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NONLINEAR TRANSMISSION OF AN ACOUSTICAL WAVE THROUGH A WEAK SHOCK

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ABSTRACT

Recent observations (Ducousso et al., Phys. Rev. Appl., L051002, 2021) demonstrated the possibility to image weak shock propagation in solids by an ultrasonic probe wave. Interaction of an acoustical wave with a steady step shock in air has been previously described (Burgers, Selected Papers, Springer, 1995, McKenzie and Westphal, Phys. Fluids, 11, 1968), without consideration for the particular case of a weak shock nor for the influence of the medium. The present study investigates the interaction of a weak shock with an incident probe wave in any fluid. No reflected wave arises. The transmitted wave, vortex and entropy modes behind the shock, and the shock front disturbance, are determined by the linearisation of the Rankine-Hugoniot relations. For a weak shock, entropy mode is negligible. The shock motion induces a Doppler effect dependant on the medium, air and water giving opposite trends. The amplitude of the transmitted wave is either increased or reduced through energy exchanges with the shock. For an incidence beyond the critical angle, instead of total reflexion, model predicts an inversion of the direction of the transmitted wave, now propagating in the same direction as the shock. This phenomenon seems specific to weak shocks.

Keywords: *weak shocks, probe wave, transmission, nonlinear acoustics*

1. INTRODUCTION

Interaction of an acoustic (probe) wave with a shock has been investigated theoretically by Burgers in 1946 [1] and Brillouin in 1955 [2] at one dimension (normal incidence), and by Mc Kenzie and Westphal in 1968 [3] at two dimensions (oblique incidence). This theory has been used namely to explore shock interaction with a turbulent flow [4]. However, the particular case of an acoustical weak shock has never been studied in detail, and the only considered medium was a perfect gas, namely air. A recent study [5] used a probe ultrasound wave to interact with and image a shock wave generated by laser in metals [6, 7]. The objective of this work is therefore to focus on the case of the nonlinear interaction of a probe, linear acoustical wave with a weak shock wave, considering any classical fluid, either gases or liquids.

2. THEORY

The investigated situation is depicted in Fig.1: an ideal step shock is moving in an inviscid fluid at speed w_s. Downstream, the fluid is at rest (density ρ_0 and sound speed is c_0). Upstream, behind the shock, there is a uniform flow at velocity v_s, and the medium parameters are modified, with density ρ_s and sound speed c_s. An acoustical wave, propagating with sound speed c_0 on the downstream side, is incident on the shock wave front with angle θ. It is transmitted to the upstream side, now propagating with speed c_s. No reflected wave can exist, as such wave would propagate at sound speed c_0 slower than the shock speed, and would be immediately overtaken by this one. However, the shock front itself can be disturbed by the sound wave. Moreover, on the upstream side, the flow motion allows existence of an entropy mode (affecting only density and entropy) and of a vorticity mode (affecting only fluid velocity in a pure rotational way). These two modes are convected by the ambient upstream fluid motion at speed v_s. Convection is also to be taken into account for the transmitted acoustical wave. Conditions satisfied by the sought after four unknown waves (transmitted acoustical wave, shock disturbance, entropy and vorticity modes) are the four Rankine-Hugoniot relations describing the mass, momentum (in the two directions) and

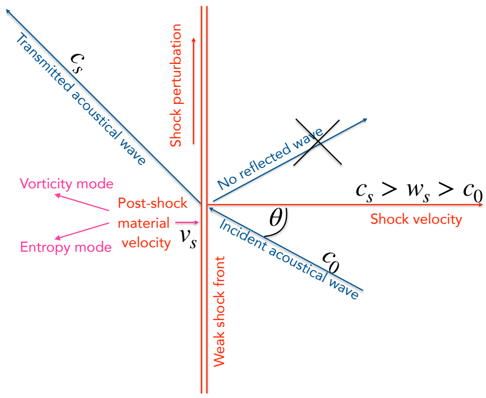

Figure 1. Transmission of an acoustical probe wave through a weak shock : sketch of geometry and various modes.

energy balance laws through the shock front. The problem can be solved by linearizing these relations around the case unperturbed by the acoustic probe wave. The wavenumber in the y-direction parallel to the shock front is the same for all waves, but the frequency is modified through transmission: a Doppler effect occurs due to the interaction of the incident wave with the mobile shock.

