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NONLINEAR ACOUSTIC MODELING IN WAVEGUIDES: FROM BRASS INSTRUMENTS TO INDUSTRIAL APPLICATIONS

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ABSTRACT

The acoustic modeling of wave propagation in musical instruments, based on linear models, is very common in the world of sound synthesis. The particular case of brass instruments implies the addition of a nonlinear term involved in the "brassiness" color of the sound. A frequency resolution algorithm called Harmonic Balance Method (HBM), was initially described by Gilbert & al (2000) to take into account this term. Beside this method, the resolution requires time domain methods such as Finite Difference Time Domain schemes (FDTD). The present topic focuses on the study of some of these time domain methods applied to brass instrument, and how they can be implemented in industrial piping systems with many other elements disturbing the acoustic field: orifices, cross-section changes, bifurcations, and lumped elements such as volumes or Helmholtz resonators. Modeling these combined effects is the motivation for this work.

Keywords: *nonlinear acoustics, waveguides, FDTD modeling, sound synthesis*

1. INTRODUCTION

In energy industries, the propagation of gas using reciprocating compressors can occasionally affect the safety of the piping system due to excessive vibratory stresses. The normal operation of these compressors induces wave pulsations in the piping system, causing vibrations induced by fluid-structure coupling. To reduce the level of these pulsations, elements such as expansion chambers, orifices, or volumes acting as added mass are integrated into the piping system. However, the position of these elements requires specific positioning and design to ensure constructive tuning of the damping over the operating frequency band of the reciprocating compressor (usually 0-300 Hz). Historically, this tuning was based on the electro-acoustic analogy from the telegrapher's equations whose linear resolution allowed, by transfer matrix, to estimate the acoustic response of the system along the piping line. But the emergence of increasingly powerful compressors operating at faster speeds (0-600 Hz) has led to the generation of important non-linear effects. These effects appear either during propagation, or in a localized manner at singularities such as orifices or section changes.

In the field of nonlinear resolution, numerical methods are often necessary to simulate wave propagation. To reduce the calculation times for industrial applications, the one-dimensional resolution has been preserved (i.e., this assumption is acceptable because the system cut-off frequency is generally above the maximum operating frequency of the compressor).

The first part deals with the time domain resolution of the weakly nonlinear propagation of waves in waveguides using differents method like FDTD (Finite Difference Time Domain) schemes. The second part deals with the nonlinear effects induced by the orifices, at the ori-

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gin of strong nonlinear and damping effects in the system. The third part concerns the validation of the hybrid model arising from these two parts and his application on an industrial case.

2. MODELING OF NONLINEAR PROPAGATION

The hypothesis of weakly nonlinear propagation ($u' \ll c_0$, where u' is the acoustic velocity and c_0 the celerity), allows to obtain the non-dissipative Burgers equation, whose literature is full of resolution methods [1–3]. A well-known quasi-analytical model is achhieved using the Burgers-Hayes method [4, 5], allowing resolution for any form of solution (see Fig.1).

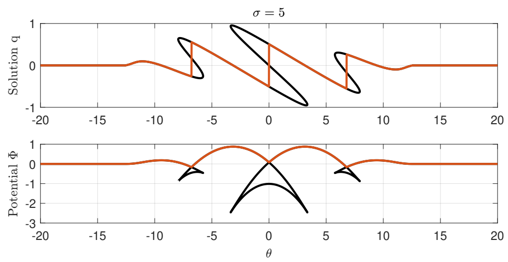


Figure 1. Solution of the propagation of a wave packet, beyond the shock formation. In black, the multivalued Poisson solution of and in red, the the physically admissible solution from the of Burgers-Hayes method.

The interest of this model lies in the possibility of evaluating the harmonic evolution of a signal during its propagation, beyond the shock formation. The resolution of the Burgers equation is more interesting from a physical point of view because it introduces a dissipative term [6]. Some analytic solutions still exist in special cases [7–9], but numerical resolution becomes essential for a generalized solution [10, 11]. For example, Lombard & Mercier [12] have proposed an explicit 2^{nd} order TVD scheme with limiters applied on the flux function, to ensure stability beyond shock formation.

