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MEASUREMENT OF PLANE ACOUSTIC WAVES USING AN OPTICAL FEEDBACK INTERFEROMETER

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ABSTRACT

When an acoustic wave propagates, pressure variations induce changes in the optical refractive index which can be measured using optical interferometers. Optical feedback interferometers (OFI) can detect this acousto-optic effect and have the advantage of not requiring external optical components. While it is known that an OFI can be used as a vibrometer, its characteristics as acoustic pressure sensor need to be clearly established. For this purpose, an experimental setup and a two-step calibration procedure were developed. Plane acoustic waves are emitted in square section waveguides suitable for optical measurement. Based on feedback interferometry theory, we first evaluate the parameters of the OFI in vibrometer mode only by measuring shaker-driven vibrations of a reflecting surface, without acoustic excitation. The determination of parameters like the optical feedback factor or the linewidth enhancement factor then makes it possible to mesure the variations of the optical path induced by acoustic excitation only (shaker swithed-off) and evaluate the pressure. Results show good agreement when compared to those obtained using a microphone.

Keywords: *acousto-optic, optical feedback interferometer, self-mixing*

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1. INTRODUCTION

Acousto-optic techniques have been established for detecting and characterizing acoustic waves through interferometric measurement of optical index variations. [1–5]. A recent development in this area is the Optical Feedback Interferometer (OFI), also known as Self-Mixing Interferometer, which has been proposed for visualizing acoustic fields [6, 7]. This interferometric technique involves a semiconductor laser which is sensitive to optical feedback. This effect is obtained when a fraction of the emitted laser light is backscattered in its cavity, resulting in a change in the laser diode's emitted power and optical wavelength, depending on the optical path of the laser beam [8]. The OFI is typically implemented using a laser diode that targets a retro-reflective surface and an embedded photodiode for the interference measurements. Compared to other types of interferometers, the OFI has the advantage of requiring less equipment and being self-aligned. However, few studies have been conducted on the OFI's performance for quantitative acoustic pressure measurement [9]. Indeed, the OFI practices an integral measurement of the acoustic wave along its laser beam [4, 5] and the noise present in the OFI signal significantly limits the detection of acoustic waves below 83 $\mathrm{dB_{SPL}}$ [10].

This paper proposes a method to calibrate an OFI for quantitative measurements of acoustic waves. In particular, the acoustic pressure of a plane wave propagating perpendicular to the laser beam. This method is based on the optical feedback equations and on the experimental evaluation of the OFI parameters using vibrometric measurements.

A reminder of the acousto-optic effect theory and the equations governing the OFI are presented in Sec. 2. Then, a method to measure the OFI parameters based on the work of Ri et al. [11] is described in Sec. 3. An experimental measurement of plane acoustic waves with the OFI is presented in Sec. 4, the results are shown and discussed in Sec. 5 and summarized in Sec. 6.

2. THEORY

This section reminds the theory of the acousto-optic effect in Sec. 2.1 and of the optical feedback in Sec. 2.2.

2.1 Acousto-optic effect

The acousto-optic effect is the modification of the optical refractive index of a medium caused by an acoustic wave. By describing the pressure of a medium as:

$$p(\boldsymbol{r}, t) = p_0 + p'(\boldsymbol{r}, t), \tag{1}$$

where $p(\boldsymbol{r}, t)$ is the pressure of the medium at a given time t and position $\boldsymbol{r}$, p_0 is the static pressure of the medium and $p'(\boldsymbol{r}, t)$ is the acoustic pressure. It is possible to describe the refractive index of a medium $n(\boldsymbol{r}, t)$ as:

$$n(\boldsymbol{r}, t) = n_0 + n'(\boldsymbol{r}, t), \tag{2}$$

where n_0 is the static refractive index and $n'(\boldsymbol{r}, t)$ is the fluctuation of refractive index caused by the acoustic waves.

