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ACOUSTICS OF A DOUBLE HELMHOLTZ RESONATORS WITH FLOW

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ABSTRACT

The acoustic behavior of a double annular resonator is studied. Compared to the associated single resonator, the double resonator shows a transmission peak near the resonance frequency where the transmission is minimal when the losses are zero. This peak is associated with an evanescent coupling between the two resonators and is called "Autler-Townes splitting". During the measurement of the device, this peak is very strongly attenuated by viscous effects, so much so that it almost disappears. When a flow is added, even a very small one, a gain is created and the peak reappears at the resonator frequency. As the mean flow velocity increases, this gain can become large enough for a whistling sound to appear. For this velocity of appearance of the whistling, the effect of the flow on the single resonator is negligible.

Keywords: *Helmholtz resonator, Autler-Townes splitting, Acoustics, Flow effect, Non Hermitian*

1. INTRODUCTION

The Helmholtz resonator has been known to have a filtering effect at the wall of an acoustic waveguide for a long time [1, 2]. Over the years, many studies have been conducted to explore the practical applications of this device in the presence of flow [3–5]. Recent studies have focused on a variant where two Helmholtz resonators are located in the wall [6–9]. When these two resonators are positioned so closely that they are only connected by the evanescent field, a phenomenon, similar to Autler-Townes splitting (ATS), appears [10]. This causes a window of transparency in a frequency band where sound waves are not expected to be transmitted. However, this effect should not be confused with acoustically induced transparency, which is due to the destructive interference of waves propagating in both directions between the two resonators [8, 9]. It is worth noting that many studies on this phenomenon neglect the effects of flow, even though this type of system is often referred to as a "ventilated barrier" [11, 12].

The present experimental study aims to investigate the effects of flow and viscosity on a double Helmholtz resonator. The ATS phenomenon is first introduced and then the impact of viscosity and flow is discussed. The experimental results demonstrate a significant change in behavior when either viscosity or flow is considered. In particular, the gain induced by flow can be so substantial that it causes whistling to occur [13].

2. DOUBLE RESONATOR

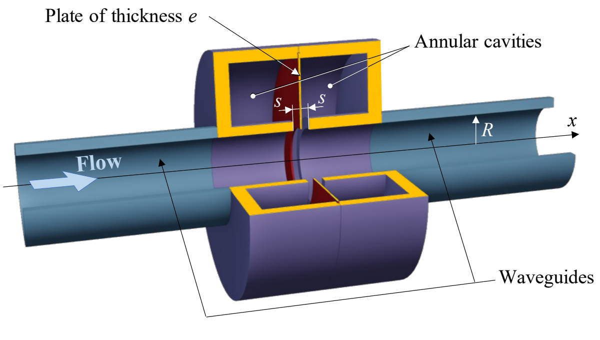

Figure 1. Schematic of the annular double resonator.

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The double annular resonator is schematized in Fig. 1. It consists of two identical Helmholtz resonators with annular necks of $s = 2.5$ mm width separated by a thin plate of $e = 0.5$ mm thickness which are connected to two cylindrical waveguides of radius $R = 15$ mm and to two annular cavities. This device is fully symmetrical.

3. ACOUSTICAL EFFECT WITHOUT FLOW

Compared to the associated single resonator (same system where the middle plate is removed), the double resonator exhibit a sharp peak in transmission near the resonance frequency where the transmission is minimum. This peak is associated to an evanescent coupling between the two half resonators (ATS effect) [9]. This effect is very clearly visible on a numerical simulation (COMSOL) without any dissipation (see magenta line in Fig. 2). When thermo-viscous effects are taken into account in the numerical simulation or when transmission is actually measured (see the green line in Fig. 2), this ATS almost completely disappears. This is due to the very high sensitivity of very sharp peaks to small dissipation (the normalized resistance of Helmholtz resonators is experimentally estimated at 0.02).

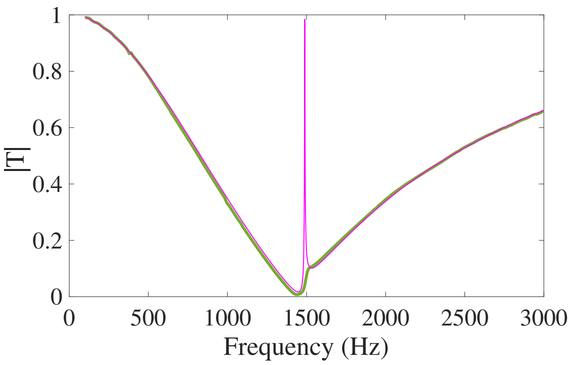

Figure 2. Absolute value of the transmission coefficient of the annular double resonator. Thin magenta line: Numerical result without viscosity and flow. Thick green line: experimental results without flow

4. EFFECT OF FLOW ON A DOUBLE RESONATOR

The first very striking effect of the flow is that the double resonator starts whistling for low values of velocity (above 13 ms^{-1}). In contrast, the single resonator, obtained by removing the separator plate, does not whistle at any velocity. The frequency of the whistling remains relatively constant and is locked close to the resonance frequency of the resonators. The Strouhal number of appearance of the whistling is given by

$$S_t = \frac{f_R\, s}{U_c} = 0.29$$

where f_R =1500 Hz is the resonance frequency and U_c = 13 ms^{-1} is the mean flow velocity where the whistling appears.