In the case of a weak shock, simple relations exist between upstream and downstream quantities, all governed by the small parameter ϵ measuring the relative variation of density ρ

$$\rho_s = \rho_0(1+\epsilon) \tag{1}$$
$$p_s = p_0 + \epsilon\rho_0 c_0^2 + (B/2A)\epsilon^2\rho c_0^2 \tag{2}$$
$$v_s = \epsilon c_0\,(1+((B/2A)-1)\,\epsilon) \tag{3}$$
$$w_s = c_0\,(1+0.5\,((B/2A)+1)\,\epsilon) \tag{4}$$
$$c_s = c_0\,(1+(B/2A)\epsilon) \tag{5}$$
$$s_s = s_0(1+O(\epsilon^3)) \tag{6}$$

where $B/2A$ is the classical parameter of acoustical nonlinearity of the ambient fluid

$$\frac{B}{2A} = \frac{1}{2}\frac{\rho_0}{c_0^2}\left(\frac{\partial^2 p}{\partial\rho^2}\right)_s(\rho_0, s_0). \tag{7}$$

Here pressure is noted p, material velocity v, sound celerity c and entropy s, with index $_0$ for the unperturbed downstream flow and $_s$ for the upstream, post-shock flow.

Eq.(6) is a classical result: the entropy jump through a weak shock wave varies as power 3 of the shock amplitude and is therefore of smaller order. Consequently here, the entropy jump and the entropy mode can be neglected, while the Rankine-Hugoniot balance law for energy can be ignored. The weak shock problem therefore involves three unknowns only. At one dimension ($\theta = 0$), the vorticity mode vanishes also and only two unknown quantities are to be determined. Detailed calculations are too lengthy to be reported here.

3. RESULTS AND DISCUSSION

3.1 Normal incidence

At normal incidence, simple closed form solutions can be obtained. The transmission coefficient T (amplitude ratio of the transmitted wave to the incident one) is found to be

$$T = [1-(B/2A-1)\,\epsilon]\,. \tag{8}$$

The Doppler effect is measured by the ratio D of the transmitted ω^{tr} to the incident ω^{inc} frequencies

$$r = \omega^{tr}/\omega^{inc} = (1-B/2A)\,\epsilon. \tag{9}$$

The velocity shock perturbation $w_a(t)$ turns out to be proportional to the velocity waveform $v_a(t)$ of the incident wave

$$r = w_a(t)/v_a(t) = (B/2A-1)\,. \tag{10}$$

Doppler effect D and deviation from unperturbed transmission $T-1$ are of course proportional to the weak shock amplitude ϵ. What is remarkable is that these laws are all proportional to the coefficient $B/2A-1$. For media with $B/2A < 1$ such as air ($B/2A = (\gamma-1)/2 = 0.2$ where γ is the ratio of specific heats for a perfect gas), the amplitude and the frequency of the wave increase through transmission, while the shock perturbation is in phase opposition to the incident probe wave velocity. On the contrary, for media with $B/2A > 1$ (such as water, $B/2A = 2.5$), the amplitude and the frequency of the wave decrease through transmission, while the shock perturbation is in phase with the incident wave velocity. For a particular medium with $B/2A = 1$, a weak shock wave would be fully transparent to an incident sound wave at normal incidence. The role of this parameter is explained by considering the phase velocity of the transmitted wave $c_\phi = c_s - v_s$. The term $-v_s$ describes convection of the wave by the post-shock flow. Using Eq.(3) and Eq.(5), one gets

$$c_\phi = c_0\,[1+(B/2A-1)\,\epsilon]\,. \tag{11}$$

Hence, for $B/2A < 1$, the transmitted phase velocity is smaller than the incident one, while it is larger in the opposite case, and this ratio c_ϕ/c_0 controls the whole behaviour of the transmission phenomenon. This is the first main result of this study, outlining the key role of the nonlinear acoustical parameter of the medium. To our knowledge, only the case of air (or perfect gases) with $B/2A < 1$ had been studied yet, without emphasis to the weak shock case. Here we point out that air and water for instance, the two most common fluids, lead to opposite behaviours. Though valid for fluids only, one can expect the above results to be at least qualitatively valid also for solids, as the behaviour of a nonlinear longitudinal wave is similar to an acoustical compression wave. For many solids, the equivalent of the nonlinear parameter $\beta = 1 + B/2A$ is significantly larger (equal to around 11 for instance in aluminium). This would therefore explain the large dip in transmitted wave amplitude observed by Ducousso et al. [5] for a laser weak shock generated in aluminium.

3.2 Oblique incidence

At oblique incidence, analytical solutions are too lengthy to be presented here and we simply show the resulting behavior of the transmission coefficient T, of the Doppler effect D and of the axial wave number of the transmitted wave k_x^{tr} normalized by the incident wave number $k_0 = \omega^{inc}/c_0$, as function of the incidence angle θ for air (in red) and water (un blue). All curves are drawn for a shock relative amplitude equal to $\epsilon = 0.1$

Transmission coefficient – Fig.(2) – in air always keeps larger than one and varies little with incidence angle. On the contrary, in water it increases with angle, keeping values smaller than one for small and moderate angles, but going beyond one for large angles (above about 73°). These differences between air and water amount to the fact that, for air, key parameter $B/2A - 1 = -0.8$ is negative and relatively small in amplitude, while it is positive in water (1.5) and twice larger in absolute value. For the Doppler effect, again variations with incidence angle are much larger for water than for air. The ratio D increases for both media, so that for water it keeps always larger than one. For air it goes above the one value for very large angles close to 80°.