For acoustic waves propagating in the air, Ciddor's model allows to calculate the refractive index fluctuations [12]. In this work, the assumption that $n'(\boldsymbol{r}, t)$ is proportional to the sound pressure is made:

$$n'(\boldsymbol{r}, t) = \beta(\lambda, p_0, T_0, \phi_h, c_{\mathrm{CO_2}}) p'(\boldsymbol{r}, t), \tag{3}$$

where β is a quantity which depends on: λ the electromagnetic wavelength, p_0 the static pressure, T_0 the temperature of the medium, ϕ_h the humidity rate of the medium and $c_{\mathrm{CO_2}}$ the molar concentration of CO_2 in the medium. This equation allows the measurement of acoustic waves by measuring the refractive index of the propagation medium with interferometers.

For laboratory conditions of $\lambda = 1309$ nm, $p_0 = 1$ bar, $T_0 = 20$ °C, $\phi_h = 0.5$ and $c_{\mathrm{CO_2}} = 440$ ppm, the value of β is 2.6×10^{-9} Pa^{-1}. Note that T_0 is the parameter with the most significant influence on the value of β with a variation of 0.011 Pa^{-1}/°C around these thermodynamic conditions.

2.2 Optical feedback

The optical feedback interferometer exploits a property of laser diodes. If a part of the photons emitted by it returns in its laser cavity by backscattering on any surface, the behavior of the laser diode changes in terms of power and wavelength [8]. This situation is schematized in Fig. 1 assuming a non-deflected laser beam, which is reasonable considering a low refractive index gradient.

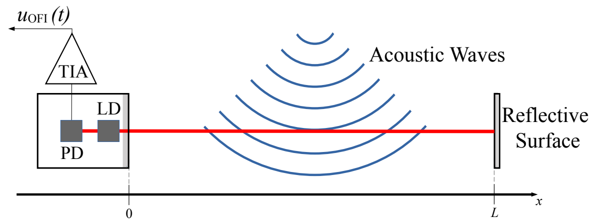

Figure 1. Scheme of an Optical Feedback Interferometer. TIA: Transimpedance Amplifier. LD: Laser Diode. PD: Photodiode.

This phenomenon known as optical feedback or self-mixing is described, in conditions where the amount of photons returning to the laser cavity is much smaller than the amount leaving it, by the following equation [8]:

$$\frac{2\pi}{\lambda_0}\mathcal{L}(t) = \Phi(t) + C \sin\left(\Phi(t) + \arctan(\alpha)\right), \tag{4}$$

where $\mathcal{L}(t)$ is the optical path outside the laser's cavity, λ_0 is the wavelength of the laser diode without optical feedback, C is the feedback parameter that depends on the amount of light reflected back into the laser cavity, α is the linewidth enhancement factor (a quantity intrinsic to the laser diode) [13] and $\Phi(t)$ is the external round-trip phase at the perturbed laser wavelength such as:

$$\Phi = \frac{2\pi}{\lambda}\mathcal{L}, \tag{5}$$

where λ is the actual wavelength of the laser perturbed by the feedback.

Different optical feedback regimes exist depending on the amount of photons returning to the laser cavity: the weak feedback when $C < 1$ and the moderate or strong feedback when $C > 1$. For the work presented here, C must not exceed 15 to be sure to avoid entering into regimes where instabilities yield into the OFI behaviour [8]. Depending on the feedback regime, solutions of Eqn. (4) are not the same [14].

Assuming a situation as described in Fig. 1, the optical path $\mathcal{L}$ can be described by:

$$\mathcal{L} = 2\int_0^L n(x)\mathrm{d}x, \quad (6)$$

where x is the coordinate along the laser beam, L its length and $n(x)$ is the optical index at position x. By combining Eqns. (2), (3) and (6), the optical path $\mathcal{L}$ is rewritten as:

$$\mathcal{L} = 2\left(n_0 L + \beta\int_0^L p'(x)\mathrm{d}x\right) \quad (7)$$

$$= 2n_0(L_0 + L_\mathrm{V} + L_\mathrm{AO}), \quad (8)$$

where L_0 is the length of the laser beam at rest, L_V is the variation of the the laser beam length due to mechanical vibration on the reflecting surface such as $L = L_0 + L_V$ and L_AO is considered as an apparent variation of the length of the laser beam caused by the acousto-optic effect [10]. Thus the OFI is sensitive to the integral of the acoustic wave along its laser beam. As the informations of the acoustic wave passing through the laser are contained in L_AO, it is therefore necessary to measure it to use the OFI as an acoustic measuring instrument.

