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LABYRINTHINE METAMATERIALS AND SORPTIVE POROUS MEDIA APPLIED TO VENTED-BOX LOUDSPEAKERS' DESIGN

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ABSTRACT

Acoustic metamaterials have shown great potential for a number of acoustic applications due to their extraordinary, tunable acoustic properties such as deep sub-wavelength sound absorption, bandgaps, and focusing to name a few. Likewise, multiscale sorptive porous media have been shown to exhibit remarkable acoustic properties, including unusually high low frequency sound absorption and enabling the enhancement of the compliance of resonator's cavities or loudspeaker's boxes. This work applies labyrinthine metamaterials and multiscale sorptive porous materials to the design of vented-box loudspeakers. An electro-mechano-acoustical model of a vented-box loudspeaker is introduced. This model takes into account the effects of the said meta- and porous material on the vented-box loudspeaker's sensitivity and electrical impedance. It is shown that by combining both types of materials, the form factor of vented-box loudspeakers can be significantly reduced, without compromising their low frequency performance. Thus, this work contributes towards the sought-after 'holy grail' of loudspeaker's design; that is to achieve 'big bass from small boxes'.

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1. INTRODUCTION

Acoustic metamaterials are artificial materials with acoustic properties that are not usually found in naturally occurring materials. Among the unusual properties of these materials one can mention deep sub-wavelength sound absorption, bandgaps, and focusing to name a few (see [1, 2] for detailed reviews). In particular, from a large number of metamaterials, structures comprising a complex pattern of channels and/or chambers that create a complex path for sound waves to propagate through have received significant attention (see, e.g., [3, 4]). A directly relevant application of these labyrinthine structures to loudspeaker systems was proposed in [5], where it was shown that the use of labyrinthine-type metamaterials allows controlling the reflection of sound from the back cavity of a driver so as to not interfere with the sound radiated by the driver forward. In this work, labyrinthine metamaterials are considered for a different purpose, as it will become apparent below.

In multiscale porous materials, e.g. double [6, 7] or triple [8, 9] porosity materials, characteristic lengths ranging from nm up to mm or larger can be identified. Provided that the characteristic lengths of the different scales of porosity are highly contrasted, local dynamics affected by nanoscale phenomena, such as adsorption/desorption, emerges. These phenomena have been shown to determine the remarkable acoustic properties of sorptive media. One of these properties is the capacity of sorptive media to increase the effective compliance of a loudspeaker/resonator's cabinet/cavity [10–12]. Whilst this is known in loudspeaker applications [10, 12], the synergistic combination of acoustic metamaterials and sorptive porous media to reduce the form-factor of loudspeaker systems remains to be fully explored.

This work applies labyrinthine metamaterials and multiscale sorptive porous materials to the design of vented-box loudspeakers systems [14, 15]. An electro-mechano-acoustical model of a vented-box loudspeaker is introduced. This model takes into account the effects of the said meta- and porous material on the vented-box loudspeaker's sensitivity and electrical impedance. It is shown that by combining both types of materials, the form factor of vented-box loudspeakers can be significantly reduced, without compromising their low frequency performance. Thus, this work contributes towards the sought-after 'holy grail' of loudspeaker's design; that is to achieve 'big bass from small boxes' [10].

2. THEORY

2.1 Acoustic properties of permeable media

It is well established that a rigid-frame permeable material can be modelled as an equivalent fluid [13]. The effective parameters that characterise long-wavelength sound propagation in the equivalent fluid are the effective complex-valued frequency-dependant density $\rho(\omega)$ and compressibility $\mathsf{C}(\omega)$, where ω is the angular frequency. Expressions of these effective parameters for materials with specific microstructure are available, for example, in [7, 13]. The leading-order asymptotic low- and high-frequency behaviour of the said effective parameters for conventional single porosity materials are given by

$$\rho(\omega \ll \omega_v) = \frac{\sigma_0}{j\omega}, \quad \rho(\omega \gg \omega_v) = \frac{\rho_0 \alpha_\infty}{\phi}, \tag{1}$$

