

# Acoustic Radiation from 3D Cylinders using the Equivalent Sources Method

Constantin Onescu

University of Pitesti, Ro-110040 Pitesti, Targu din Vale Str. 1, Romania, [costi\\_onescu@yahoo.com](mailto:costi_onescu@yahoo.com)

The paper treats the acoustic radiation of a 3D cylinders using the numerical method called equivalent sources method. In generally the velocity distribution on the cylinder surface can be found with the help of finite element method or by measurements. The surface velocity reconstruction with minimum errors is made by optimizing the positions of equivalent simple sources (monopoles or dipoles) and using different methods to solve matrix equation. The computing time is optimized by minimizing the number of sources without decrease the accuracy of the results. To establish the correct number of surface control points the error of radiated acoustic power can be minimized.

## 1 Introduction

The modelling of sound radiation of a complex structure is a difficult problem. The well known method for solving acoustic radiation of vibrating bodies is BEM (Boundary Element Method [1]). It knows that exist many problems with the numerical implementation of Helmholtz integral in BEM and in the last decades many authors improved BEM (CHIEF method, boundary element multigrid method [4] etc).

The Helmholtz integral is shown in equation (1):

$$\iint_S \left[ p(M_s) \frac{\partial g(M, M_s)}{\partial n_s} - \frac{\partial p(M_s)}{\partial n_s} g(M, M_s) \right] ds = p(M)$$

where  $g(M, M_s) = \frac{1}{4\pi r} e^{jkr}$  is the Green function in free field,  $r$  is the distance between points  $M$  (exterior to structure) and points  $M_s$  (structure).

The equivalent source method is a relatively new numerical method in acoustics and in this paper is used for the calculus of radiation of vibrating bodies. The method consist in replacing of the surface of vibrating body with a equivalent set by simply sources located inside of the radiator. The method is also called "Wave superposition method [2,3]" and when the equivalent sources are multipoles the method is called "Method of multipole synthesis".

This method appears like an alternative to BEM (Boundary Element Method) and some authors considers to be superior to the usual boundary integral formulations of the exterior acoustic problem for several reasons [2,4]. One of them, because the source and the collocation points never coincide, is avoided the singularity of the Green functions.

The central problem of the method is the number and the positioning of the sources for a minimum error.

The advantage of this method is that the accuracy of solution for a radiator with a given surface boundary condition can be directly quantified.

The equivalent source method is well suited to treat field problems of Neumann type, although it can be applied to other boundary problems. It enables the synthesis of a sound field created or modified by the presence of solid body which stay in contact with the acoustic medium across interface surface(s). In order to simplify the computation, the substitute sources are made infinitely small, to avoid sound scattering on these sources. The exact matching of the two will be possible only in rare simple cases, so the adjustment will consist in most cases in getting as close to the original velocity as possible using the difference of the original and the synthesised velocity as the guiding criterion.

## 2 The equivalent sources method

For the equivalent sources method the non-solving problem is optimal positioning of these sources to obtain the minimum velocity and pressure errors, also for the acoustic power.

To simplify the calculus, the small order sources are used (monopoles or dipoles) and the acoustic pressure generated by the simple sources is given in (2).

$$p(M, \omega) = \iint_S j\omega\rho_0\mu(M_0)g(M/M_0)ds \quad (2)$$

where  $M_0$  denote the position points of equivalent sources and  $\mu(M_0)$  the sources distribution density. The requirement that the synthetic velocity match the prescribed one is given by the (3).

$$\iint_S \mu(M_0)\nabla g(M_s/M_0)ds = \hat{v}_n \quad (3)$$

The equation (3) in fact is a Fredholm integral equation of first kind. These equation are known to be ill-posed and requires the use of regularization techniques (truncated singular value or Tikhonov regularization) or the others techniques to solve the matrix equation like conjugate gradient method.

Koopmann [2] proofs that starting from equation (2) we can obtain the Helmholtz integral equation (1).

For the case of  $M$  points on the surface (control points) equation (3) can be expressed in a matrix form:

$$[v_n] = [T][Q] \tag{4}$$

For the general case when  $N \neq M$  we obtain the solution of the equation (4) is:

$$[Q] = \left[ [T]_{mn}^* [T]_{mn} \right]^{-1} [T]_{mn}^* [v_n] \tag{5}$$

unde  $T^*$  is conjugate transpose matrix of  $T$ .