The diode laser power under optical feedback is modeled by the equation [15] :

$$\mathcal{P}(t) = \mathcal{P}_0\left(1 + m\cos(\Phi(t))\right), \quad (9)$$

where $\mathcal{P}(t)$ is the power of the laser diode, $\mathcal{P}_0$ is the power of the diode laser without optical feedback and m is the modulation index which is proportional to C and depend also of the laser length L and the photon lifetime in the laser cavity. In this work, the variations of L will be small enough to consider m as a constant.

The power output of a laser diode $\mathcal{P}(t)$, is typically determined in a practical setup by the use of an embedded photodiode (PD in Fig. 1). The current generated by the PD is then transformed into a voltage signal with a transimpedance amplifier (TIA in Fig. 1) to generate the OFI signal $u_\mathrm{OFI}(t)$. This signal being proportional to $\mathcal{P}(t)$, it can be described as :

$$u_\mathrm{OFI}(t) = U_0 + \upsilon\cos(\Phi(t)), \quad (10)$$

where U_0 and υ are two constants.

3. EVALUATION OF THE OFI PARAMETERS TO DETERMINE $\mathcal{L}$

The output signal of the OFI depending on $\Phi(t)$, the first step to measure the acoustic waves propagating through the laser beam is to determine the optical path $\mathcal{L}(t)$ using Eqns. (4) and (10). First of all, $\Phi(t)$ has to be computed from Eqn. (10). It is therefore necessary to determine U_0 and υ. This process is described in Sec. 3.1. Once $\Phi(t)$ is measured, it is necessary to evaluate the values of C and α to solve Eqn. (4). The method for the measurement of these two quantities is described in Sec. 3.2. All these parameters are specific to the OFI and the setup disposition.

3.1 Determining U_0 and υ

By varying $\Phi(t)$ by more than 2π so that $\cos(\Phi(t))$ reaches the values of -1 and 1 at some point, υ and U_0 can be identified as :

$$\upsilon = \frac{u_\mathrm{max} - u_\mathrm{min}}{2}, \quad (11)$$

and,

$$U_0 = \frac{u_\mathrm{max} + u_\mathrm{min}}{2}, \quad (12)$$

where u_max is the maximum of $u_\mathrm{OFI}(t)$ which is reached when $\cos(\Phi(t)) = 1$ and u_min is the minimum of $u_\mathrm{OFI}(t)$ which is reached when $\cos(\Phi(t)) = -1$.

Experimentally, varying $\Phi(t)$ by more than 2π is easier by varying $L_\mathrm{V}(t)$ (i.e. the length of the laser beam), instead of $L_\mathrm{AO}(t)$ because of the small value of β . The use of a shaker on the reflective surface to vary $L_\mathrm{V}(t)$ is the solution chosen in this paper to measure υ and U_0.

3.2 Determining C and α

Several methods are described in the literature for the characterization of C and α when an OFI is used as a vibrometer [14, 16–18]. The method used in this work is the one described by Ri et al. [11] which has several advantages:

- This method can be applied to the same measurements as those used for the evaluation of U_0 and υ described in the previous section.
- As the value of υ is proportional to C, the sensitivity of the OFI to acoustic waves increases with it. This method allows to measure C values in strong feedback regime and thus to optimize the OFI sensitivity.

In moderate and strong feedback ($C > 1$), the OFI signal $u_\mathrm{OFI}(t)$ exhibits hysteresis and discontinuities when the variation of $\Phi(t)$ is more than 2π [8, 14]. These properties are such that the OFI signal has particular shape represented in Fig. 2.

