DOI: 10.61782/fa.2023.0103

forum acusticum 2023

INVESTIGATION OF COHERENT MOTIONS AND NOISE RADIATION IN TWIN SUPERSONIC JETS USING HIGH-SPEED SCHLIEREN IMAGES

Iván Padilla-Montero[1*] **Daniel Rodríguez**[1] **Vincent Jaunet**[2]
Stève Girard[2] **Damien Eysseric**[2] **Peter Jordan**[2]
[1] ETSIAE - Universidad Politécnica de Madrid, 28040 Madrid, Spain
[2] Institut Pprime, CNRS - Université de Poitiers - ISAE - ENSMA, 86036 Poitiers, France

ABSTRACT

Spectral proper orthogonal decomposition is applied to high-speed Schlieren data sets obtained for a supersonic twin-jet configuration. Relevant features in the SPOD energy spectra are identified and described, showing the amplitude functions of the associated modes and their mechanics of propagation. At overexpanded conditions, a screech resonance dominates the noise generation in the system, involving downstream-propagating Kelvin-Helmholtz (KH) waves and upstream-propagating freestream acoustic waves. In the perfectly-expanded regime, energetic tones are also observed in the spectrum, which suggest the existence of acoustic resonances of a different nature than screech. However, only signatures of Kelvin-Helmholtz structures and downstream-propagating acoustic waves are identified in the obtained Schlieren data for this case.

Keywords: *twin jets, Schlieren, supersonic, SPOD*

1. INTRODUCTION

Modern propulsion systems employed by rocket launchers and high-speed aircraft often feature twin- or multi-jet engines. The governing noise-generation mechanisms in such systems are not fully understood and constitute a state-of-the-art topic. Closely-spaced jets are known to interact at the hydrodynamic and acoustic levels, giving rise to more complex flow structures than single jets. Early experiments on twin-jet configurations were able to identify two different mechanisms which can result in a reduction of the far-field noise with respect to isolated jets [1]. On the one hand, for closely-spaced nozzles, the sound sources can be modified and reduced by means of the interaction between the turbulent mixing regions and near pressure fields of each jet. On the other hand, one jet can produce acoustic shielding on the other. In contrast, modern experimental observations reveal increased noise levels in the plane perpendicular to the jet centers, even beyond those corresponding to two linearly superposed single jets [2].

Different investigations on single round jets agree on the fact that radiated sound is highly directional for both subsonic and perfectly-expanded supersonic jets [3, 4], and that it is strongly correlated with large-scale, low-frequency coherent fluctuations in the mixing layers [5]. Such structures were found to bear resemblance to instability waves in harmonically-forced supersonic jets, opening the door to employing linear instability theory to model the sound-emitting structures in turbulent jets [6–8]. Although being widely successful, linear stability models are not straightforward to implement and apply to complex flows such as twin jets.

Despite its theoretical foundation being established decades ago, spectral proper orthogonal decomposition (SPOD) [9] has recently arisen as a powerful data-driven technique capable of extracting flow structures that are coherent both in space and in time. Numerous works have applied SPOD successfully to single-jet flow fields, employing both numerical and experimental data sets [10, 11]. However, the application of SPOD to twin-jet flow

*Corresponding author: ivan.padilla@upm.es.

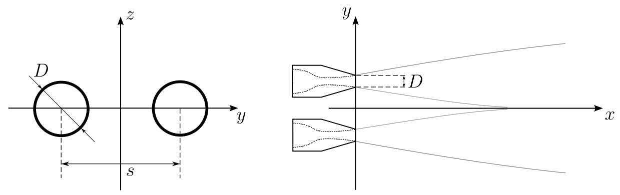

Figure 1. Sketch representing the studied twin-jet configuration and geometrical parameters.

fields is still scarce and very recent in literature [12, 13], and it has the potential to elucidate unknown noise-generation mechanisms that might be exclusive to twin-jet systems and guide their modeling using stability theory.

This work describes coherent structures obtained by means of SPOD based on high-speed Schlieren data sets for supersonic twin jets generated by convergent-divergent nozzles. The paper is organized as follows. Sections 2 and 3 discuss the twin-jet geometry and the experimental setup under consideration, while section 4 provides details on the SPOD methodology applied to the Schlieren data sets. SPOD results are presented in section 5 and conclusions are issued in section 6.

