **w**, i.e, $\mathbf{w} = \mathbf{R}_s^{-1}\mathbf{R}_d$
Iteration Process
1. Identify the location of all the sidelobe peaks within the specified design region. 2. Determine the response values f_i for the residual pattern. $f_i = (\varepsilon - \lvert c_i \rvert)\frac{c_i}{\lvert c_i \rvert}$ 3. Solve $\Delta\mathbf{w}$ for by LCMV's condition. $\Delta\mathbf{w} = \mathbf{R}_s^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}_s^{-1}\mathbf{C})^{-1}$. 4. Update the weight. $\mathbf{w} \leftarrow \mathbf{w} + \Delta\mathbf{w}$ 5) Obtain the array pattern by $P_y(\theta) = \mathbf{w}^H\mathbf{v}(\theta)$. If it is satisfactory, stop; otherwise, go to step 1.

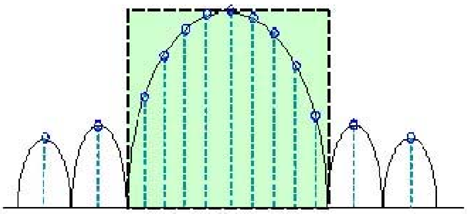


Figure 1 : Mainlobe in Philip's method

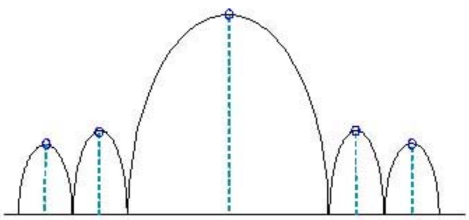


Figure 2 : Peaks in Tseng's method

The reason for using the initial weighting vector of Philip's method is to minimize a variety of mainlobe beamwidth by decreasing the sidelobe level to the desired value. As shown in the Figure 1 and 2, Tseng's method consider only one peak in the mainlobe in the same way as the sidelobe, calculates the difference between the synthesized value $P_y(\theta)$ and the desired value $P_d(\theta)$ in the peaks. In contrast, Philip's method defines the mainlobe region as $N_L \le \theta_n \le N_R$, and then calculates the difference at selected all points in the mainlobe. Thus, due to these differences, we believe that it would be possible to maintain the mainlobe beamwidth in the proposed algorithm despite the decreasing of the sidelobe level.

3 Simulations

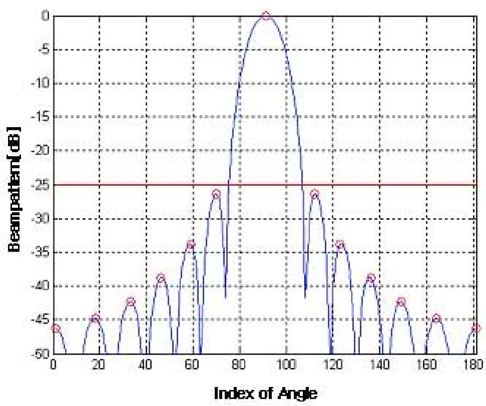
In this section, we present the simulation result using the proposed algorithm. The conditions of the simulation are the same as those of Tseng's and Philip's algorithm. Namely, the array consisted of 18 elements and the desired value of sidelobe is -25dB and iteration be made to continue until the value of sidelobe peaks optimized at the desired pattern. In the Figure, the peaks of lobe are marked with small circle and solid line denotes the desired value of sidelobe.

The simulation result using this proposed algorithm is shown in Figure 3. Figure 3(a) shows the initial beampattern, and Figure 3(b) shows the beampattern iterated by the proposed algorithm.

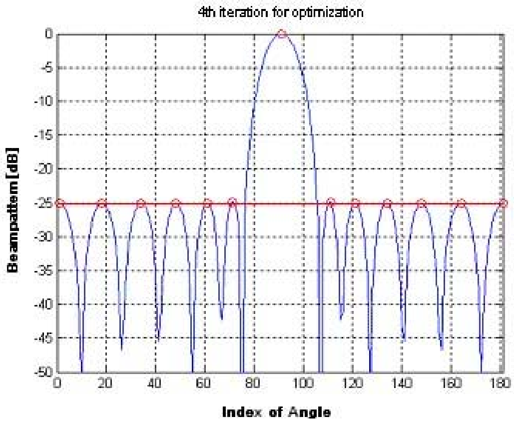
As shown in Figure 3(b), the beampattern using the proposed algorithm is precisely optimized at the desired sidelobe level(-25dB) with only four iteration as opposed to Tseng's method which took 91 iterations. Furthermore, the precision for desired sidelobe level can be seen to be more superior than Philip's method.