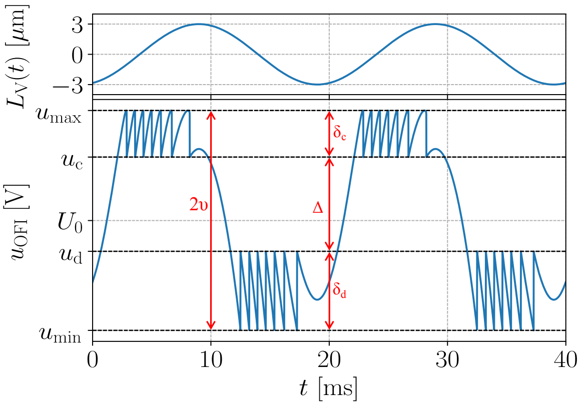

Figure 2. Simulation of the OFI signal $u_{OFI}(t)$ based on [14] algorithm. The length of the laser beam varies such as $L_V(t) = \delta_L \cos(2\pi f t)$ and $\delta_L = 3\ \mu$m, $f = 50$ Hz, $C = 9$, $\alpha = 6$, $\lambda_0 = 1309$ nm, $L_0 = 40$ cm.

It can be observed that each time the OFI signal reaches its maximum (and respectively its minimum), the signal becomes discontinuous and returns at a value that called u_c (respectively u_d). The temporal gap between 2 discontinuities corresponds to a displacement of $\lambda/2$ of the reflecting surface.

According with Fig.2, ones denotes:

$$\Delta = u_c - u_d, \tag{13}$$

and,

$$\delta = \delta_d - \delta_c = (u_d - u_{min}) - (u_{max} - u_c). \tag{14}$$

Ri et al. approximate the values of C and α by the equation:

$$4X + (\delta^2 - \Delta^2 - 4)X + \Delta^2 = 0, \tag{15}$$

with:

$$X = \sin^2(\arctan(\alpha)), \tag{16}$$

or,

$$X = \sin^2\left(\frac{\gamma + C\sin(\gamma) + 2\pi C}{1 + C}\right), \tag{17}$$

with,

$$\gamma = \arccos\left(\frac{-1}{C}\right). \tag{18}$$

Solving these equations allows for example to calculate a value of $C = 9.33$ and $\alpha = 6.03$ for the signal simulated in Fig. 2.

It can be noted that this method has the advantage of not requiring to know precisely $L_V(t)$ in particular its amplitude and its frequency.

Once the values of U_0, υ, C and α are measured, all the unknowns of the problem are identified to compute $\mathcal{L}(t)$ using Eqns. (4) and (10). Thus, if the variations of $\mathcal{L}(t)$ are caused by $L_{AO}(t)$ variations, it is possible to measure the acousto-optic effect with an OFI.

4. EXPERIMENTAL SETUP

The experimental setup used to measure the parameters of an OFI and acoustic waves is presented. It is schematized in Fig. 3. Sec. 4.1 presents the OFI used in this paper and Sec. 4.2 the acoustic source.

4.1 The Optical Feedback Interferometer (OFI)

The OFI's laser diode (LD) is a Thorlabs© L1310P5DFB with a wavelength of $\lambda_0 = 1309$ nm and a maximum power of 5 mW. A embed photodiode (PD) is used to measure the LD power $\mathcal{P}(t)$, which generates a current. The latter is converted into a voltage $u_{OFI}(t)$ with a transimpedance amplifier (TIA in Fig. 3) Femto© DLPCA-200 with a gain set to 10^4 V/A. A Thorlabs© LDC205C current driver powers the LD which is maintained at a constant temperature of 12 °C with a Thorlabs© TED200C temperature controller.

A Thorlabs© C110TMD-C lens, with a focal length of 6.24 mm, collimates the beam. The latter is directed onto a reflective tape placed at $L_0 = 40$ cm from the LD, which backscatters photons into its cavity. The reflective tape is attached to a PCBpiezotronics 352C65 accelerometer which is mounted on a Brüel & Kjaer© 4810 shaker allowing to vary $L_V(t)$ and evaluate the parameters of the OFI. The accelerometer is linked to a PCBpiezotronics 482C05 conditioner and allows an independent measurement of $L_V(t)$.

