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PARTICLE MANIPULATION WITH ACOUSTIC WAVES BASED ON HERTZ-MINDLIN MECHANICS

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ABSTRACT

A particle on a substrate can be moved by the dry friction force excited via a surface acoustic wave. The effect is referred to as vibrational transportation or Rayleigh linear motor and is used in a number of industrial applications. A traditional theory of vibrational transportation considers a particle as a material point that cannot be strained and also cannot induce any strain in the substrate. A known result consists in the fact that such a particle, with a certain choice of system parameters, can move against the surface wave.
Here we use another approach based on the Hertz-Mindlin mechanics in which a deformable axisymmetric body moves on a deformable substrate. The contact zone in this case is not a point but rather a circle that generally contains a smaller circle of stick and surrounding annulus of slip. Using a previously developed numerical tool we show that such a body can also move in the direction of the wave which allows one more elaborate particle positioning. The steady drift in one of these directions can be observed (or not) depending on a combination of two system characteristics only that incorporate excitation parameters, elastic properties and particle inertia.

Keywords: *vibrational transportation, particle manipulation, Rayleigh wave, Hertz-Mindlin mechanics, Method of Memory Diagrams.*

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1. INTRODUCTION

It is well known that nonlinear contact frictional interaction can be used for transporting small solid objects on a substrate under the action of a surface acoustic wave. This principle is now applied to a number of practical cases: separation of granular mixtures, transport of grains in silos and conveyors, powder dosage in pipes and, in general, is of interest for food, chemical, pharmaceutical and coating industries [1-3]. Future applications are possible such as delivery of solid agents for defect healing, noninvasive excretion of undesirable particles, activation of micromachine components, and others.
A particle can move when there exists some asymmetry between left and right (Fig. 1). The mechanism is based on the fact that a surface wave simultaneously excites oscillatory normal and tangential particle motions with some phase shift. Depending on this shift, the average normal compression during right and left excursions of the tangential motion is different. Then the presence of dry friction can result in a preferred movement direction characterized by a lower compression.
The effect has been experimentally observed many times (e.g. in [1-4]). At the same time, theoretical description of the process is of interest since it can help optimize real systems once predicts the resulting behavior in a particular situation determined by a number of parameters such as wave characteristics, material properties, and geometric sizes and shapes.
A traditional model of the effect [5,6] is based on representation of the particle as a material point which is rigid by definition and an assumption that particle-substrate contact does not induce any deformation in the substrate. It was shown previously [6] that in such a system the particle can detach and experience multiple bouncing, be stuck, or move with some constant average velocity against the

surface wave. The latter regime is suitable for the purpose of vibrational transportation.
There is another opportunity to describe the system in a more precise way offered by the recently developed contact models [7-10] based on the Hertz-Mindlin mechanics [11]. The key feature of the approach is a finite size of the body as well as the presence of finite contact displacements in the system, i.e. deformability of both body and substrate. In this communication, we show that the deformability effect impacts dramatically the observed motion regimes. In particular, we demonstrate that a material point can controllably move only against the wave while a deformable body is able to slide in the both directions.
It is also important to mention that the both models are based on the Coulomb friction law written either for concentrated forces (material point), either for stress distributions in the contact zone (deformable body of finite size).

2. EQUATIONS FOR A PARTICLE ON THE SUBSTRATE EXCITED BY RAYLEIGH WAVE

Here we consider a motion of an axisymmetric particle positioned on a substrate in which a Rayleigh wave

$$\begin{aligned} u_x(t) &= -A_y r(\nu)\sin(2\pi f t - k_r x) \\ u_y(t) &= A_y\cos(2\pi f t - k_r x) \end{aligned} \tag{1}$$

is excited. Here A_y is the vertical amplitude, f is the wave frequency, k_r is wavenumber, $r(\nu)$ is the known ratio of the horizontal and vertical amplitudes that depend on ν, Poisson's ratio. The equations of motion for bulk coordinates (x, y) of the small body of a mass m read

$$\begin{aligned} m\ddot{x} &= -T(b,a) \\ m\ddot{y} &= Mg - N(a) \end{aligned} \tag{2}$$

where $T(b, a)$ and $N(a)$ are friction and normal reaction forces, respectively, that depend on normal a and tangential b contact displacements (see Fig. 1). Here by introducing $M>m$ we take into account that the particle can be precompressed by an external force F_{ext}, in addition to the gravity force mg, i.e. $M=m+F_{ext}/g$.
Eqn. (2) should be supplemented by an obvious geometric relationship in the form

$$\begin{aligned} x &= b + u_x(t) \\ y &= a + u_y(t) \end{aligned}. \tag{3}$$

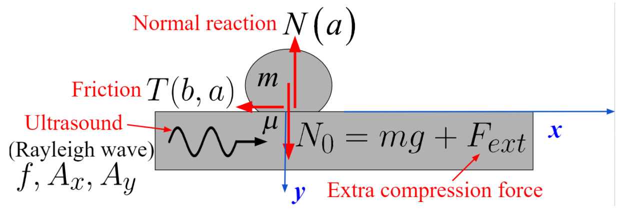

Figure 1. Axisymmetric particle on a substrate under the action of Rayleigh wave.