$$\mathsf{C}(\omega \ll \omega_t) = \mathsf{C}_0 = \frac{\phi}{P_0}, \quad \mathsf{C}(\omega \gg \omega_t) = \frac{\phi}{\gamma P_0}, \tag{2}$$

where P_0 is the equilibrium pressure, ρ_0 is the density of air, γ is the adiabatic exponent (equal to 1.4 for air), The permeable material's parameters are the porosity ϕ, static flow resistivity σ_0, tortuosity α_∞, viscous characteristic frequency ω_v, and thermal characteristic frequency ω_t. Physically, σ_0 measures the resistance of the material to fluid flow, ϕ is the fraction of the saturating gas in the material, α_∞ is a measure of how intricate the fluid-saturated path a sound wave follows in the material is, ω_v determines the transition from viscosity- to inertia-dominated oscillatory fluid flow in the material, and ω_t characterises the transition from isothermal to adiabatic sound propagation in the material. Some of these parameters will be used to model the acoustic behaviour of the port, made of a labyrinthine metamaterial, of a vented-box loudspeaker. Some of these parameters will be used to model the acoustic behaviour of the port, made of a labyrinthine metamaterial, of a vented-box loudspeaker. It is stressed that this labyrinthine metamaterial-type port design can also be thought as a folded tube design.

The effective parameters of multiscale sorptive material with highly contrasted permeabilities (see e.g. [6–9] for more details, including general expressions as well as particular ones for materials with specific multiscale microstructure) exhibit mathematically the same asymptotic behaviour shown in Eqn. (1) and Eqn. (2), with the difference being that ϕ in Eqn. (2) must be interpreted as an apparent porosity [9,11], i.e. $\phi \to \Phi$. It is stressed that Φ can take values larger than one because of sorption phenomena occurring in the nanopores of the sorptive material [9]. Hence, a larger effective compressibility can be observed which can result in a larger compliance of a loudspeaker's box when it is partially filled with a sorptive material, as it will be shown in what follows.

2.2 Vented-box loudspeaker system

Fig. 1 shows a diagram of a vented-box loudspeaker system. Its port is made of a labyrinthine metamaterial while its box is partially filled with a sorptive material.

The formulation of the electro-mechano-acoustical model introduced in this work closely follows that for a classical vented-box loudspeaker presented in [16]. Here, only the final results are presented, together with highlighting the differences between the classical case and the present one.

The pressure radiated by the system at a distance r is given by

$$\tilde{p}(r) = \frac{\tilde{e}_g B\ell S_D \rho_0}{(R_g + R_E) M_{MS}} \frac{e^{-jk_0 r}}{4\pi r} G(s), \tag{3}$$

where k_0 is the wave number in air, $B\ell$ is driver's force factor, $S_D = \pi a^2$ is the effective area of the diaphragm and a its effective radius, R_E is the electrical resistance

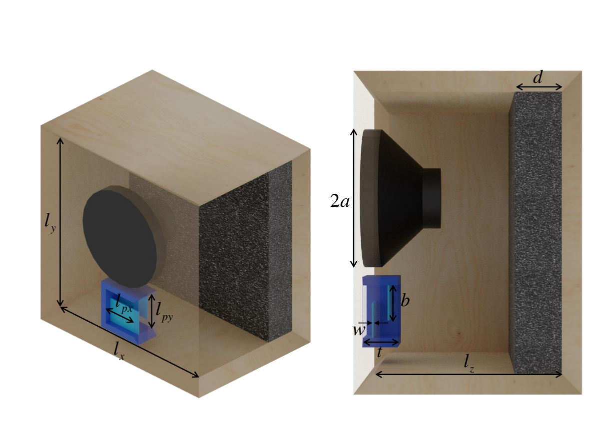

Figure 1. Diagram of the geometry and dimensions of a vented box loudspeaker with a labyrinthine metamaterial-type port and a multiscale sorptive material infill (in blue).

of the driver, M_{MS} is the combined diaphragm and air-load mass, $\tilde{e}_g$ is the open-circuit voltage supplied by the amplifier which has an output electrical resistance R_g.

