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THE PRE-HISTORY OF 20TH CENTURY ACOUSTICS: THE LEGACY OF LORD RAYLEIGH

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ABSTRACT

Rayleigh's two-volume work "The Theory of Sound" was the crowning glory of 19th century acoustics, and it set the agenda for everything that followed in the 20th century. The talk will review Rayleigh's life and key contributions, which covered structural vibration as well as airborne acoustics. His interest in wave theory also extended to optics: he famously explained why the sky is blue. The talk will include images from the surviving laboratory at Terling Place, the ancestral home of the Rayleigh family. Apparatus associated with some of Rayleigh's iconic experiments in acoustics can still be seen there. A flight of stone steps in the grounds was an early inspiration for the theory of diffraction gratings, and this was followed up by an ultrasonic experiment involving a periodic array of muslin discs with variable spacing.

Keywords: *Acoustics, Lord Rayleigh, Wave theory*

1. INTRODUCTION

Lord Rayleigh (1842–1919) was a giant of 19th-century science. He was the "godfather" of British acoustics, and he also worked on all the other major areas of physical science of his day. He wrote some 450 scientific papers. He was the first to explain why light from the sky is blue (and also polarised) [1, 2], and how it is that seabirds can soar for long distances without flapping their wings [3]. He shared the Nobel prize in 1904 for the discovery of argon.

But this paper centres around his two-volume work "The Theory of Sound", first published in 1877-78 [4]. Volume 1 deals with structural vibration, Volume 2 with airborne sound. Both volumes give a complete summary of knowledge in his day, much of the material consisting of original contributions by himself. A second edition came out in 1894, and a telling detail is that for this revised edition he added sections on the "new-fangled" electrical circuits which had come to prominence between the two dates. The general theory he had developed for discrete vibrating systems could be applied directly to capacitor/resistor/inductance circuits — an interesting inversion of the tendency in the 1950s and later to analyse vibration problems using electrical circuit analogies.

2. HISTORICAL MATERIAL

When the Cavendish Laboratory was first established in Cambridge, James Clerk Maxwell was the first holder of the Cavendish chair. But Maxwell died tragically young, and the University of Cambridge prevailed on Rayleigh to do a 5-year stint as Cavendish professor of physics, to get the new lab on its feet. He succeeded spectacularly in this task, setting the Cavendish lab on the road to becoming the global force it is today. After that, Rayleigh needed to return to his family estate at Terling Place in Essex, so in order to continue his research activities he converted various spaces in the house into laboratories. These are still present, together with quite a lot of Rayleigh's experimental apparatus — for example the sensitive balance used for the work on the density of air which led to the discovery of argon.

One example concerns work on diffraction gratings. In the grounds of Terling Place is a flight of stone steps, in a setting with no other strong acoustical reflections (see Fig. 1). If you clap your hands in front of these steps, you hear a tonal echo. According to family legend, this may have inspired the young Rayleigh to investigate the reflection of sound and light by periodic structures.

He developed the theory, and one experiment used to

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Figure 1. Steps at Terling Place.

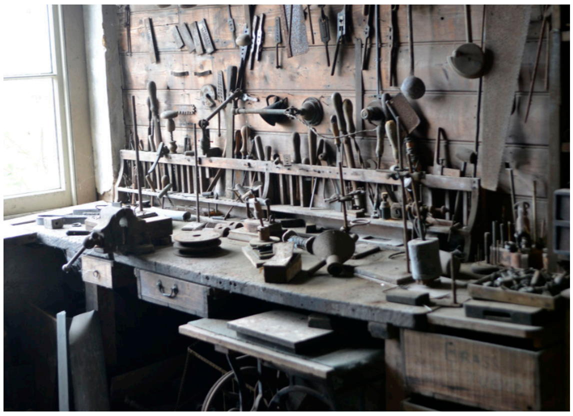

Figure 3. Rayleigh's laboratory workshop.

test the predictions is shown in Fig. 2. A set of muslin discs is fixed to a "lazy-tongs" arrangement so that the regular spacing can be varied. The rig was no doubt built in the workshop shown in Fig. 3. In those days before electronic test equipment, he used as his ultrasonic sound source a "bird call" whistle, and to detect the intensity of the resulting sound field he used a "sensitive flame". With this primitive equipment he was able to verify the predictions of his theory. Two "bird calls" can be seen in Fig. 4, resting on the lid of their storage box still bearing Rayleigh's handwritten list of the frequencies and wavelengths.

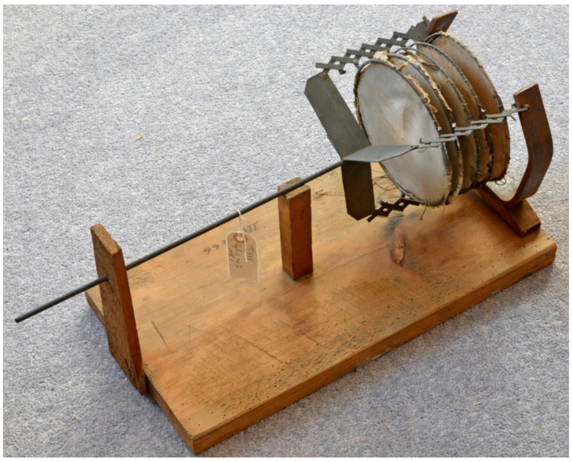

Figure 2. Rayleigh's diffraction experiment.