The calculus of matrix pseudo-inverse is made in the sense of the minimisation of the root mean square error. The transfer matrix  $i,j$  element represents the complex amplitude of the normal surface velocity at the  $i$  point due to unit-strength elementary source at the  $j$  point [7]:

$$T_{ij} = \frac{1}{4\pi} \frac{jk|r_j - r_i|^{-1}}{|r_j - r_i|^2} e^{jk|r_j - r_i|} \cos\theta_{ij} \tag{6}$$

where  $|r_j - r_i|$  the distance between the control point and the equivalent source and  $\theta_{ij}$  the angle between the vector  $r_j - r_i$  and the outer normal to the surface.

### 3. Positioning of the equivalent sources

Many authors consider that the placement of the equivalent sources inside of radiator should be done over a interior surface with the same shape like the vibrating surface.

In this case it is possible to have problems with the eigen frequencies of this interior surface [4].

To avoid this disadvantage authors like Pavic [7] proposed a "greedy search" algorithm which find the optimum positions for the sources which produce the lowest velocity errors.

From [7] it observes that in the case of low frequencies the "greedy search" algorithm put the sources over a surface near boundary points, and approximately the same shape like boundary. For high frequencies the positioning of the sources is given near center of the boundary surface, in this case are more discrete points that are not distributed over a surface.

This paper proposes a "equivalent surface search" algorithm of the interior surfaces. The equivalent sources distributed on the surface are monopole type. The main steps of this search procedure are given bellows:

- a) A large number of interior surfaces similar with body vibrating surface is defined.
- b) Each interior surface is checked from the surface velocity reconstruction point of view and a single surface is found for the smallest deviation between original and that created by equivalent sources.
- c) The surface found before is checked from point of view of fictitious eigenvalue with the influence on the synthetic pressure.

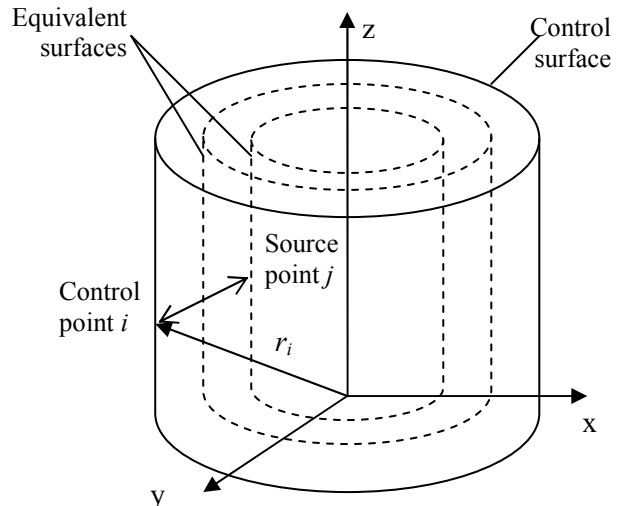


Figure 1: The positioning of equivalent sources on a surface and the control surface of vibrating cylinder.

In this paper two type of positioning source are used and compared:

- the positioning selected by the "greedy search" algorithm [7];
- the positioning of sources over an interior surface of vibrating cylinder.

### 4. Application to cylinder

#### 4.1 The "greedy search" algorithm

To check the validity of the method, the acoustic radiation of a cylinder is studied. Only the cylindrical surface of the cylinders is vibrating, the other faces are immobile. Constant velocity is applied and the wavenumber is  $kr=2,5$ . For exemplification are shown two cases in which the number of control surface points, the number of candidate points and the number of equivalent sources differ.

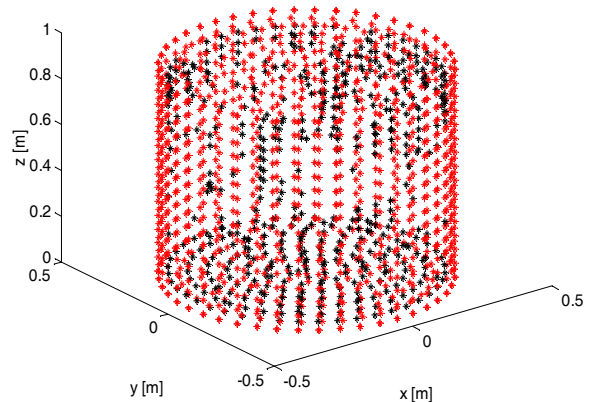


Figure 2: The control surface of the cylinder (red points) and the equivalent sources (black) for the first case