2. TWIN-JET CONFIGURATION

The twin-jet configuration studied in this work is illustrated in Fig. 1. Two identical round convergent-divergent nozzles are considered. The center of each of the nozzles is located at the y-axis ($z = 0$) and the spacing between the axisymmetry axis of each of them is given by the distance s. The quantity D denotes the nozzle exit diameter, and the exit of each of the nozzles is located at $x = 0$. The nozzle geometry has been designed at Institut Pprime (CNRS-Université de Poitiers-ISAE-ENSMA), and features a truncated ideal contour (TIC) profile with an exit diameter of $D = 0.025$ m and an exit-to-throat area ratio of $A_e/A_t = 1.225$.

The jets are non-isothermal (not heated) and are subject to different nozzle pressure ratios with a total temperature of 300 K. The pressure ratio considered, that is, the ratio of the total pressure in the reservoir (p_0) to the ambient pressure (p_∞) is defined in terms of the isentropic Mach number (M_j) through the isentropic relation $p_0/p_\infty = (1 + 0.5(\gamma - 1)M_j^2)^{(\gamma-1)/\gamma}$.

The theoretical perfectly-expanded condition is determined by the isentropic flow assumption, which for the nozzle under consideration is $M_j = 1.57$. In practice, however, owing to viscous effects and the truncation of the nozzle contour, this condition is never reached. The closest to the perfectly-expanded condition obtained in the experimental configuration (by means of flow visualization) corresponds to a slightly smaller value of M_j, namely $M_j = 1.54$. This value has been replicated succesfully by means of RANS calculations.

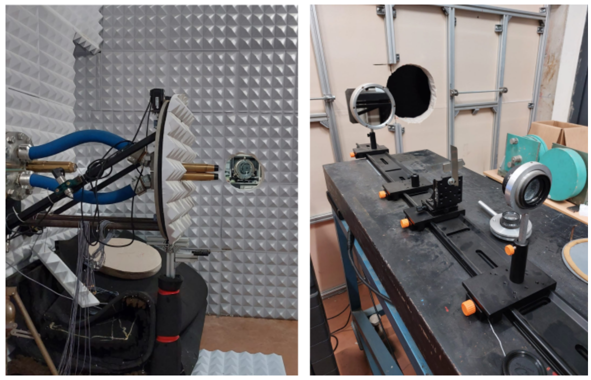

Figure 2. Pictures of the hardware setup for the experiments. (left) General view of the twin-jet system; (right) part of the Z-type Schlieren setup.

3. EXPERIMENTAL SETUP

Experimental investigations on the twin-jet configuration described above have been performed at the PROMÉTÉE platform of Institut Pprime (CNRS-Université de Poitiers-ISAE-ENSMA). The facility employed is the T200 [14] compressible wind tunnel, which is powered by the platform's 200 bar compressed air network and can reach operational conditions up to $M_j = 2$ for the employed nozzle geometry. The left picture in Fig. 2 displays the twin-jet experimental system.

Schlieren measurements have been performed for different jet spacing and total pressure conditions. Part of the Schlieren setup is shown in the right picture in Fig. 2. It consists of a Z-type configuration featuring two parabolic mirrors, two flat mirrors and a vertical knife edge, oriented to visualize streamwise density gradients. A Phantom v2640 camera is used to record 30,000 images with a resolution of 352×512 pixels at a sampling frequency of $f_s = 68$ kHz for each test. An exposure time of 0.8 µs is used, which is sufficiently small to freeze the convected disturbances while ensuring enough contrast in the image.

In this work, results are shown for a jet spacing of

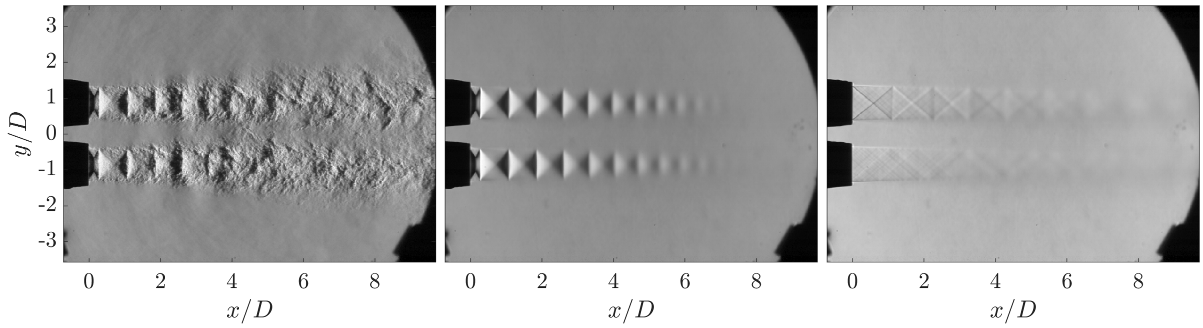

Figure 3. (left) Sample instantaneous Schlieren field for $M_j = 1.26$; (middle) mean Schlieren field for $M_j = 1.26$; (right) mean Schlieren field for $M_j = 1.54$. Jet spacing $s/D = 1.76$.