Burgers equation simulate the weakly nonlinear propagation of waves including a bulk losses term (interaction between particles: viscosity, thermal conductivity and molecular relaxation). But the guided wave propagation implies additional friction of the particles against the walls of the waveguide [13, 14]. This term is called viscothermal losses and lead to the generalized Burgers equation :

$$\frac{\partial q}{\partial \sigma} - q\frac{\partial q}{\partial \theta} = -\frac{T}{\epsilon}\frac{\partial^{\frac{1}{2}} q}{\partial \theta^{\frac{1}{2}}} + \frac{\mathrm{S}}{\epsilon}\frac{\partial^2 q}{\partial \theta^2}, \tag{1}$$

where the term $\frac{T}{\epsilon}$ corresponds to the ratio between nonlinear effects and boundary layer dissipation effects, with :

$$T = \frac{\alpha}{k_0}, \tag{2}$$

where $k_0 = \omega/c_0$ and α is the wall attenuation coefficient.There is no analytical solution to this equation. Menguy & Gilbert [15] proposed a numerical resolution method in the frequency domain called the Harmonic Balance Method (HBM). Lombard & Mercier proposed a time domain numerical scheme using a fractional step called Strang Splitting [16]. This scheme seperates the equation into a propagative part (Burgers equation), whose resolution is carried out by TVD scheme already mentioned, and a relaxation part (diffusion equation), dealing with the fractional derivative term associated with viscothermal losses at the wall. The latter is solved by the Yuan-Agrawal method [17], transforming a fractional differential equation into an ordinary differential equation. Figure 2 illustrates the effect of adding these loss terms, in the evolution of a wave packet beyond shock formation, in a 50 mm diameter pipe.

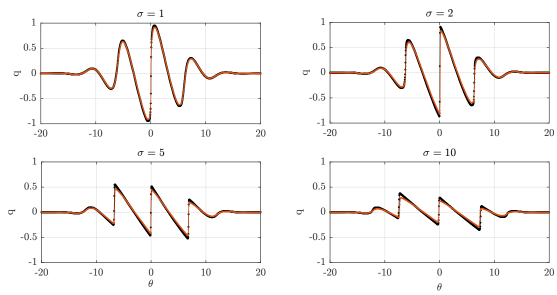


Figure 2. Solution of the propagation of a wave packet, beyond the shock formation. In black the lossless Burgers solution obtained from the Burgers-Hayes method, in red, the generalized Burgers solution obtain from the FDTD scheme proposed by Lombard & Mercier.

This equation is valid for a progressive wave. However, the modeling of wave propagation in a piping system implies a non-zero stationarity rate. In the case where the reflection rate of a termination is independent of the incident wave, Harisson & Bilbao showed numerically that in the weakly nonlinear case, the decoupling of the incident and reflected waves is reasonnable [18]. But in the case of high pulsating levels, the response of an orifice is nonlinear, meaning that its impact depends on the incident waves on both sides. In other words, the reflected waves thus become dependent on the incident waves. A direct resolution of the 1D Navier-Stokes equations is necessary to combine nonlinear wave propagation with the nonlinear behavior of orifices. Gascon & Corberan [19] proposed a 2^{nd} order TVD scheme to solve these 1D Navier-Stokes equations in a proper way.

3. LOCALIZED NON-LINEARITIES THROUGH AN ORIFICE

From the assumption of an incompressible and one-dimensional fluid, it is possible to obtain the unsteady formulation of Bernoulli's equation, using the motion conservation equation. Using also the mass conservation equation, Cummings [20] proposed a simple equation connecting the acoustic velocity to the pressure difference on both sides:

$$p_1 - p_2 = \frac{1}{2}\rho_0 \left(\frac{1-\zeta^2\Upsilon_a^2}{\Upsilon_a^2}\right)\left(2V_{0(o)} + |v_{a(j)}|\right) v_{a(o)} + \rho_0 l \frac{v_{a(o)}}{t}, \quad (3)$$

with ζ the section ratio, Υ the vena contracta coefficient, $V_{0(o)}$ the steady flow, and l a length associated with the added mass effect provided by the volume of fluid contained through the orifice. This equation can be solved numerically using the 4^{th} order Runge Kutta's theorem. By considering the orifice as a point element of the FDTD scheme, it is possible to combine the nonlinear propagation model with this localized model, in order to obtain a hybrid model adapted to industrial issues [21].

4. RESULTS

The validation of this hybrid model involves comparison with experimental data. Cummings performed measurements on very narrow orifices, maximizing localized nonlinear effects [22] [20]. Two measurements, made on an orifice leading to an anechoic termination, were of particular interest for the authors: the output signal of a high level impulse wave without steady flow; thto demonstrate its usefulness in non-linear modellinge output signal of a high level periodic wave with steady flow.