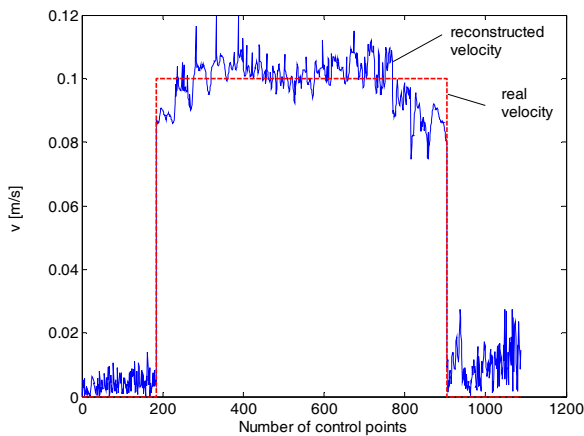


Figure 3: Comparison between the prescribed and reconstructed normal surface velocity for the first case

For the first case the number of surface control points is 1088 and the number of candidate points is 3540 (Figure 2). With the “greedy search” algorithm 500 equivalent source are chosen and the reconstruction of surface velocity is given in Figure 3. In the figure 4 is presented the level of the acoustic pressure on the vibrating surface of the cylinder.

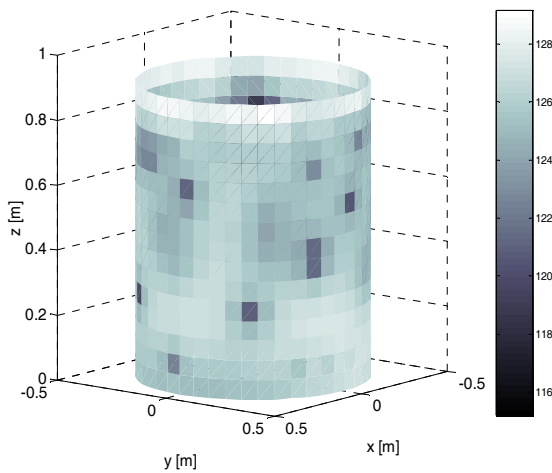


Figure 4: The sound pressure level on the vibrating surface of the cylinder for the first case

In the second case we have  $kr=10$ , the number of the control points is 1043, the number of candidate source is 2655. Are chosen 300 source to reconstruct the velocity field at the boundary of the cylinder.

In the cases presented above the velocity on the lateral surface of cylinder is constantly and the value is 0.1 m/s. In the next cases we will treat the vibration of cylinder with nonconstantly velocity vibration.

Increasing the frequency is necessary to increase the number of control points. In generally number of control points are 4...6 points par wavelength. Pavic [7] used

the discretisation error in study of acoustic radiation body to minimise the acoustic power errors in the equivalent sources method modeling.

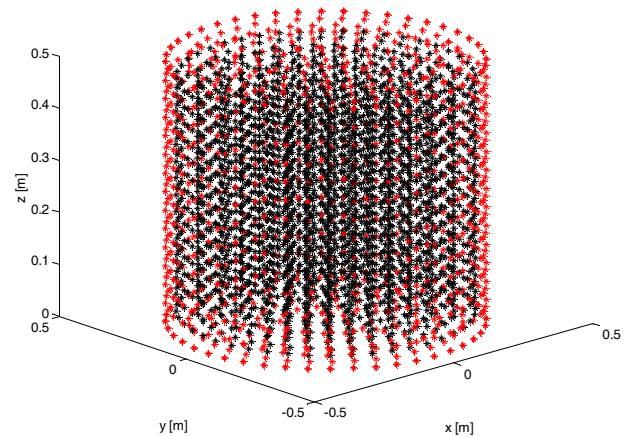


Figure 5: The control surface of the cylinder (red points) and the candidate source points for the second case

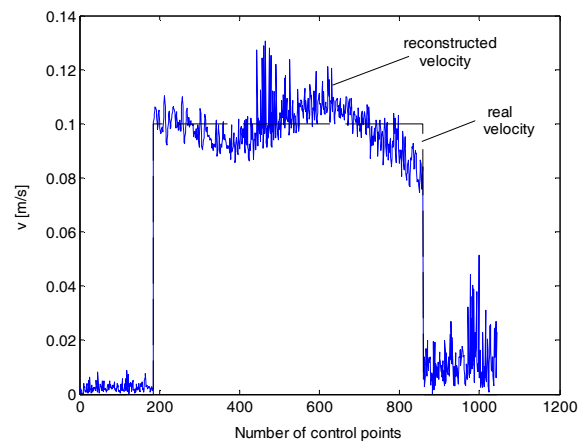


Figure 6: Comparison - the prescribed and reconstructed normal surface velocity for the second case