$s/D = 1.76$, which corresponds to the closest possible nozzle spacing allowed by the design. Two different total pressure conditions are considered, namely $M_j = 1.26$ (overexpanded regime) and $M_j = 1.54$ (perfectly-expanded regime).

4. SPECTRAL PROPER ORTHOGONAL DECOMPOSITION OF THE SCHLIEREN DATA

The spectral proper orthogonal decomposition methodology described in [9] is considered. The sampling frequency at which the Schlieren images are recorded yields a Nyquist frequency of $f_s/2 = 34$ kHz, which corresponds to Strouhal numbers of $St = fD/u_j = 2.36$ and $St = 1.93$ for $M_j = 1.26$ and $M_j = 1.54$, respectively. Based on microphone acoustic measurements performed at a higher sampling frequency, these values of Strouhal number are believed to be high enough so that the data sets are well time-resolved for the frequencies governing the physical mechanisms of interest.

Given the symmetry of the twin-jet configuration, the coherent structures (modes of oscillation) are expected to have symmetric or antisymmetric amplitude functions across the y axis. To exploit this property, the Schlieren images are split by the line at $y = 0$ and two new data sets are created, namely, a symmetric Schlieren data set $\sigma_s = (\sigma_u + \sigma_l)/2$ and an antisymmetric one $\sigma_a = (\sigma_u - \sigma_l)/2$, with σ_u and σ_l respectively denoting the upper ($y > 0$) and lower ($y < 0$) halves of the original Schlieren data.

Each data set is divided into 57 Hamming windows of 1024 snapshots each with a 50% overlap. A Cartesian weighted 2-norm is employed for the SPOD inner product.

5. RESULTS

The left image in Fig. 3 shows a sample instantaneous Schlieren field obtained for $M_j = 1.26$. The middle and right pictures correspond to the mean fields obtained by averaging 30,000 instantaneous Schlieren images for $M_j = 1.26$ and $M_j = 1.54$, respectively.

The mean Schlieren field for $M_j = 1.54$ features very weak shock and expansion waves in the potential core of both jets. In practice, due to viscous effects and geometrical imperfections, weak discontinuities remain in the flow field also for the perfectly-expanded condition. Such waves are more visible in the top jet than in the bottom jet. It was verified that a small imperfection in the form of a step was present in the contour of the top nozzle. Interchanging the top and bottom nozzles, the inverted pattern was reproduced, indicating that it is the cause of the observed asymmetry. This discrepancy between the top and bottom jets has only been observed at the perfectly-expanded condition. At $M_j = 1.26$, the mean field for both jets shows an excellent symmetry, as illustrated in the middle picture of Fig. 3.

5.1 Overexpanded condition

The symmetric/antisymmetric SPOD spectra obtained for the overexpanded condition ($M_j = 1.26$) are represented in Fig. 4. The strong energy peaks found in both spectra correspond to the fundamental screech resonance and its respective harmonics. In this regime, the noise radiated by the twin-jet system is dominated by the screech phenomenon (see for example [11, 15]). The screech tone energy for the symmetric data set is one order of magnitude larger than for the antisymmetric one, indicating that

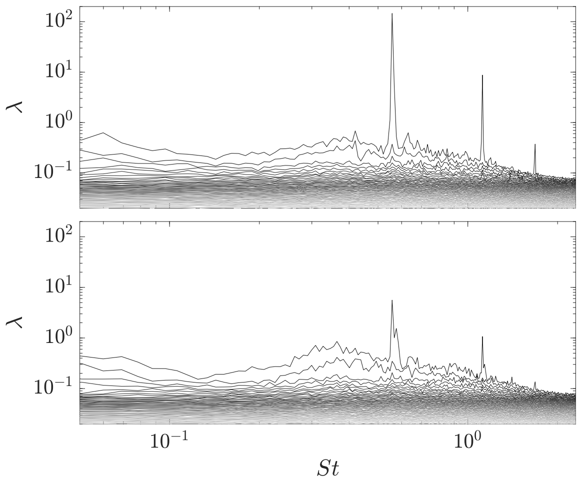

Figure 4. SPOD spectra obtained for $M_j = 1.26$: (top) symmetric data set; (bottom) antisymmetric data set. Each line corresponds to one SPOD mode. A grayscale is used to range from the most energetic mode (black) to the least energetic one (white).

the preferred screech mode for this configuration features a symmetric fluctuation.