The response function $G(s)$ reads as (with $s = j\omega$)

$$G(s) = \frac{s^4}{s^4 + G_3 s^3 + G_2 s^2 + G_1 s + G_0}, \tag{4}$$

where the coefficients G_i (with $i = 0..3$) are given by

$$G_3 = \frac{\omega_s}{Q_{TS}} + \frac{\omega_B}{Q_L}, \tag{5}$$

$$G_2 = \left(1 + \frac{C_{AS}}{C_{AB}}\right)\omega_S^2 + \omega_B^2 + \frac{\omega_S \omega_B}{Q_{TS} Q_L}, \tag{6}$$

$$G_1 = \frac{\omega_S \omega_B^2}{Q_{TS}} + \frac{\omega_S^2 \omega_B}{Q_L}, \tag{7}$$

$$G_0 = \omega_S^2 \omega_B^2. \tag{8}$$

In these equations, ω_S, Q_{TS}, and C_{AS} are the resonance frequency, total Q, and equivalent acoustic compliance of the driver, respectively. The resonance frequency of the port-box is denoted as ω_B, while Q_L is a $Q-$factor that encapsulates the losses in the system due to leakage and/or in the port as well as in the sorptive material. The former is given by

$$\omega_B = \sqrt{\frac{1}{M_{AT} C_{AB}}}. \tag{9}$$

Here, M_{AT} is the acoustic mass of the port and C_{AB} is the acoustic compliance of the box partially filled with a sorptive material. In this work, the labyrinthine metamaterial port is modelled as an equivalent fluid. For the range of frequency of interest, and taking into account usual local dimensions of the port, its dynamic density can be approximated by its asymptotic limiting value shown in Eqn. (1) for $\omega \gg \omega_v$. Hence, the acoustic mass of the port is given by

$$M_{AT} = \frac{\alpha_\infty}{\phi} \rho_0 t \frac{(1 + \delta/t)}{S_p}, \tag{10}$$

where S_p is the cross-section of the port, δ is an end correction that is dependent on the geometry of port, and ϕ is the fraction of air in the port. Eqn. (10) reduces to the classical expression for M_{AT} for a constant cross-section port since $\phi = 1$ and $\alpha_\infty = 1$ with, for example, δ becoming $0.84\sqrt{S_p}$ for a straight tubular port. Furthermore, Eqn. (10) shows that a labyrinthine metamaterial port exhibits a larger M_{AT} than that of a port with same length but of constant cross section. This is because in a labyrinthine metamaterial port one has that $\phi < 1$ and $\alpha_\infty > 1$. Thus, it follows from Eqn. (9) that the resonance frequency of the port-box can be lowered.

The effective acoustic compliance of the partially filled box is given by

$$C_{AB} = V_B[(1 - q)\mathsf{C}_a + q\mathsf{C}_0], \tag{11}$$

where V_B is the volume of the box minus the volume of the port, q is the filling fraction, $\mathsf{C}_a = 1/\gamma P_0$ is the adiabatic compressibility of air, and $\mathsf{C}_0 = \Phi/P_0$ is the low-frequency approximation of the compressibility of the sorptive infill material. As demonstrated in [9], the apparent porosity Φ can take values larger than one due to sorption effects in the nanopores of the sorptive material, leading to an increased effective compressibility of fluid equivalent to the sorptive infill. The physical origin of this phenomenon is as follows. Sorption is a term that encompasses a physical or chemical process in which the fluid molecules are adhered onto (adsorption) and release from (desorption) a surface. Adsorption, which is strong in nanopores, leads to a local increase of the fluid density in the vicinity of the fluid-solid interface. This, together with isothermal effects in the larger pores of the sorptive material, results in a larger effective compressibility of the fluid equivalent to the sorptive material when compared with that of conventional permeable material in which sorption is negligible. This atypical behaviour of the effective compressibility, described in detail in [9], opens the possibility of achieving values of C_{AB} that are larger than those achieved by either partially filling a box with a conventional porous material or by no placing a porous material in the box. Hence, it follows from this analysis and Eqn. (9) that a further reduction in ω_b is possible. It also is highlighted that Eqn. (11) shows that the C_{AB} of i) an empty box is retrieved when $q = 0$ and ii) a box partially filled with a conventional porous material is retrieved when considering that Φ is the porosity of the infill material (see Eqn. (2) and its discussion in § 2.1). On the other hand, it is emphasised that the approximation of the effective compressibility of the sorptive infill in Eqn. (11) holds for frequencies lower than any of its characteristic frequencies that determine inner diffusion processes, as discussed in detail in [9].