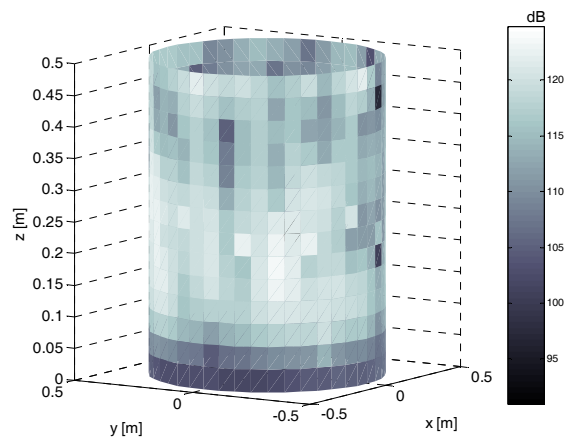


Figure 7: The sound pressure level on the vibrating surface of the cylinder for the second case

### 4.2 The surface choice algorithm

To obtain in a simplified way an optimum distribution of interior sources the method described to the paragraph 3 is applied. In fact the control surface and the candidate source points have the same coordinates like in paragraph 4.1 only the way in that it makes the chosen of sources differs. The method is applied for the second case (the control surface and candidate sources from Figure 5), when  $kr=10$ , the number of control points is 1163 and the number of simply sources on the equivalent surface is 300. From Figure it observes that the equivalent surface in this case tends to symmetry axis of the cylinder.

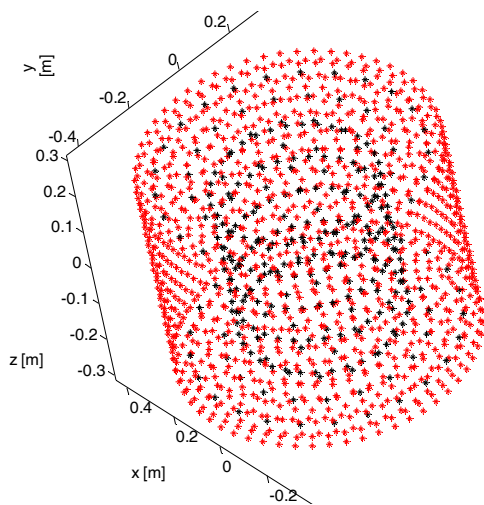


Figure 8: The control surface (red points) and the equivalent source surface (black points) for the second case ( $kr=10$ )

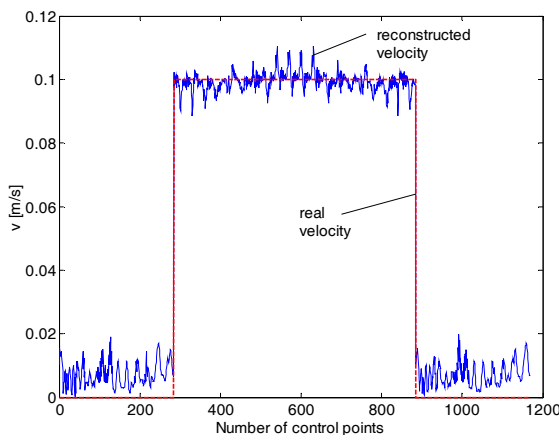


Figure 9: Comparison - the prescribed and reconstructed normal surface velocity for the equivalent surface.

The algorithm presented in paragraph 3, leads to better results regarding the surface velocity reconstruction comparative with the "greedy search" algorithm for this case.