4.2 The acoustic source

In order to simplify the integral expression of $L_{AO}(t)$ (see Eqn. (8)), it was chosen to measure acoustic plane waves propagating perpendicular to the laser beam of the OFI. To generate these plane waves, a parallelepipedic tube of dimensions $45 \times 430 \times 25$ mm^3 closed at one of its extremities is used as an acoustic waveguide. A loudspeaker is placed at its other extremity. It is powered by a Visaton© AMP 2.2 LN amplifier (AMP in Fig. 3).

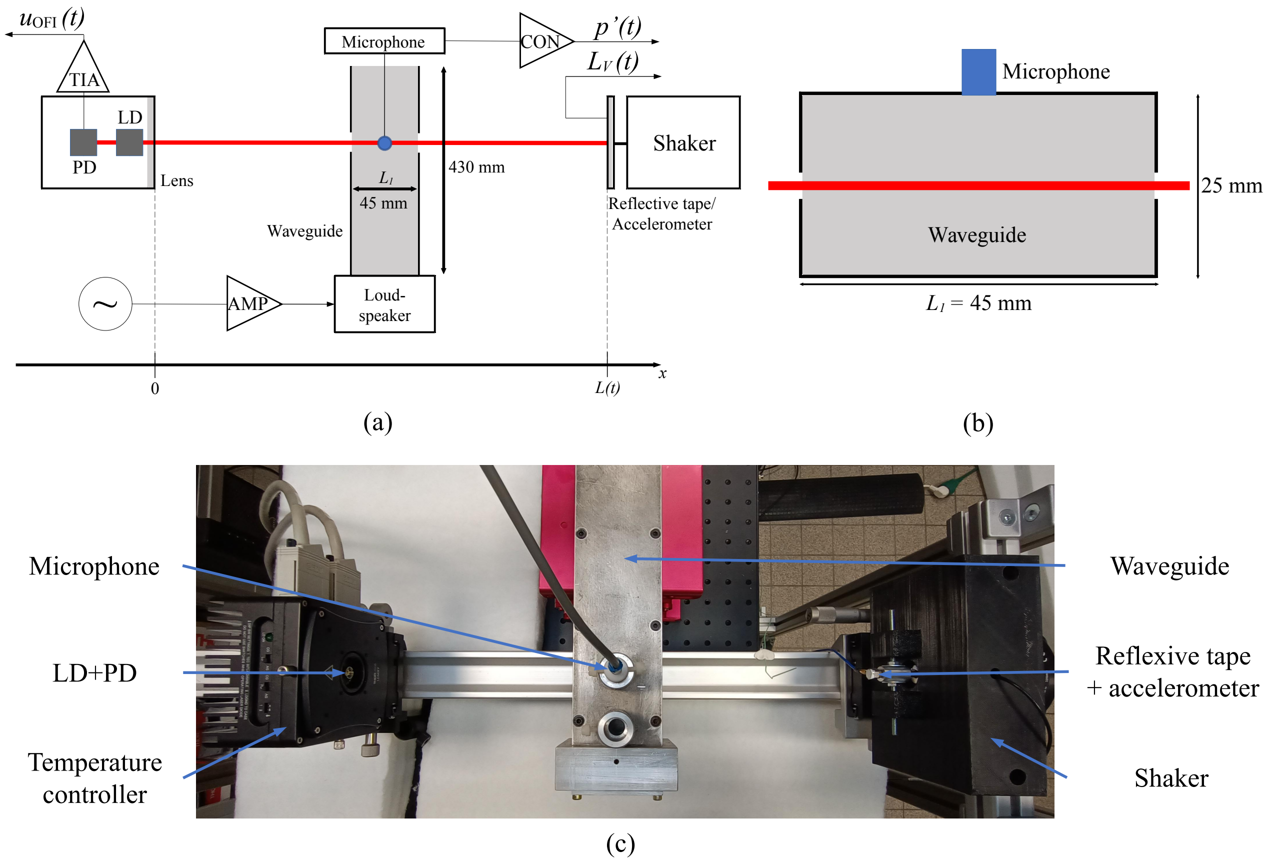

Figure 3. (a) Scheme of the experimental setup. AMP: power amplifier of the loudspeaker. CON: microphone conditioner. TIA: transimpedance amplifier. (b) Scheme of the waveguide cross section through which the laser passes. (c) Photography of the experimental setup.