Finally, the electrical impedance Z_E of the vented-box loudspeaker is obtained through

$$\frac{Z_E}{R_E} = 1 + \frac{s\omega_S}{Q_{ES}} \frac{s^2 + (\omega_B/Q_L)s + \omega_B^2}{s^4 + E_3 s^3 + E_2 s^2 + E_1 s + E_0}, \quad (12)$$

where Q_{ES} is the electrical Q factor of the driver and the parameters E_i (with $i = 0..3$) take the same mathematical form as G_i but with Q_{TS} being replaced by the mechanical Q factor of the driver Q_{MS}.

3. RESULTS

The sensitivity of a vented-box loudspeaker system is predicted with Eqn. (3) while its electrical impedance with Eqn. (12). Three systems are considered, namely (a), (b), and (c). Their geometries and their descriptions are presented as inset diagrams and in the caption of Fig. 2, respectively.

The driver is identical in all the systems and has the following parameters: $R_E = 6$ Ohms, $\omega_S = 2\pi \times f_s$ with $f_s = 64.1$ Hz. $C_{AS} = V_{AS}/\gamma P_0$ with $V_{AS} = 7.5$ liters, $B\ell = 4.39$ Tm, $Q_{ES} = 0.37$, $Q_{MS} = 4.43$, and $Q_{TS} = 0.34$. The dimensions of the box are $l_x = 15$ cm, $l_y = 15$ cm, and $l_z = L_z$ with $L_z = 20$ cm (systems (a) and (c) in Fig. 2) or $L_z = 10$ cm (system (c) in Fig. 2). The box is either partially filled with a sorptive material with apparent porosity [9]: $\Phi = 2.2563$ [systems (a) and (b) in Fig. 2], or a conventional porous material with porosity 0.9 [system (c) in Fig. 2]. For the three systems $q = 0.25$. In system (c), the square port has a length of $t = 12$ cm and lateral dimensions $l_{px} = l_{py} = 3$ cm (i.e. $S_p = l_{px}l_{py}$), while in the systems (a) and (b) the port is made of a labyrinthine metamaterial (see also Fig. 1). The latter has a length that is six times smaller than that of the classical system (c) (i.e., $t = 2$ cm). The other dimensions of the port are $b = 2l_{px}/3$ and $w = 3$ mm. These parameters lead to an internal porosity of 0.93 and a tortuosity of $\alpha_\infty = 4.64$. For simplicity, Q_L has been set to the usual value of 7, while $R_g = 0$, $r = 1$ m, and $\tilde{e}_g = \sqrt{8}$ V.

Fig. 2 shows the sensitivity of the three vented loudspeaker systems as a function of frequency. It is observed that the systems (b) and (c) have comparable low-frequency response, despite the fact that the box volume in system (b) is half that in system (c). This is explained by both the increase in compliance achieved by inserting the sorptive material in the box and the larger acoustic mass of the labyrinthine metamaterial port as a consequence of its tortuous geometry. These trends in sensitivity are consistent with the fact that the port-box resonance frequency f_B is reduced by increasing both the acoustic compliance of the box and the acoustic mass of the port, as can be seen in Fig. 3 where the magnitude of the electrical impedance $|Z_E|$ of the two systems is shown. In particular, f_B is the frequency at which $|Z_E|$ is minimum in between its two characteristic peaks. Having exemplified the potential of reducing the form factor of a vented loudspeaker systems by using metamaterial-based ports and sorptive infills, the discussion is now focused on system (a), which has the same box volume than that of system (c) but a shorter port as well as a sorptive instead of a conventional porous material infill. It is clear that the system (a) has an extended low frequency response.

