DOI: 10.61782/fa.2023.0196

forum acusticum 2023

2.5D BOUNDARY ELEMENT METHOD FOR THE DETECTION OF MOVING SOURCES

Holger Waubke[1*] **Prateek Soni**[1] **Christian Kasess**[1] **Wolfgang Kreuzer**[1]
[1] Acoustics Research Institute, Austrian Academy of Sciences, Austria

ABSTRACT

Methods to detect acoustic sources on a moving train using holography and the 2.5D BEM are investigated. For these methods it has to be assumed that the cross-section of the train is constant and that the train is infinitely long. The movement of the train leads to a shift in the frequency wavenumber domain that is compensated by switching from the fixed coordinate system of the microphone array to the moving coordinate system of the train.

Keywords: *source localization, moving source, spectral approach, boundary element method, wavenumber domain.*

1. INTRODUCTION

Acoustic source localization is needed to derive detailed source models used, e.g., in noise mapping software. Especially for railway traffic, knowing the position of possible noise sources is important for determining the necessary height of noise barriers. Also, the localization of sources is important for the design of measures that reduce the noise emission.

The aim of the LION project "Localization and identification of moving noise sources" is to derive a detection model that also includes a moving source. To simplify this setting for this manuscript, the noise source is assumed to be mono-frequent and to move along the x-direction with constant velocity v, that is known from a separate measurement.

This manuscript is split up into two parts. The first part is the extension of the acoustic holography to deal with a moving source. The second part is to introduce the formulation of the boundary element method in 2.5D that can be used to derive a transfer matrix that later can be used to determine the sound source.

2. ACOUSTIC HOLOGRAPHY

2.1 Theory of acoustic holography

The acoustic holography is an intermediate step in this project. The axis along the movement x is transformed using the Fourier Transformation In contrast to the 2.5D BEM, the vertical coordinate z is also transformed using the Fourier Transformation [1] in the holographic approach.

Along the height of the train z, the source is assumed to be represented by a pressure distribution $A(z)$, the source is assumed to be mono-frequent with angular frequency $\Omega : p_s(t) = \cos(\Omega t + \phi_0)$. The phase of the source at time $t = 0$ is ϕ_0 and the x-coordinate of the source at that time is $x(0) = x_0$. The source moves along the x direction with a fixed velocity v, thus, the x coordinate of the source at time t is $x(t) = \delta(x - x_0 - vt)$. At the measurement position, it is assumed that the measured sound pressure is already given in the frequency domain ω. As the sound source is moving, the Doppler effect needs to be considered, and therefore, we distinguish between the source frequency Ω and the receiver frequency ω.

For the holography the pressure distribution is investigated in the wavenumber-frequency domain $k_x = \mathcal{F}(x), k_z = \mathcal{F}(z)$, and $\omega = \mathcal{F}(t)$, where $\mathcal{F}(.)$ denotes the Fourier integral transformation [1].

**Corresponding author*: holger.waubke@oeaw.ac.at.

The Fourier integral transformation of the source pressure with respect to the time t leads to two Dirac delta distributions in the wavenumber frequency domain. The argument of the Dirac delta distribution depends on the velocity v, the wavenumber k_x and the two angular frequencies ω and Ω.

With this notation the sound pressure at the receiver is given in the wavenumber-frequency domain as:

$$\begin{aligned}
&\hat{p}(k_x,k_z,\omega) = \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{A}(k_z)\cos(\Omega t+\phi_0)\cdot \\
&\qquad \cdot\, \delta(x-x_0-vt)e^{-\mathrm{i}\omega t}e^{-\mathrm{i}k_x x}dxdt = \\
&= \int_{-\infty}^{\infty} \tilde{A}(k_z)\frac{e^{\mathrm{i}(\Omega t+\phi_0)}+e^{-\mathrm{i}(\Omega t+\phi_0)}}{2}\cdot \\
&\qquad \cdot\, e^{-\mathrm{i}k_x(x_0+vt)}e^{-\mathrm{i}\omega t}dt = \\
&= \int_{-\infty}^{\infty} \frac{\tilde{A}(k_z)}{2}e^{-\mathrm{i}k_x x_0}e^{\mathrm{i}\phi_0}e^{-\mathrm{i}(\omega-\Omega-k_x v)t}dt + \\
&\quad + \int_{-\infty}^{\infty} \frac{\tilde{A}(k_z)}{2}e^{-\mathrm{i}k_x x_0}e^{-\mathrm{i}\phi_0}e^{\mathrm{i}k_x x_0}e^{-\mathrm{i}(\omega+\Omega-k_x v)t}dt
\end{aligned} \tag{1}$$

and thus,

$$\hat{p}(k_x,k_z,\omega) = \begin{cases} \pi e^{-\mathrm{i}(k_x x_0-\phi_0)}\tilde{A}(k_z) \text{ for } \omega = \Omega + k_x v, \\ \pi e^{-\mathrm{i}(k_x x_0+\phi_0)}\tilde{A}(k_z) \text{ for } \omega = -\Omega + k_x v, \\ 0 \text{ otherwise,} \end{cases} \tag{2}$$

with $\tilde{A}(k_z) = \int_{-\infty}^{\infty} A(z)e^{-\mathrm{i}k_z z}dz$ and $\mathrm{i}^2 = -1$.