The OFI laser beam is able to pass through the guide with two side holes. A reference microphone is flush-mounted above the beam to measure the acoustic pressure in the same section crossed by the laser (see Fig.3). The microphone employed is a 1/4" GRAS© 40BH linked to a Brüel & Kjaer© NEXUS 2690-A-0F2 conditioner (CON in Fig. 3).

The cut-off frequency of this waveguide below which the waves propagating in it are plane is approximately 3500 Hz. Thus, for acoustic waves with a frequency lower than this cutoff frequency and by neglecting the acoustic waves radiating from the holes, $L_{\mathrm{AO}}(t)$ can be written as:

$$L_{\mathrm{AO}}(t) = \frac{\beta L_1}{n_0} p'(t) \tag{19}$$

where L_1 is the dimension of the waveguide through which the laser passes and $p'(t)$ the acoustic pressure in the measured section.

5. RESULTS AND DISCUSSIONS

In this section, an experimental measurement of the OFI parameters is presented (Sec. 5.1), followed by a comparison of microphone and OFI measurements of sinusoidal acoustic waves in the waveguide (Sec. 5.2).

5.1 Evaluation of the OFI parameters

A vibrometric measurement of the reflecting tape is made with the OFI. The shaker is powered to vibrate sinusoidally at a frequency of 200 Hz. The vibration amplitude is set to have a signal similar to Fig. 2 with about ten discontinuities per period. The OFI signal $u_{\text{OFI}}(t)$, measured during 5 s and sampled at 200 kHz, is represented in Fig. 4. To reduce measurement errors caused by signal noise, the values of u_{max}, u_{c}, u_{d} and u_{min} are measured by averaging the ordinates of $u_{\text{OFI}}(t)$ located just before and after the discontinuities. Each of the measured values for the averaging is represented by colored dots in Fig. 4.

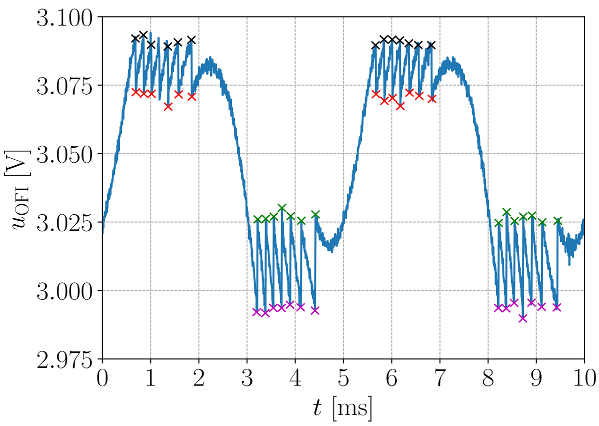

Figure 4. Sample of $u_{\text{OFI}}(t)$ acquisition for the measurement of OFI parameters. The ordinates of the black points are those taken into account for the measurement of u_{max}, those in red for the measurement of u_{c}, those in green for the measurement of u_{d} and those in purple for the measurement of u_{min}.

The Tab.1 summarizes the OFI parameters measured in this situation using the equations described in Sec. 3.

Table 1. OFI parameters measured on the signal in Fig. 4.

U_0 (V)	υ (V)	C	α
3.043	0.048	9.64	6.65

5.2 Acoustic waves measurements

The value of C being very sensitive to the alignment of the laser beam, it is not possible to modify the experimental setup at the risk of changing it and making its previous measurement obsolete. Then, once the OFI parameters are measured, the vibrating pot is turned off and the waveguide is excited by a 610 Hz sine wave. This frequency corresponds to a resonance frequency of the waveguide allowing to obtain a high sound level of 150 dB_{SPL} in the section crossed by the laser.

The OFI, accelerometer and microphone signals are measured during 1 s and sampled at 10 kHz. The OFI signal is represented on the top of the Fig. 5.