In that respect, the proposed algorithm satisfies the specification of the design easily, and not only consuming lesser computation time but also guaranteeing precision about the desired value.

Figure 4 are different expression about two dimensional case.

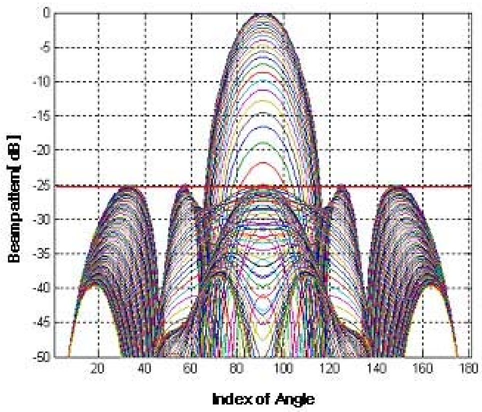


(a) Initial beampattern

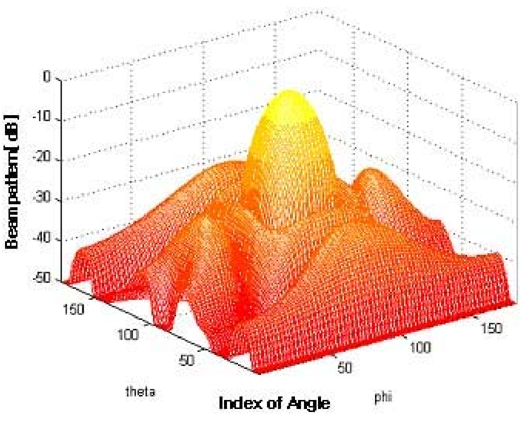


(b) beampattern after 4th iteration

Figure 3 : Beampattern by the proposed algorithm (18 ULA)



(a)



(b)

Figure 4 : Simulation result of the proposed algorithm (5×5 planar array)

3.1 HPBW(Half-Power BeamWidth)

Important factors that determine the performance of the antenna are Gain and HPBW. Here, HPBW(Half-Power BeamWidth) means the interval degree between two points $|P(\theta)|^2 = 0.5$ or $|P(\theta)| = 1/\sqrt{2}$. Namely, it means the degree at -3dB point from the maximum power. This is a way to measure the directivity of beamformer and a effective beamformer will have a narrow beamwidth or will have a similar value to desired pattern.

In this section, we will compare each HPBW for the three methods(i.e. Philip, Tseng and the proposed algorithms). The conditions for the simulation are follows. All of the methods have 18 arrays, the desired sidelobe level is -25dB and the desired region of mainlobe gives 10 degree on both sides from the center. The result is shown in Figure 5.

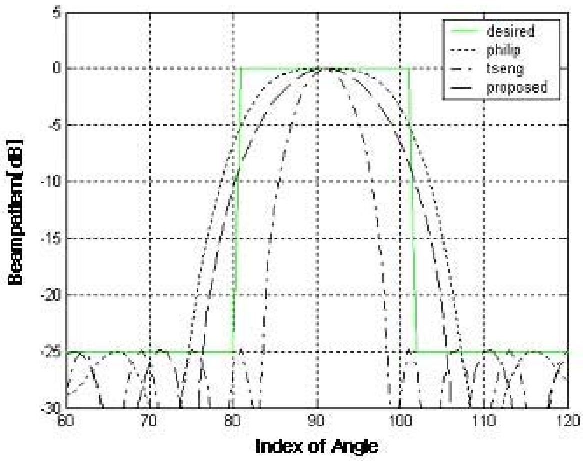


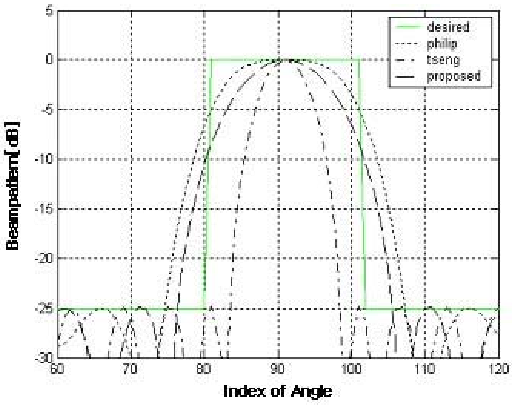
Figure 5: The comparison of HPBW (18 ULA ,Desired region : ±10 degree from center)