This means, that there is a strict link between Ω and ω. For a given frequency ω at the measurement point, the mono-frequent source signal needs to have a frequency of either

$$\Omega_+ = \quad \omega - k_x v \tag{3}$$

or

$$\Omega_- = -\omega + k_x v = -\Omega_+. \tag{4}$$

To full the homogeneous Helmholtz equation in the full wavenumber-frequency space $k_x^2 + k_y^2 + k_z^2 + k^2 = 0$, thus

$$k_y = \sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}. \tag{5}$$

The main step of the acoustic holography is the projection of $\hat{p}(k_x,k_z,\omega)$ from the measurement array to the source plane by using

$$\hat{p}(k_x,k_z,\omega)_{y=y_0} = \hat{p}(k_x,k_z,\omega)_{y=0}e^{-\mathrm{i}k_y y_0}. \tag{6}$$

2.2 Numerical simulation

In real life situations, one only has access to sampled data with small support. Therefore, the Discrete/Fast Fourier transformation (DFT, FFT) needs to used instead of a Fourier integral transformation introducing unwanted artefacts like periodic fields. One of the biggest problems, is the fact that the space where the microphones are placed is very small in its extension. If, for example, the microphone grid is given by a rectangular 8×8 grid in the x and z directions with a distance of 0.1 m between microphones, the resolution is very limited and a DFT using 8 points in each direction is far from the optimal case given by the Fourier integral transform. To increase the spectral solution one option is to use zero padding in both directions to reduce wrap around effects, but the assumption of having no acoustic field in the measurement plane except at the area of the microphone positions is very unrealistic. An additional problem is the distance between measurement plane and source plane, which for realistic situations with respect to noise measurements for trains can become very large (in the example used it was set to 10.4 m). In this case evanescent waves with a imaginary wavenumber k_y (see Eq. (5)) are dropped to avoid amplification of signal noise.

In Fig. 1 the magnitude of the calculated source distribution is shown for the above scenario based on a simulated pass-by. In the time domain the signals are windowed with a Hanning window. The window length was chosen so that, with a given velocity (v = 100 km/h), the source can travel from one end of the spatial window (8 m) to the other end. At $t = 0$ the source is placed at $z = 0$ starting at $x = -4$ m. As one can easily see, no sources can be immediately detected with this simple approach. One reason for this behavior lies to the far distance and the deletion of evanescent waves (near field components). Thus, we suggest to used the 2.5D boundary element method as an alternative.

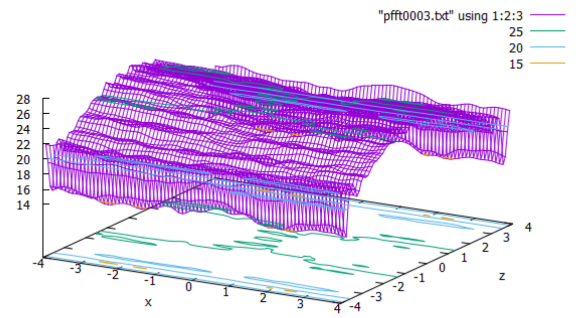

Figure 1. Projection of the pressure to the measurement plane with moving coordinate x and vertical coordinate z filtered from 500 to 2000 Hz.

3. INVERSE 2D BOUNDARY ELEMENT METHOD

3.1 2.5D BEM with a moving source

Under the assumption of a constant cross section and an infinitely long object, the Fourier integral transform $x \rightarrow k_x$ is used to reduce the 3D boundary integral equation with (acoustic) wavenumber k_{3D} into a series of 2D boundary problems with different acoustic (wavenumbers) $k_{2D} = \pm\sqrt{k_{3D}^2 - k_x^2}$ [2]. Compared to standard 2D approaches, the 2.5D BEM also allows to use point sources in 3D instead of coherent line sources.

As in Sec. 2, a mono-frequent source with angular frequency Ω that moves along the x-axis with a velocity v is assumed. Again, this source can be implemented in the model by using $\delta(x - x_0 - vt)$ in the defintion of this source which leads to (acoustic) wavenumbers

$$k_{3D} = \frac{\omega}{c} = \frac{\Omega - k_x v}{c}$$
$$k_{2D} = \sqrt{k_{3D}^2 - k_x^2} = \sqrt{\frac{\Omega - k_x v}{c} - k_x^2}, \qquad (7)$$

where c is the speed of sound in the medium.