The axial wavenumber – Fig.(3) – indicates the axial direction of the transmitted wave. In classical reflexion at a fluid/fluid fixed interface, when the sound celerity of the transmission medium is larger than the incident one, k_x^{tr} decreases while incidence angle increases, until

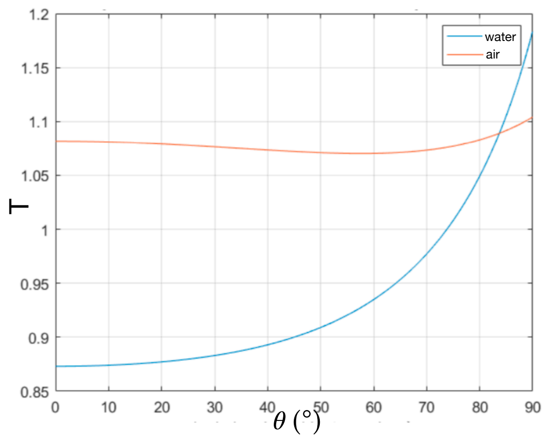

Figure 2. Transmission coefficient T versus incidence angle in degrees. Red line : in air. Blue line : in water.

reaching zero. Beyond is the phenomenon of total reflection: the axial wave number gets a pure imaginary number and the transmitted wave transforms into an evanescent one. On the contrary, the reflection coefficient is equal to one: on average, all the incident energy is reflected. Such phenomenon cannot occur here as there is no reflection, but one could imagine all the incident energy could be transferred to the shock disturbance and/or to the vorticity mode. This is not what happens. As c_s is larger than c_0, there indeed exists some critical angle, but beyond it the axial wavenumber k_x^{tr} changes of sign and gets positive. This happens only for grazing incidence, slightly below 80° for water and at almost 90° for air. The transmitted wave then propagates *towards* the shock. However, the phase velocity remains always lower than the shock speed w_s, which means that the transmitted wave always remains behind the shock front in the post-shock flow. It thus appears as a post shock perturbation of the medium lagging behind the shock. Such a phenomenon is of course not possible in case of a fixed interface, but is here physically admissible because of the shock motion. Note that for shocks of larger amplitude, imaginary wavenumber can be recovered [3]. With the present weak shock theory, this is recovered for values around ϵ=0.215 in water and ϵ=0.92 in air. At least for air, these amplitudes are nevertheless beyond the approximation of weak shocks ($\epsilon << 1$) and higher-order approximations would be necessary to consider.

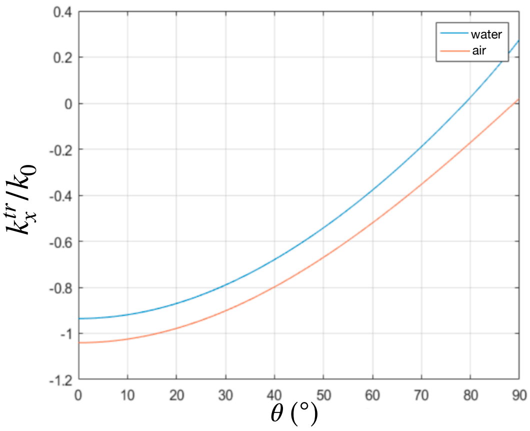

Figure 3. Normalized transmitted axial wave number k_x^{tr}/k_0 versus incidence angle in degrees. Red line : in air. Blue line : in water.

4. CONCLUSION

We investigated the transmission of an acoustical probe wave of small amplitude through a counter-propagating shock wave. We focused our attention to the little explored case of a weak shock wave, so that the shock itself can be considered as a (nonlinear) acoustical wave. In this preliminary study, we obtained two main results. At normal incidence, we outlined the influence of the medium of propagation, with opposite behaviours observed for air and for water. Up to now, only the case of perfect gases was explored. We expect the water-type behaviour to be observed also for solids, as indicated by recent experiments. However, the theory has to be adapted to this case. For oblique incidences, the phenomenon of total reflection with a transmitted evanescent wave is replaced by the appearance of a transmitted wave propagating towards the interface instead of away from it, but at a slower velocity and therefore always lagging behind it in the post-shock region. This phenomenon is however limited to shocks of small or moderate amplitudes, and a higher-order theory is necessary to quantify this limitation in terms of shock amplitude. Various behaviours show more sensitivity to the incidence angle for water than for air, and we expect this to be even more true in case of solids.

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6. ACKNOWLEDGMENTS

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