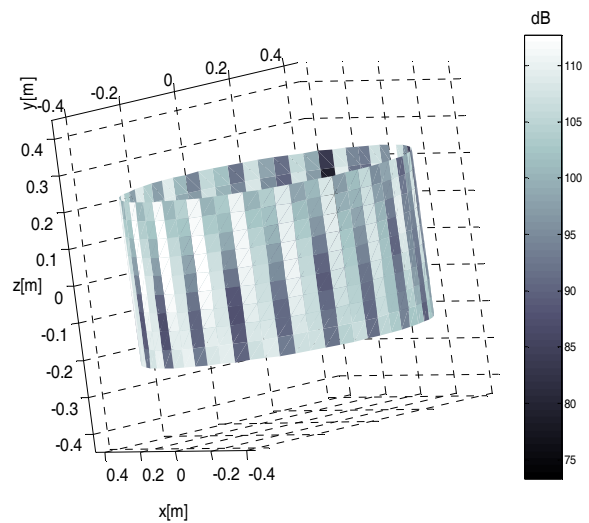


Figure 10: The sound pressure level on the vibrating surface of the cylinder obtained with equivalent surface

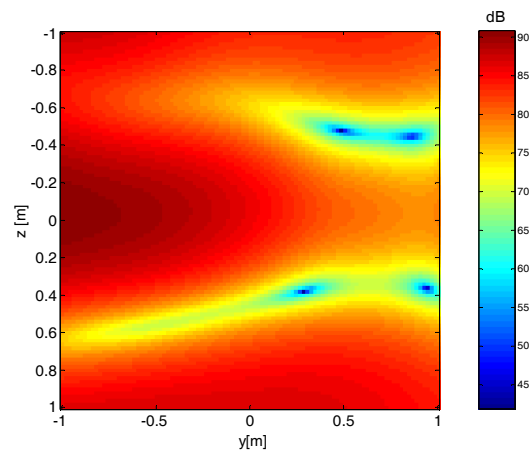


Figure 11: The acoustic pressure on a YZ plane situated at 2 m from cylinder axis.

Regarding acoustic pressure are obtained bad results with this method because at some frequencies can appears eigen modes of equivalents surfaces, means for equivalent cylinder for these cases (Figure 10 and 11).

### 5. Conclusion

The study of the cylinder radiation with the help of the equivalent sources method in that the sources are monopole give good results if it tries to optimize the positioning of this monopoles. In the case of a complex field velocity even in the case of cylinder it necessary to change the type of the equivalent sources (using of dipoles or multipoles) or to apply the weighting function to solve the integral equation avoiding the numerical instabilities[5].

With the surface choice algorithm proposed in this paper it improves the time of calculus which is

comparable with the ‘greedy search’ algorithm time for the same number of equivalent sources to minimize the errors of the surface velocity reconstruction. It is possible to develop the optimisation algorithm for source positioning using multiple surface, not one equivalent surface. Also this surface choice algorithm can be applied to study acoustic radiation of the complex structures like engine block.

## Acknowledgement

I want to thank to Prof. Dr. Goran Pavic from the INSA Lyon, France, for my initiation in the field of the equivalent source method in acoustics. Also, I am grateful to Dr. Quentin Leclere from INSA Lyon for his help in the MATLAB programming.

This paper is supported by the Romanian Ministry of Education and Research, contract CNCSIS TD177/2005.

## References

- [1] C. Lesuer, ‘*Rayonnement acoustique des structures*’, pp. 355-402 Editions Eyrolles, Paris, (1988)
- [2] G. Koopmann, L. Song, J. Fahline, ‘A method for computing acoustic fields based on the principle of wave superposition’, *J. Acoust. Soc. Am*, Vol. 86(6), pp. 2433-2438 (1989).
- [3] M. Ochmann, ‘Calculation of sound from complex machine structures using the multipole radiator synthesis and the boundary element multigrid method’, *Revue française de mécanique, Numero special*, pp. 457-471 (1991).
- [4] C. G. Provatidis, N. K. Zapiropoulos, ‘A modified indirect boundary element method for solving two-dimensional sound radiation problems’, *Forsch Ingenieurswes*, Vol. 68, pp. 8-18 (2003)
- [5] M. S. Magalhaes, R. Tenenbaum, ‘Sound sources reconstruction techniques: A review of their evolution and new trends’, *Acta Acustica united with Acustica*, 90(2), pp.199-220 (2004)
- [6] P. R. Stepanishen, S. Ramakrishna, ‘Acoustic radiation from cylinders with a plane of symmetry using internal multipole line source distributions’ *J. Acoust. Soc. Am*, Vol. 93(2), pp. 658-672 (1992).
- [7] G. Pavic, ‘An engineering technique for the computation of sound radiation by vibrating bodies using equivalent sources’, *InterNoise*, Prague (2004)
- [8] C. Onescu, G. Pavic, ‘An application of the equivalent source method in modelling of sound radiation’, *Int. Conf. of Romanian Society of Acoustics*, Bucharest (2004)