One advantage of the BEM approach is, that compared to holography and beamforming the structure of the scatterer can be included in the model as long as the scatterer has constant cross section. By assuming different velocity boundary conditions along the cross section of the scatterer, the BEM is used to derive a transfer function between source at the scatterer and sound pressure at the microphones. Note, that the sources do not necessarily need to be restricted to one specific BEM element, thus, several source distribution functions can be easily included in this approach. The advantage of the 2.5D approach lies in the fact, that solving 2D problems (even if there are many of them) puts less strain on computer power then solving the full 3D approach, and that parallelization is straight forward.

3.2 Theorem of the inverse 2D BEM

To determine the position of the source, the transfer function derived with the 2.5D method needs to be inverted in either the x or the k_x domain which, in general, is only possible if an appropriate regularization is used. Schumacher [3,4] uses a Tikhonov-regularization together with the L-curve method for the determination of the regularization parameter for the inverse BEM in 3D. However, for the 2.5D BEM, the determination of the regularization parameter is time consuming, because the L-curve is different for every frequency ω_{3D} and every wavenumber k_x. As an alternative, one can assume that only a few dominant sources are present, thus, methods used in compressive sensing like the Orthogonal Matching Pursuit (OMP, [5, 6]) algorithm can be used. Preliminary results using the 2.5D inverse BEM approach in the x-domain show the potential of this method. Using the example above, Fig. 2 shows the result of th inversion using a Tikhonov-regularization and a single data window. Different parameters such as the choice of frequencies used, the regularization parameter, and the windows length can affect the results quite strongly. The first component of the OMP (black circle) finds the exact position as in this particular case the source was place exactly on a source grid point.

4. OUTLOOK

The detection of moving acoustic sources on trains in the frequency wavenumber domain using acoustic holography gave disappointing results for the obvious reason that the measurement plane has to be placed too far from the source plane. Therefore, the 2.5D BEM approach was introduced. This approach has the advantage that also the structure of the scatterer, reflection of sound from the ground, and different materials on the scatterer can be taken into account. Preliminary results are promising but there are still open questions that have to be answered in

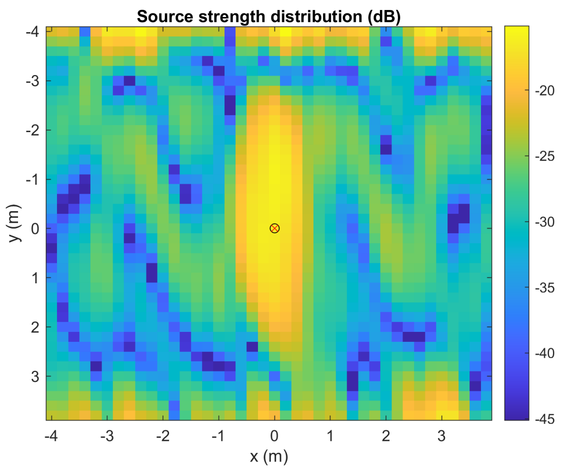

Figure 2. Result of the inverse 2.5D-BEM for a mono-frequent point source with a frequency of 1000 Hz moving at 100 km/h and derived from an 8x8 microphone array at 10.4 m. The red cross shows the true position and the circle the result of the OMP.

the future, e.g., whether the inversion of the transfer matrix benefits from going to the k_x domain, the effect of the window on the transfer function, and how parameters such as window length, number of sources, and choice of frequencies affect the inversion, in particular concerning the difference in x and z resolution. Furthermore, incorporating broad band noise is also a topic for future reseach.

5. ACKNOWLEDGEMENTS

This work is supported by Austrian Science Fund (FWF), project I 4299 N32 ”Localization and identification of moving noise sources” (LION).

6. REFERENCES

[1] N. Tse, L. Schlussler, J. Fontana, and G. Wade, “Image reconstruction of acoustic holograms for moving objects,” in *Proc. SPIE 0118, Optical Signal and Image Processing*, 1977.

[2] D. Duhamel, “Efficient calculation of the three-dimensional sound pressure field around a noise barrier,” *J. Sound Vibr.*, vol. 197, pp. 547–571, 1996.

[3] A. Schuhmacher, *Sound source reconstruction using inverse sound field calculations*. PhD thesis, Technical University of Denmark, 2000.

[4] A. Schumacher, J. Hald, K. Rasmussen, and P. Hansen, “Sound source reconstraction using inverse boundary element calculations,” *J. Acoust. Soc. Am.*, vol. 113, pp. 114–127, 2003.

[5] S. Sahoo and A. Makur, “Signal recovery from random measurements via extended orthogonal matching pursuit,” *Transactions on Signal Processing, IEEE*, vol. 63, pp. 2572–2581, 2015.

[6] Y. C. Pati, R. Rezaiifar, and P. S. Krishnaprasad, “Orthogonal matching pursuit: recursive function approximation with applications to wavelet decomposition,” in *Conference Record of the Asilomar Conference on Signals, Systems & Computers*, vol. 63, pp. 40–44, 1993.