Suppose now that the displacement a_0 produced by the total compression force $N_0=mg+F_{ext}$ is known. Then it is convenient to express the friction force through its normalized value $T^*_{MMD}=T/(\mu N_0)$ where μ is the coefficient of friction. The subscript MMD is related to the Method of Memory Diagrams (MMD) explained in more detail in the next section. The use of the MMD also implies that the normalized tangential displacement equals $b^*=b/(\theta\mu a_0)$ with $\theta=(2-\nu)/(2(1-\nu))$. Finally, introducing other normalized variables marked with asterisks as

$$\begin{aligned} & m^* = (a_0 m f^2)/(Mg), \quad t^* = ft \\ & (a^*, x^*, y^* A_y^*) = (a, x, y, A_y)/a_0 \end{aligned} \tag{4}$$

we rewrite the equations of motion Eqn. (2) in the form

$$\begin{aligned} m^*\ddot{x}^* &= -\mu T^*_{MMD}(b^*, a^*) \\ m^*\ddot{y}^* &= 1 - N^*(a^*) \\ \theta\mu b^* &= x^* + A_y^* r(\nu)\sin 2\pi t^* \\ a^* &= y^* - A_y^*\cos 2\pi t^* \end{aligned} \tag{5}$$

A remarkable feature of these normalized equations is that they contain only two essential parameters, m^* and A_y^*. Other two parameters, μ and ν, are considered here as material constants. It is worth noting that terms containing k_r in Eqn. (1) are omitted since the Rayleigh wavelength is much longer than all dimensions related to contact.
The key point of our approach is the numerical solution to Eqn. (5) using the MMD sketched below.

3. ON THE METHOD OF MEMORY DIAGRAMS

The MMD belongs to a family of semi-analytical methods [7-10] of contact mechanics based on the Hertz-Mindlin (or Cattaneo-Mindlin) solution [11]. The Hertz-Mindlin mechanics describes contact stresses and displacements in a system consisting of two elastic spheres with friction. The spheres are subject to an oblique bulk force (N, T) which results in appearance of contact displacements (a, b). It is shown that for $|T| < \mu N$ the contact zone represents a circle of stick [0, s] surrounded by an annulus of slip [s, c]. In the considered situation of partial slip, contact forces and displacements are linked with relationships:

$$T = T_c^s \equiv \frac{2\mu E}{3R\left(1-\nu^2\right)}\left(c^3 - s^3\right),\ b = b_c^s \equiv \frac{\mu\theta}{R}\left(c^2 - s^2\right) \quad (6)$$

Here R is the radius of the sphere (the second sphere has an infinite radius), and E is the Young modulus of the materials considered identical for simplicity. The normal reaction curve N=$N(a)$ is given by the Hertz solution:

$$N = \frac{2\mu E}{3R\left(1-\nu^2\right)} c^3,\ a = \frac{1}{R} c^2 \quad (7)$$

The MMD represents a generalized and automated version of the Hertz-Mindlin mechanics valid for arbitrary history of displacements $a(t)$ and $b(t)$, for bodies of any axisymmetric shapes, and for contact modes comprising not only partial slip but also full sliding and contact loss. The solution is given as a superposition of expressions $T_c^{s_i}$ and $b_c^{s_i}$ of the kind of Eqn. (6) with various parameters s_i that depend on history. Following the evolution of $a(t)$ and $b(t)$, the "memory points" s_i are continuously created and erased. Hence, the solution remains analytical i.e. extremely rapid but its numerous parameters s_i are obtained via an algorithm.
The details can be found in [10,12]. The essential conclusion for this study is that the notation T_{MMD} (b, a) is defined for an arbitrary displacement history and can be calculated provided the dependence N=$N(a)$ is known.

4. TRADITIONAL SOLUTION FOR A MATERIAL POINT

In [6] the corresponding solution for a material point positioned on a substrate is obtained analytically. Here it is meaningful to reproduce it by modifying the numerical solution of Eqn. (5) that would provide an additional verification.
First of all, the assumption of undeformability immediately suggests that a^*=0. Then $y^* = A_y^* \cos 2\pi t^*$ provided no detachment occurs. The solution for b^* is also easy to obtain. In the case of stick, the tangential displacement does not change, and $T^* = -m^* \ddot{u}_x^*(t)/\mu$ as it follows from Eqn. (5). If now the value $|T^*|$ exceeds μN, slip occurs. Then the friction force is given by Coulomb friction law, and a small increment in b^* at a current step represents a result of the uniformly accelerated motion.