The leading symmetric SPOD mode at the fundamental screech frequency ($St = 0.56$) is illustrated in the top contour plot of Fig. 5. Upstream noise radiation, characteristic of the screech phenomenon, can be identified together with coherent structures in the potential core and shear layers. To make such structures clearer, the amplitude function of the SPOD mode is Fourier-transformed along the streamwise direction (x), obtaining a distribution of the mode energy as a function of the streamwise wavenumber k. Then, either the positive or the negative streamwise wavenumbers are suppressed, and the Fourier transform along x is inverted to reconstruct the mode with only the positive or the negative k contributions [15]. The component reconstructed with only the negative values of k represents waves with negative phase velocity $c_{ph} = \omega/k$ (with $\omega = 2\pi f$), whereas the component reconstructed with $k > 0$ represents waves with positive phase velocity. This procedure yields the middle and bottom contour plots depicted in Fig. 5, respectively showing the downstream and upstream-propagating components of the screech mode. The $k > 0$ component mainly consists of coherent structures in the potential core and the shear layers of each jet, which manifest

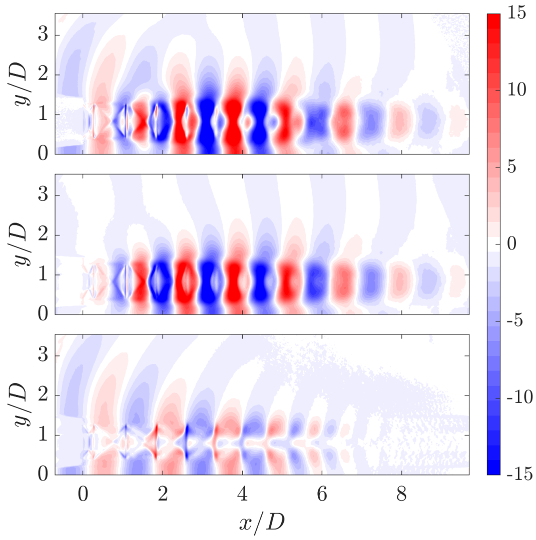

Figure 5. (top) Leading symmetric SPOD mode obtained for $M_j = 1.26$ and $St = 0.56$; (middle) $k > 0$ component; (bottom) $k < 0$ component.

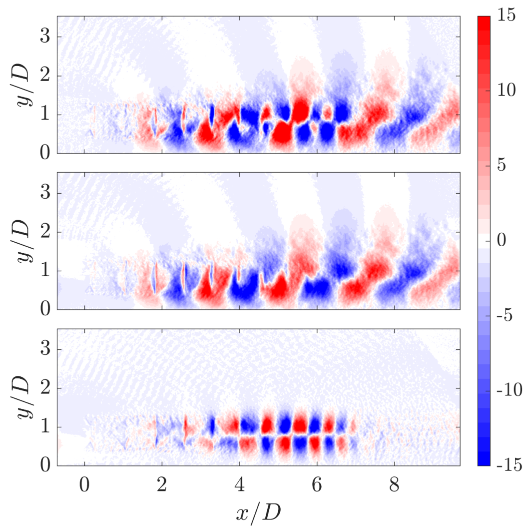

Figure 6. (top) Leading antisymmetric SPOD mode obtained for $M_j = 1.26$ and $St = 0.36$; (middle) $k > 0$ component; (bottom) $k < 0$ component.

as a toroidal ($m = 0$) [16] Kelvin-Helmholtz fluctuation of the jet columns, slightly modulated by the presence of the shock cells. On the other hand, the $k < 0$ component

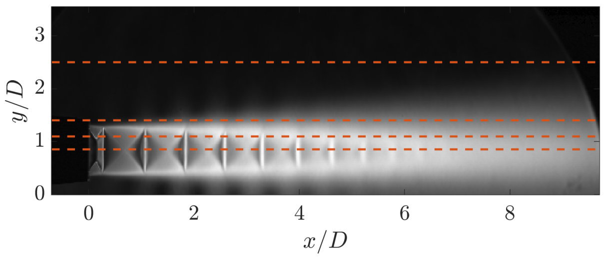

Figure 7. Root mean square of the fluctuation component obtained from the Schlieren data at $M_j = 1.26$. Only the upper half is shown for clarity. The orange dashed lines indicate the y locations at which the different maps in Fig. 8 are obtained.