3. RAYLEIGH'S SCIENTIFIC LEGACY

An important theme in Rayleigh's legacy, still relevant to teaching undergraduates today, is his talent for making telling use of general principles to cut through the detail of a complicated problem, and reveal the main structure in the simplest way.

An example is his pioneering use of dimensional analysis. Consider a bending "Euler beam". The only dimensioned parameters that enter the problem are the mass per unit length m, the bending rigidity EI (Young's modulus times the second moment of area of the beam cross-section), and the length L. Rayleigh pointed out that there is only one way to combine these to make a quantity with the dimensions of frequency, so *any* natural frequency ω_n of *any* such bending beam must be determined by an equation of the form

$$\omega_n = \frac{K}{L^2}\sqrt{\frac{EI}{m}} \tag{1}$$

where K is a non-dimensional constant.

Another famous example is "Rayleigh's principle", bounding the eigenvalues of a discrete vibrating system in terms of potential and kinetic energy expressions. Rayleigh introduces this important principle in the early chapters of Volume 1, and subsequently uses it to draw both quantitative and qualitative conclusions for many problems. But this is the tip of an iceberg. Rayleigh made extensive use of maximum/minimum variational principles for many purposes: to derive governing equations, to resolve a controversy about the correct boundary conditions at a free edge of a thin bending plate, to carry out a wide range of perturbation analyses.

Figure 4. ‘Bird calls” and Rayleigh’s handwriting.

There are many other examples of the “Rayleigh mindset” dotted through The Theory of Sound. The chapters on “Vibrating systems in general” contain many general theorems such as the eigenvalue interlacing theorem (now commonly known as the Sturm sequence property). In the form relevant to vibration, this states that if you change a discrete linear system by adding or removing any single constraint, the natural frequencies before and after the change must interlace one-for-one. Rayleigh made clever use of this result to make simple deductions about the behaviour of complicated systems.

A final example from Volume 1: in the second edition, material was added about the vibration of “curved plates or shells”. Rayleigh made the important observation that if a shell is geometrically capable of *inextensional* deformation (no stretching or shear in the middle surface) then by the energy-minimisation principle, those deformations are likely to give a good approximation to low-frequency vibration modes. He applied this idea to hemispherical shells and the vibration of church bells.

In the field of wave theory, Rayleigh’s work on diffraction gratings has already been mentioned. Volume 2 of The Theory of Sound also contains many examples of a style of analysis that was later to be formalised as the method of “matched asymptotic expansions”, and play a very significant role in 20th century acoustical theory.

He was equally interested in wave problems in optics. He was the first to suggest that the iridescent colours of certain crystals, and also of some insects, such as butterfly wings, were the result of “structural colour” rather than pigment [5]. He suggested that the microstructure of the crystal or the butterfly wing either had layers (like the muslin disc rig) or approximately periodic surface features (like the flight of steps). His hypothesis was correct, and it was the subject of further research by his son, the 4th Lord Rayleigh [6]. Figure 5 shows some iridescent insects collected by the Rayleighs, and still in their study at Terling Place. Structural colour is the reason these are still as bright now as on the day they were collected: pigment would have faded.

Figure 5. Iridescent insects, collected by Rayleigh and his son and still to be found in a desk drawer in their study at Terling.

As well as being a master of acoustical theory, Rayleigh was also closely involved with practical applications. An example, connected with his work on wave theory and diffraction, concerns foghorns (used before the days of GPS to warn ships of dangerous waters during foggy weather). Rayleigh was consulted about a serious problem: ship’s captains had reported that from certain directions they were unable to hear the foghorn, even when they were quite nearby. He correctly diagnosed the problem as resulting from null directions in the sound radiation pattern of large horns. The solution was to build the horns like the example shown in Fig. 6, with an elliptical section. The horizontal dimension needs to be small compared to the wavelength of sound, to ensure omnidirectional radiation with no nulls, but it is an advantage if the vertical dimension is larger, to concentrate the sound in the horizontal plane where it is needed. The authorities did not believe this counter-intuitive design until he built a demonstration model and showed them!

Figure 6. An elliptical foghorn near Whitby, from a design by Rayleigh.

Of course, there are things that Rayleigh did not know about, back in 1877. To modern eyes, the most obvious missing ingredient in The Theory of Sound is the frequency domain viewpoint. The idea of frequency response functions, let alone the ability to measure them, had to await the development of electronic test gear in the 20th century. Similarly, although Rayleigh's treatment of the general linear theory of discrete systems is very sophisticated, it does not use the language of matrices: those only became familiar in the world of physics with Heisenberg's version of quantum mechanics, long after Rayleigh's time. At a more trivial level, the reader is struck by the fact that the partial derivative symbol had not yet been introduced. But these things do not detract from the power and magic of the two books: The Theory of Sound is one of a very select group of books of that age which are still valued as working textbooks, and not just as historical curiosities. That, perhaps, is the real mark of Rayleigh's greatness. If you haven't read these these books, I commend them to you.

4. ACKNOWLEDGMENTS

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5. REFERENCES

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