Figure 3 illustrates the results obtained experimentally by Cummings and those obtained numerically by the hybrid model. It appears that without flow the model is very accurate, and slight deviations appear with added steady flow. However, given the narrowness of the orifices (approximately 1/9 of the diameter of the tube), and the technical complexity that such a measurement represents, these results are very satisfactory.

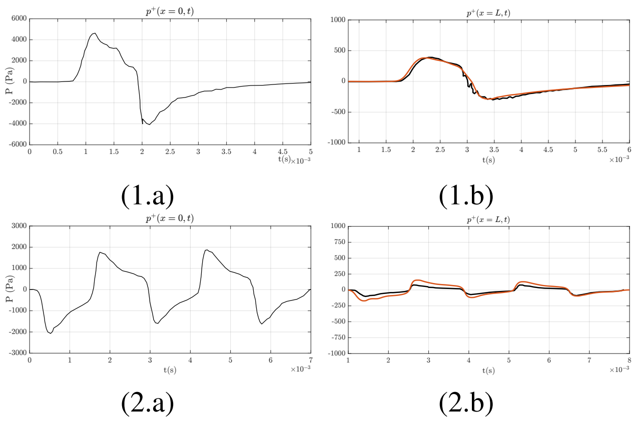


Figure 3. Left column, incident signal to the left of the orifice, right column, transmitted signal to the right of the orifice. The first and second line correspond respectively to the impulse source without flow and to a periodic source with flow (V_0 = 50 m.s^{-1}). The black lines correspond to the values measured by Cummings, the red lines correspond to the results of the hybrid FDTD model.

A second analysis consists of modeling a pipe with two expansion chambers, between which an orifice is positioned (fig.4). The presence of the chambers generates the formation of stationary waves around the orifice. Note that the higher the speed through an orifice, the more it impacts the surrounding pressure field. Thus, the impact of an orifice will be maximum on a pressure node and minimum on a pressure anti-node.

In the present case, a very high level signal is generated at the pipeline input. Without orifice, this signal is distorted during its propagation and diffused in the pipeline due to the presence of the chambers. Thus the monochromatic input signal is transformed into a

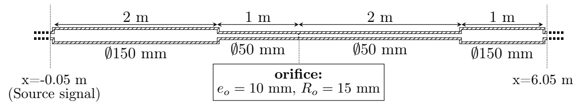
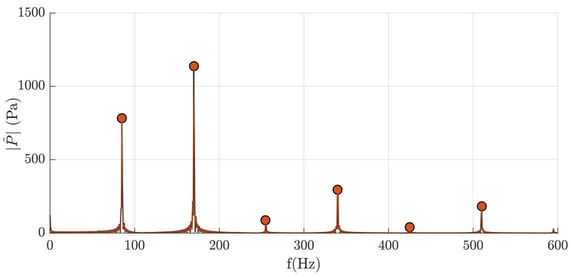
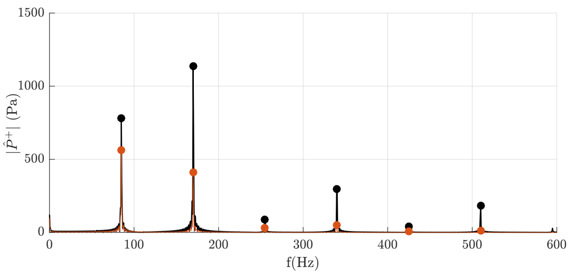


Figure 4. Geometry of the pipe including the two expansion chambers and the orifice (e_o = 10 mm and R_o = 150 mm) located in a waveguide (x = 3 m and R = 250 mm).

harmonic output signal, whose timbre is shaped by the pipeline configuration. When an orifice is added to a pressure anti-node (see Fig.4(a)), the signal is not impacted by the orifice. The orifice is acoustically transparent. When the orifice is moved over a pressure node (see Fig.4(b)), its impact becomes very significant and plays a role of damping over the entire width of the spectrum.



(a)



(b)

Figure 5. Pressure spectrum at the pipe outlet. (a) orifice positioned on a pressure node, (b) orifice positioned on a pressure node. the black and red lines correspond respectively to the pipe without and with orifice.

4.1 Conclusion

This document describes in successive bricks, some models allowing to simulate the nonlinear propagation of acoustic waves. These models provide a bridge between industrial piping system issues and wind instrument sound synthesis. A hybrid model is also presented to incorporate orifices in piping system simulations. This model is validated experimentally and tested on an industrial case study to demonstrate its usefulness in nonlinear modelling.

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