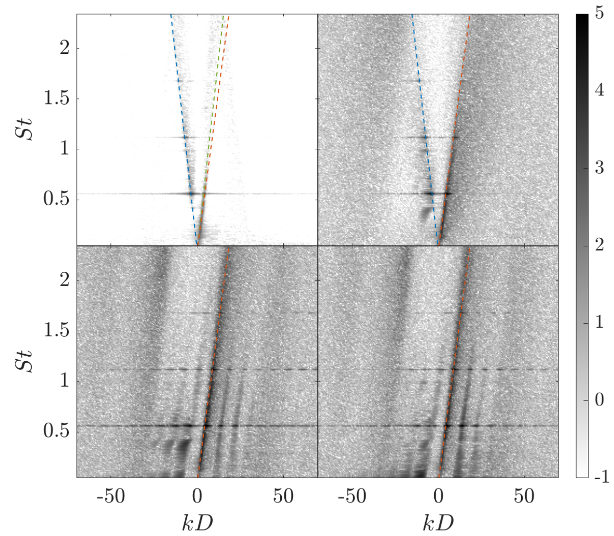

Figure 8. Contour maps of the leading SPOD mode energy as a function of St and k at different y locations ($M_j = 1.26$, symmetric case): (top-left) $y/D = 2.5$; (top-right) $y/D = 1.4$; (bottom-left) $y/D = 1.1$; (bottom-right) $y/D = 0.85$, i.e. nozzle axis. Energy is represented on a logarithmic scale.

contains the distinct upstream acoustic radiation, as well as energetic structures at the shock cell locations.

In addition to screech tones, a broadband increase in energy is found in the range $St \approx 0.23$-0.45, more clearly visible in the antisymmetric SPOD spectrum. This frequency range is characterized by mixing-layer noise driven by Kelvin-Helmholtz instabilities. Nevertheless, a look at the SPOD mode structure at $St = 0.36$, shown in Fig. 6, reveals also the existence of a trapped acoustic subsonic mode evolving within the jet, which features a negative phase speed ($k < 0$ component), and takes the form of a flapping ($m = 1$) [16] oscillation of the jet core. This type of mode was originally described by [17] and is also observed in the numerical analysis of [10] for a single supersonic jet. For subsonic jets, [18] described its central role in experimentally-observed acoustic resonances.

To gain additional understanding on the directions of wave propagation in the flow, the energy obtained by Fourier-transforming the leading SPOD modes along x is represented as a function of St and k for different lines at constant values of y/D (see Fig. 7) [18]. This produces the contour maps illustrated in Fig. 8, each of them obtained by plotting the data at one of the $y/D =$ const. lines shown in Fig. 7. The right half of these maps ($k > 0$) contains those energetic wave structures that have a positive phase speed, whereas the left half contains waves with negative phase speed. Those energetic structures that have a positive slope in the k-St plot feature a positive group velocity $c_g = \partial\omega/\partial k$, and therefore propagate energy downstream in the flow. To the contrary, those waves with signatures that have negative slope are waves with negative group velocity and as a result propagate energy upstream. This information can be exploited to obtain an overview of the different mechanisms that contribute to the energy of a given SPOD mode. For the M_j under consideration, only the contour maps for the symmetric SPOD calculation are shown. The maps corresponding to the antisymmetric results are qualitatively identical to the symmetric ones.

Outside of the jet ($y/D = 2.5$), signatures of freestream acoustic waves propagating upstream and downstream can be observed along most of the studied frequency range, together with weak KH fluctuations visible below $St \approx 0.6$. The distinction between freestream acoustic waves and KH waves is illustrated by the dashed lines. The orange line has a phase speed of $0.8u_j$, which agrees very well with the KH signature, whereas the blue and green lines have phase speeds respectively equal to $\pm c_\infty$, which denotes the freestream speed of sound. The energetic horizontal bands correspond to the fundamental and harmonic screech tones.

Within the outer shear layer ($y/D = 1.4$), a more energetic KH band is visible, together with a stronger signature of upstream-propagating freestream acoustic waves.

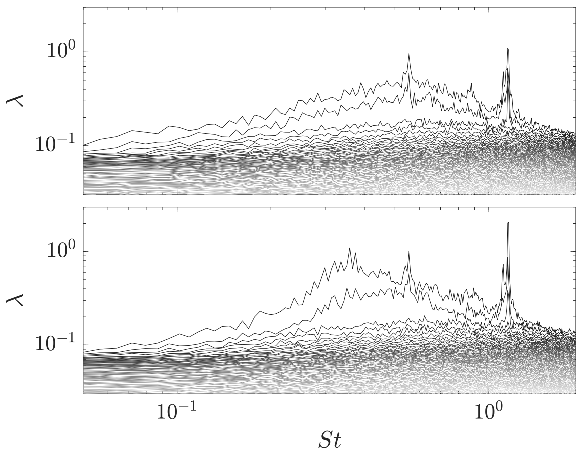

Figure 9. SPOD spectra obtained for $M_j = 1.54$: (top) symmetric; (bottom) antisymmetric.