As a final remark, it must be mentioned that in the low-to-high frequency transition, the systems (a) and (b) exhibit smaller sensitivity than the system (c). This is an aspect that requires further research and improvement. Moreover, this work is restricted to linear phenomena. This means that non-linear effects, such as turbulence in the port, cannot be assessed with the developed theory.

4. CONCLUSIONS

This work applied labyrinthine metamaterials and multi-scale sorptive porous materials to the design of vented-box loudspeakers systems. A simple electro-mechano-acoustical model of a vented-box loudspeaker was introduced. This model takes into account the effects of the said meta- and porous material on the vented-box loudspeaker's sensitivity and electrical impedance. It was shown that the combination of both types of materials in the design of vented-box loudspeakers can result in an extension of their low frequency end or a significant reduc-

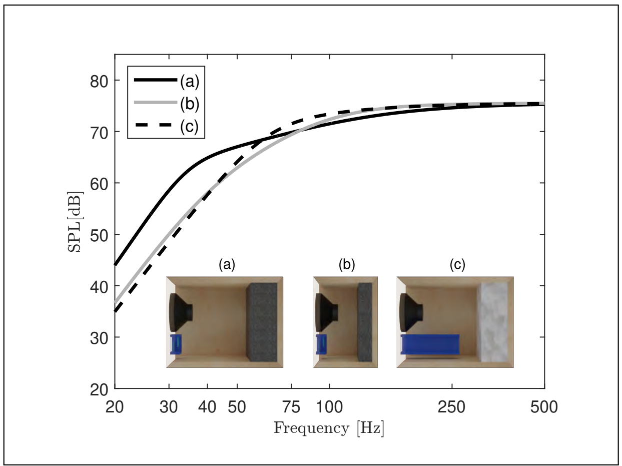

Figure 2. Sensitivity of different vented-box loudspeaker systems as a function of frequency. (a) Vented box system with a labyrinthine metamaterial-type port and a multiscale sorptive material infill. (b) Reduced form factor vented box system with a labyrinthine metamaterial-type port and a sorptive porous material infill. (c) Vented box system with a large constant cross-section port and a conventional porous material infill.

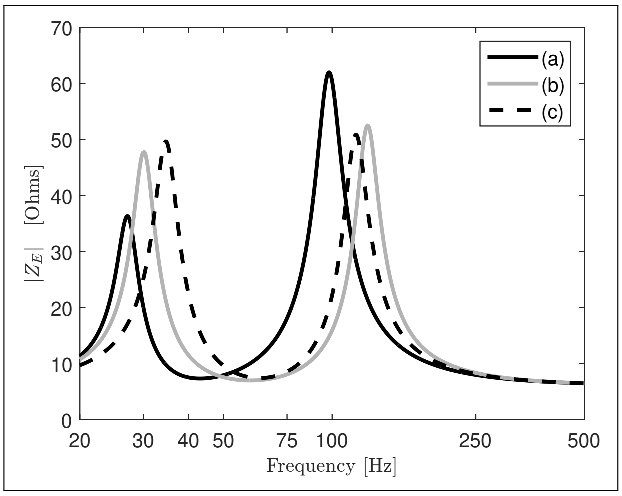

Figure 3. Frequency-dependant magnitude of the electrical impedance of the different vented box systems shown in Fig. 2.

tion of their form factor, without compromising their low frequency performance. This work paves the way for further exploration on the synergistic use of complex materials, such as metamaterials and multiscale porous materials, in the design of loudspeaker systems with the aim of achieving 'big bass from small boxes'.

5. ACKNOWLEDGMENTS

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