The second remarkable point is that the transmission peak, which had disappeared due to viscous effects in the neck in the measurements without flow, reappears in the presence of flow, see Fig. 3. We can also note the difference between downstream propagation (solid line) and upstream propagation (dashed line). This difference is the testimony of a loss of reciprocity due to the flow. Naturally, the transmission measurements are performed for velocities lower than the velocity where the whistling appears for the problem remains linear ($U < U_c$).

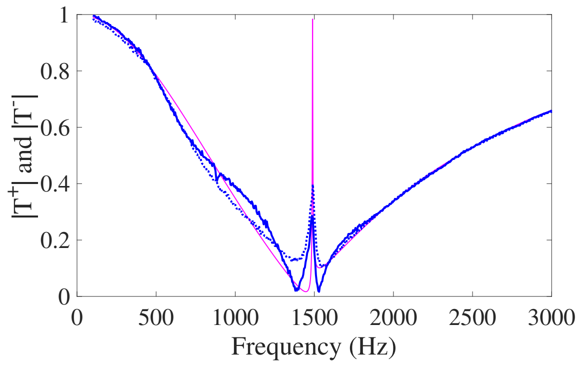

Figure 3. Absolute value of the transmission coefficient of the annular double resonator. Thin magenta line: Numerical result without viscosity and flow. Thick blue line: experimental results with flow ($M = 0.034$, continuous line in flow direction, dashed line against the flow)

Nevertheless, the effect in the presence of flow does not seem to be of the same nature as the one without flow. Indeed, without flow, the ATS effect appears due to a pressure jump at the resonator which is proportional to the average velocity between upstream and downstream of the resonator [9]. This effect is combined with the more classical effect of velocity jump proportional to the average pressure in front of the resonators (the proportionality coefficient is the input impedance of the resonators). This description using two coefficients comes from the full symmetry of the device. With flow, this symmetry is broken and the pressure jump is no longer related to the average velocity and a more complex behavior appears.

Another way to look at the problem is to consider the convected scattering matrix $\mathsf{S_c}$.

$$\begin{bmatrix}(1+M)p_2^+\\(1-M)p_1^-\end{bmatrix} = \mathsf{S_c}\begin{bmatrix}(1+M)p_1^+\\(1-M)p_2^-\end{bmatrix}$$

with

$$\mathsf{S_c} = \begin{bmatrix}T_c^+ & R_c^-\\ R_c^+ & T_c^-\end{bmatrix}$$

where the subscript 1 (resp. 2) indicates the upstream (resp. downstream) value of the plane wave propagated to the middle of the double resonator, the superscripts + and - indicate the downstream and upstream propagations. The two eigenvalues of the matrix $\mathsf{I} - \mathsf{S_c^*S_c}$, where I is the identity matrix and the star stands for the complex conjugate, correspond, respectively, to the maximum and minimum energy dissipation of the device divided by the incident energy (a negative value indicates an energy production) [14]. These eigenvalues are computed from the experimental values with and without flow and plotted in Fig. 4.

Without flow (green lines), the dissipation of the double resonator is between 0 and 5% of the incident energy except for a bump around the resonance frequency where the dissipation is between 5 and 22% of the incident energy. In the presence of flow, several phenomena appear. The first is a positive bump around 975 Hz showing that the flow allows to dissipate up to 28% of the incident energy. The maximum of this positive bump changes in amplitude and frequency as a function of the mean flow velocity U and the frequency shift of the maximum is such that the Strouhal number $S_t = fs/U = 0.20$ is constant. The second point is a rapid decay of the minimum curve from 1325 Hz indicating a gain in the double resonator. This gain stops abruptly at the resonance frequency to produce again an important loss.

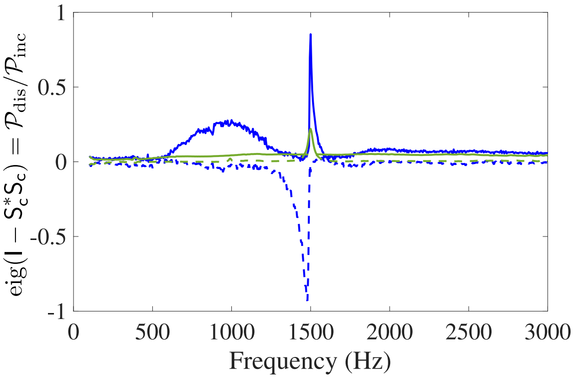

Figure 4. Eigenvalues of the matrix $\mathsf{I} - \mathsf{S_c^*S_c}$. Green: $M = 0$, Blue: $M = 0.034$. The solid line is the maximum value, while the dashed line is the minimum value.

5. CONCLUSIONS

When dealing with metamaterials, resonant systems are often used. It is therefore particularly important to take into account the effects of viscosity and flow in sufficient detail. These two effects can significantly alter the expected acoustic effects [15, 16]. In this paper, this is illustrated by the case of a double Helmholtz resonator. The expected band of transparency due to Autler-Townes splitting disappears almost completely in experiments due to viscosity, even though dissipation is very low. When a very low flow velocity is added, the transparency band reappears experimentally due to the gain generated by the flow. However, at low flow velocities, a whistling is produced, making the device unusable for passive sound attenuation.

6. REFERENCES

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