Inside the potential core ($y/D = 0.85, 1.1$), however, only downstream-propagating waves are visible in the processed data sets, which correspond to energetic KH instabilities and their interaction with the shock cells present in the core of the jets, which appear as the weaker signatures parallel to the main KH band. Since no upstream-propagating waves are visible inside the potential core, the present analysis suggests that the screech resonance mechanism in the studied configuration might be established between the KH waves and the upstream-propagating freestream acoustic waves that have support outside of the jet as well as inside the shear layers.

The energy signature that can be observed inside the jet core and within the shear layer between $St = 0.23$-0.45 and $kD = -12$ to -3 corresponds to the trapped subsonic acoustic mode depicted in Fig. 6. The contour maps show that this mode is propagating downstream for the studied conditions (i.e., it has positive c_g). Therefore, it is not believed to participate in the screech mechanism at the current condition, and it is mainly energetic at frequencies below the fundamental screech frequency.

Although not shown here, k-St maps within the inner shear layer ($y/D \approx 0.24$) and in the symmetry plane between jets ($y = 0$) show very similar signatures to those in the outer shear layer and outside of the jet, respectively.

5.2 Perfectly-expanded condition

In the following, SPOD results are presented for the perfectly-expanded condition ($M_j = 1.54$). Fig. 9 depicts the symmetric and antisymmetric spectra obtained for this configuration. In this case, both data sets yield similar energy levels, which feature a broadband energy increase in the range $St = 0.1$-1, consistent with mixing-layer noise, together with tonal peaks. The highest energy is encountered for a tone at $St \approx 1.15$, followed by another one at $St \approx 0.55$, whose amplitude functions are displayed in Fig. 10 for each symmetry. The $k > 0$ and $k < 0$ components of these SPOD modes are not reported because all the energy is found to be contained only in the $k > 0$ component. Both the symmetric and antisymmetric modes show similar coherent structures with support in the core as well as in the shear layers of the jets. Downstream acoustic radiation is also clearly visible. The SPOD amplitude signatures obtained for this configuration can be identified with the toroidal and flapping oscillations of each jet predicted by plane-marching parabolized stability equations [16].

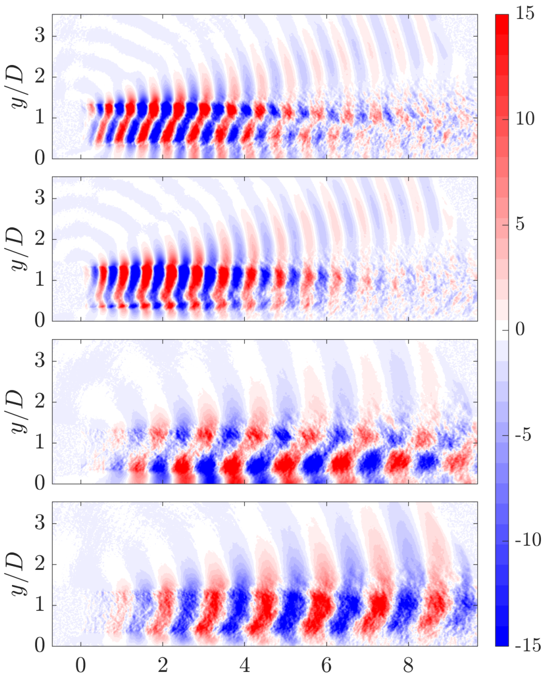

Figure 10. Leading SPOD modes obtained for $M_j = 1.54$: (first) symmetric $St = 1.15$; (second) antisymmetric $St = 1.15$; (third) symmetric $St = 0.55$; (fourth) antisymmetric $St = 0.55$.

Figs. 11 and 12 also illustrate the wave propagation dynamics for this condition at different locations via k-

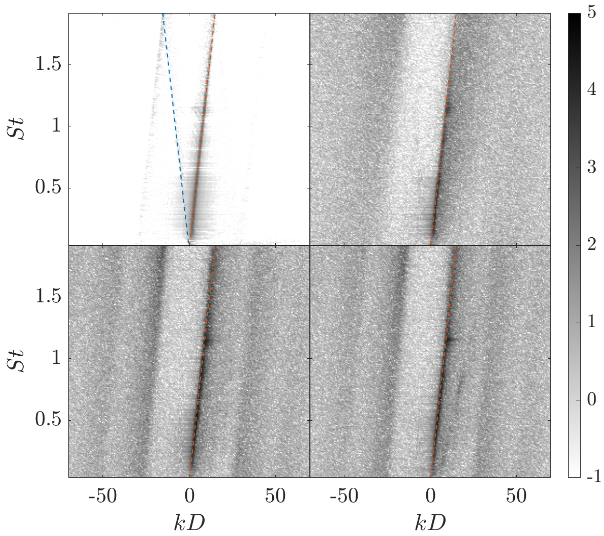

Figure 11. Contour maps of SPOD energy as a function of St and k at different y locations ($M_j = 1.54$, symmetric case). Same y locations as in Fig. 8.