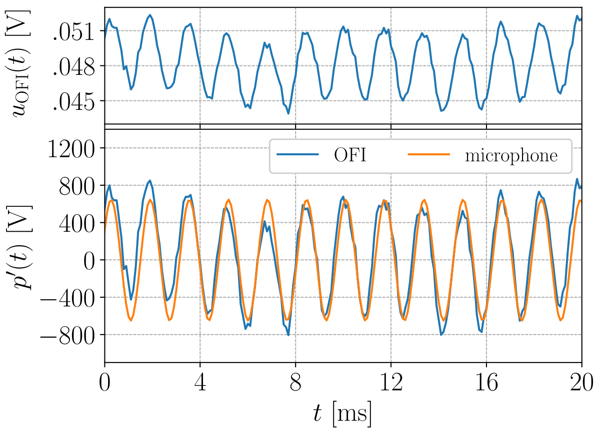

Figure 5. Measurements of $p'(t)$ obtained from the OFI signal $u_{\text{OFI}}(t)$ (in blue) compared to those obtained with the microphone (in orange).

It can be seen that the signal of the OFI does not present discontinuities because variations of $\Phi(t)$ caused by the acoustic waves are lower than 2π. The value of $\mathcal{L}(t)$ can then be calculated using Eqns. (4) and (10).

The value of $(L_{\text{V}}(t) + L_{\text{AO}}(t))$ is computed with Eqn. (8) by recovering the alternative part of the signal. $L_{\text{V}}(t)$ is subtracted from the previous value with the accelerometer measurements. These small variations in the length of the laser beam are a consequence of the reflexive tape being directly mounted on the mobile part of the shaker. It is set in motion by the acoustic waves radiating from the waveguide creating nanometers $L_{\text{V}}(t)$ fluctuations.

The OFI measurements of $p'(t)$ is computed from Eqn. (19) and is compared to the one made with the microphone in Fig. 5. The similarity between the measurements made with the OFI and the microphone are confirmed by measuring 610 Hz acoustic waves of different amplitudes. Fig. 6 shows the amplitude of the 610 Hz acoustic wave measured with the OFI (denoted P_{OFI}) compared to that measured with the microphone (denoted P_{micro}) for different measurements.

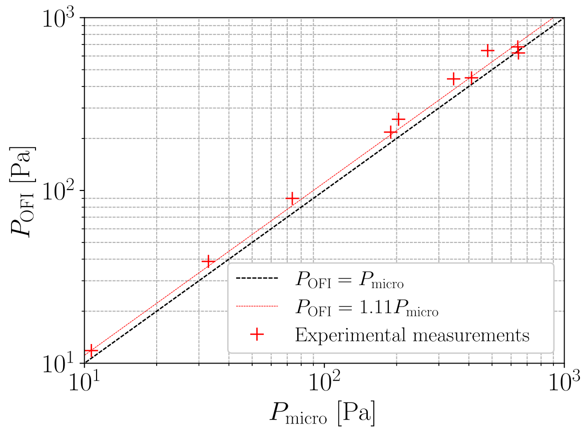

Figure 6. Amplitude of the 610 Hz acoustic waves measured by the OFI P_{OFI} in relation to those measured by the microphone P_{micro}. The black dashed line represents points where $P_{\text{OFI}} = P_{\text{micro}}$. The red dashed line is a linear regression of the experimental data of equation $P_{\text{OFI}}/P_{\text{micro}} = 1.11$ with a $r^2 = 0.96$.

This similarity between the OFI and the microphone measurements validates a good efficiency of this method for the measurement of sinusoidal acoustic waves for amplitudes between 10 and 700 Pa. The noise in the signal prevents the OFI from detecting lower amplitude acoustic waves [10].

6. CONCLUSION

The evaluation of all the variables of Eqns. (4) and (10) allowed to measure the acoustic pressure of an acoustic waves propagating perpendicular to the laser beam OFI in a waveguide. This has been verified experimentally at a frequency of 610 Hz for amplitudes between 10 and 700 Pa.

Future work will show the application to different acoustic sources, frequencies and levels.

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