5. DEFORMABLE PARTICLE UNDER RAYLEIGH WAVE ACTION: RESULTS

It is convenient to solve the set of ordinary differential equations Eqn. (5) for a deformable particle using the Adams-Bashforth method with the initial conditions b^*=0 and a^*=1. To comply with infinite in time Rayleigh wave solution, the latter was multiplied by a ramp function staring from 0 and gradually reaching 1.
The result is depicted in Fig. 2 for a spherical particle of a certain radius that fully defines the normal loading curve. Depending on m^* and A_y^*, the two governing parameters of the system, four different motion regimes can be encountered. In the case of weak oscillations around x^*=0 (blue curve), the difference in compression during positive and negative tangential excursion is not enough to launch a steady drift. The green line corresponds to detachment of the particle and a series of bouncing. In that case, a slight change in parameters modifies the behavior considerably. In addition, the motion can depend, for instance, on local surface features. Therefore, this regime is hardly usable for practical vibrational transportation. The most remarkable feature; however, is the existence of two drift regimes, in the direction of the wave and against it. This indeed offers an opportunity to manipulate a particle and move it in the desired direction.

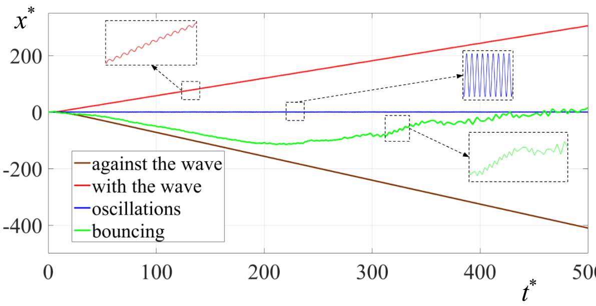

Figure 2. Four principal motion regimes for a deformable spherical particle. The insets zoom the behavior at the scale of the Rayleigh wave period.

Fig. 3 represents phase diagrams i.e. sets of points (A_y^*, m^*) at which various motion regimes take place. It can be seen that the account for particle's deformability drastically impacts the result. In particular, a "heavy" deformable particle ($m^* \gg 1$) does not necessarily detach as the material point does, and the drift against the wave is more common.

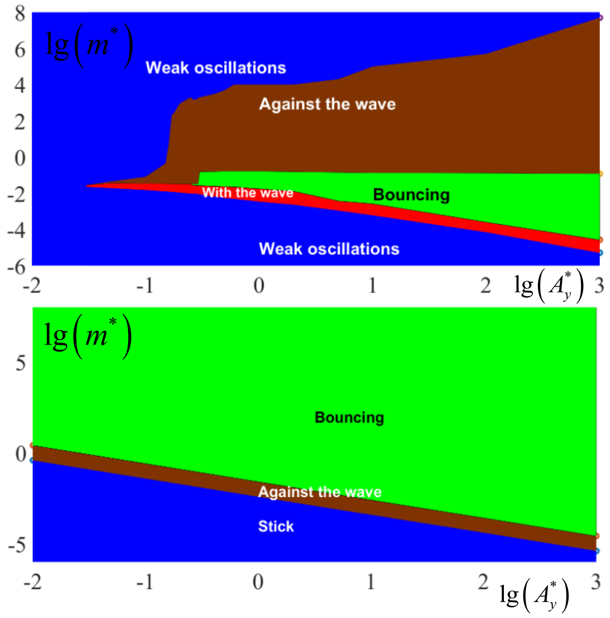

Figure 3. Phase diagram for different motion regimes of a spherical deformable particle (top) and of a material point (bottom) for μ=0.1, ν= 0.3. The material point never moves with the wave.

6. DISCUSSION

In this communication, we apply a previously developed method of contact mechanics to modeling motion of a particle on the substrate under a combined action of friction and the Rayleigh wave. It is shown that the account for particle's deformability and its finite size drastically changes the diagram of encountered regimes. The deformable particle can move with the wave and against the wave while a material point is capable of moving against the wave only.

At the same time, an important limitation arises from the fact that a particle of finite size can roll which is not considered here, whereas a material point cannot do so.

Revisiting vibrational transportation problems can be of use for modern applications in micromachines as well as state-of-the-art particle manipulation.

7. ACKNOWLEDGMENTS

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