In Figure 5, solid lines(—) represent the desired pattern. Also, each pattern of Philip's, Tseng's and the proposed algorithm are represented as dotted line(…), dashdot line(-·) and dashed line(--) respectively. All patterns are iteratively calculated to the convergence point, but Philip's pattern can be to not converged regardless of the continuous iteration. Therefore, in the case of Philip's pattern, the 300th iteration value is used for comparison.

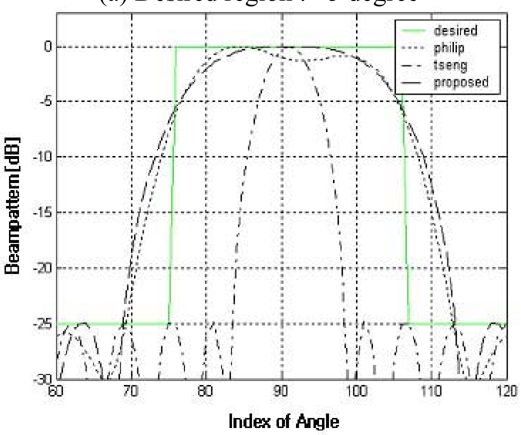
Let us consider each beamwidth of all patterns at the -3dB point from maximum power(0dB). As shown Figure 5, the Half-power beamwidth of Tseng's pattern(-·) is the narrowest. The proposed pattern can be seen to be narrower than that of Philip's. From these patterns, we can say that Tseng's pattern is very effective from a performance point of view, but it has a defect in that it cannot adapt to changes of the desired pattern because it cannot control the mainlobe shape. Thus, it may be difficult to find a signal in the wideband for the beamformer using Tseng's algorithm. Although the mainlobe shape can be controlled by adding or reducing the array elements, if the number of array elements is fixed, beamwidth of mainlobe will not have any influence on the shapes.

On the other hand, Philip's and the proposed pattern are more efficient in that it is able to control mainlobe shape according to designer's intention without having to add or reduce the array elements.

Additionally, the HPBW comparison result of the three patterns is shown in Figure 6 and Table 2. This comparison are simulated at various desired mainlobe region(i.e ±5, ±15 degree) and under the same condition as in Figure 5.



(a) Desired region : ±5 degree



(b) Desired region : ±15 degree

Figure 6 : The comparison of HPBW and BW_{NN}

Table 2: HPBW and BW_{NN} in 18 ULA

Desired	± 5		± 15	
	HPBW	BW_{NN}	HPBW	BW_{NN}
Tseng's method	6.45	14.89	6.45	14.89
Philip's method	7.89	20.66	25.24	43.61
Proposed method	11.06	25.97	24.82	44.26

As shown in Table 2, Philip's and the proposed algorithm have similar half-power beamwidth without a large difference, also distance to first null(twice this distance is BW_{NN}) are similarly calculated. Here, it is worthy of notice that unlike at ±15 degree for Philip's algorithm, the beam shape of the proposed algorithm in the mainlobe center is very stable without any distortion.

3.2 Computational complexity

The following is the comparison result of the computational complexity.

Table 3: Comparison of computational complexity

	MAD
Tseng's method	$M^3 + (N+1)M^2 + NM$
Philip's method	$4M^3 + M^2$
Proposed method	$4M^3 + M^2$

** MAD's stands for number of multiplies

In Table 3, each computational complexities to converge at desired sidelobe level are shown using the iteration number and MAD's. Total value that multiply the iteration number by MAD's will make it easy to understand the advantages of the proposed algorithm.

4 Conclusion

In this paper, we presented a new adaptive beampattern synthesis algorithm with which the desired sidelobe level can be adjusted by applying Tseng's method which is effective in controlling sidelobe level and the desired beamwidth are maintained by using Philip's method.

With this newly proposed algorithm, the shape of sidelobe is kept very stable and the computational complexity is greatly reduced compared with previous methods.

References

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