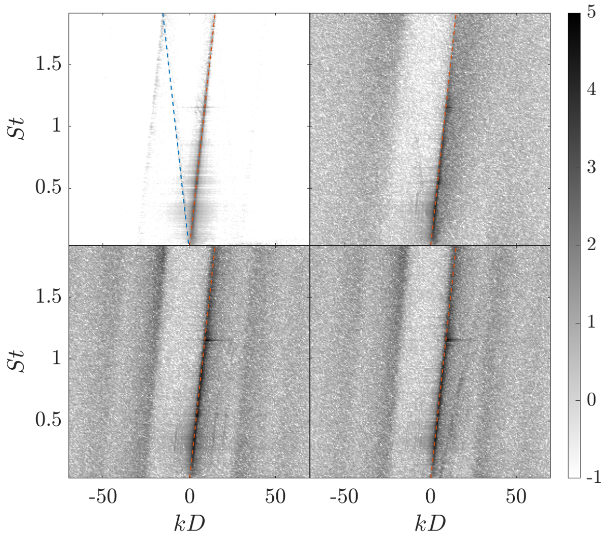

Figure 12. Contour maps of SPOD energy as a function of St and k at different y locations ($M_j = 1.54$, antisymmetric case). Same y locations as in Fig. 8.

St maps. For this M_j, the maps suggest that the fluctuation energy is fully dominated by KH structures. No upstream propagation of energy is visible inside the cores for this case either, only in the form of freestream acoustic waves outside of the jets. It is not evident whether upstream-propagating freestream acoustic waves are supported in the shear layers (top-right maps in Figs. 11 and 12), however, very weak signatures appear to be present intermittently buried under the background noise. These signatures are probably indicative of a very weak screech resonance occurring in the top jet due to the remaining feeble shock cells. In addition, traces with group velocity identical (i.e., parallel) to the main KH band are visible in the bottom-left picture of Fig. 12, which reflect an interaction between the KH waves and the weak shock structures that remain present in the jets for this case (see Fig. 3).

Tones at a fixed frequency of 20 kHz ($St = 1.15$ for $M_j = 1.54$) have also been observed for other nozzle pressure ratios (note the small peak visible at $St = 1.41$ in Fig. 4 for $M_j = 1.26$ in the antisymmetric case). This is an indication that the phenomenon associated to them is not of hydrodynamic origin but probably of acoustic one, and discards the possibility that they are related to a weak screech mechanism generated by the top jet at $M_j = 1.54$. Nonetheless, it has not been possible to identify a resonance mechanism with the present analysis as no distinct upstream-propagating waves could be observed at these conditions with the available Schlieren data. One possibility is that the signature of such upstream waves is too weak in this case to be observed in the current measurements. Linear stability models will be employed to investigate this phenomenon in the future.

6. CONCLUSIONS

Coherent sctructures in twin supersonic jets are investigated by means of spectral proper orthogonal decomposition of high-speed Schlieren data. Two different nozzle pressure ratios are considered for a jet spacing of $s/D = 1.76$, namely, an overexpanded condition ($M_j = 1.26$) and the perfectly-expanded condition ($M_j = 1.54$). For each case, SPOD calculations are carried out employing Schlieren images obtained at a high sampling frequency. Through a Fourier transformation of the SPOD modes in the streamwise direction, the directions of energy propagation in the obtained SPOD structures are also assessed.

At overexpanded conditions, a screech resonance is found to be the main source of sound generation in the twin-jet system, involving downstream-propagating Kelvin-Helmholtz waves and upstream-propagating freestream acoustic waves which have support inside the shear layers as well as outside of the jets. No signatures of upstream energy propagation have been observed inside the jet cores in this study. In the perfectly-expanded regime, energetic tones are also observed in the SPOD spectrum, which suggest the existence of resonances of different nature than screech. However, only signatures of Kelvin-Helmholtz structures and downstream-propagating acoustic waves could be clearly identified in the studied SPOD modes for this case. Predictions based on linear stability theory will be performed in the future to gain additional insight on the possible interactions that give rise to the observed energetic tones.

7. ACKNOWLEDGMENTS

The work of D.R. and I.P.M is funded by the Government of the Community of Madrid through the Program of Excellence in Faculty (V-PRICIT line 3) and the Program of Impulse of Young Researchers (V-PRICIT lines 1 and 3, Grant No. APOYO-JOVENES-WYOWRI-135-DZBLJU). I.P.M also thanks financial support from the European Union's NextGenerationEU fund.

8. REFERENCES

[1] R. A. Kantola, "Acoustic properties of heated twin jets," *J. Sound and Vibration*, vol. 79, no. 1, pp. 79–106, 1981.

[2] R. Bozak and B. Henderson, "Aeroacoustics experiments with twin jets," No. AIAA Paper 2011-2790 in 17th AIAA/CEAS Aeroacoustics Conference (Portland, Oregon), 2011.

[3] D. G. Crighton and P. Huerre, "Shear-layer pressure fluctuations and superdirective acoustic sources," *J. Fluid Mech.*, vol. 220, pp. 355–368, 1990.

[4] C. Tam, "Supersonic jet noise," *Annu. Rev. Fluid Mech.*, vol. 27, no. 1, pp. 17–43, 1995.

[5] P. Jordan and T. Colonius, "Wave Packets and Turbulent Jet Noise," *Annu. Rev. Fluid Mech.*, vol. 45, no. 1, pp. 173–195, 2013.

[6] D. G. Crighton and M. Gaster, "Stability of slowly diverging jet flow," *J. Fluid Mech.*, vol. 77 (2), pp. 397–413, 1976.

[7] K. Gudmundsson and T. Colonius, "Instability wave models for the near-field fluctuations of turbulent jets," *J. Fluid Mech.*, vol. 689, pp. 97–128, 2011.

[8] D. Rodríguez, A. Sinha, G. A. Brès, and T. Colonius, "Inlet conditions for wave packet models in turbulent jets based on eigenmode decomposition of large eddy simulation data," *Physics of Fluids*, vol. 25, p. 105107, oct 2013.

[9] A. Towne, O. T. Schmidt, and T. Colonius, "Spectral proper orthogonal decomposition and its relationship to dynamic mode decomposition and resolvent analysis," *Journal of Fluid Mechanics*, vol. 847, pp. 821–867, jul 2018.

[10] O. T. Schmidt, A. Towne, G. Rigas, T. Colonius, and G. Brès, "Spectral analysis of jet turbulence," *J. Fluid Mech.*, vol. 855, pp. 953–982, 2018.

[11] D. Edgington-Mitchell, X. Li, N. Liu, F. He, T. Y. Wong, J. MacKenzie, and P. Nogueira, "A unifying theory of jet screech," *J. Fluid Mech.*, vol. 945, pp. 1–24, 2022.

[12] J. Jeun, G. J. Wu, S. K. Lele, A. Karnam, F. Baier, and E. J. Gutmark, "Twin rectangular jet screech and coupling: Numerical study and validation," *AIAA Scitech 2021 Forum*, no. January, pp. 1–21, 2021.

[13] P. Nogueira, M. Stavropoulos, and D. Edgington-Mitchell, "Wavepacket coupling in screeching twin-jets," *Annual Conference of the Australian Acoustical Society 2021*, pp. 123–130, 2021.

[14] A. Savarese, P. Jordan, S. Girard, E. Collin, M. Porta, and Y. Gervais, "Experimental study of shock-cell noise in underexpanded supersonic jets," in *19th AIAA/CEAS Aeroacoustics Conference*, (Reston, Virginia), p. 78, 2013.

[15] D. Edgington-Mitchell, V. Jaunet, P. Jordan, A. Towne, J. Soria, and D. Honnery, "Upstream-travelling acoustic jet modes as a closure mechanism for screech," *J. Fluid Mech.*, vol. 855, pp. 1–12, 2018.

[16] D. Rodríguez, "Wavepacket models for supersonic twin-jets," in *AIAA AVIATION 2021 FORUM*, (Reston, Virginia), pp. 1–16, American Institute of Aeronautics and Astronautics, aug 2021.

[17] C. K. W. Tam and F. Q. Hu, "On the three families of instability waves of high-speed jets," *Journal of Fluid Mechanics*, vol. 201, p. 447–483, 1989.

[18] A. Towne, A. V. Cavalieri, P. Jordan, T. Colonius, O. Schmidt, V. Jaunet, and G. A. Brès, "Acoustic resonance in the potential core of subsonic jets," *Journal of Fluid Mechanics*, vol. 825, pp. 1113–1152, 2017.