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PHYSICS-INFORMED INTERPOLATION OF DIRECTIONAL CHARACTERISTICS OF SOUND SOURCES

Mathias Lemke[1*] **Arne Hölter**[2] **David Ackermann**[2] **Stefan Weinzierl**[2]

[1] Computational Fluid Dynamics Group, TU Berlin, Germany
[2] Audio Communication Group, TU Berlin, Germany

ABSTRACT

Radiation characteristics of sound sources are important for sound reinforcement and virtual acoustics. Usually, the determination is done using discrete microphone measurements and subsequent interpolation. In this paper, a physically informed method for the interpolation of directional characteristics is presented and discussed by means of synthetic analyses. The influence of parameters, such as the number of microphones used, on the quality of the directivity interpolation is investigated. In addition, a comparison with a conventional interpolation method and an investigation of the influence of measurement noise are presented. The method is capable of determining complex directivity characteristics and is robust to noise.

Keywords: *sound sources, directivity, interpolation, numerical twin, monopole synthesis*

1. INTRODUCTION

Directivity is an important parameter of acoustic or electroacoustic sound sources [1–3]. A precise representation is important, for example, in the planning of sound reinforcement systems, in room acoustic simulations and in the auralization of virtual acoustic environments. The determination of the characteristics of real sources is typically done experimentally using discrete microphone measurements [4–6]. In almost all cases, these measurements must be interpolated, either for spatial upsampling to higher resolution representations of the data, for spatial resampling to a different sampling grid, or use in simulations of sound propagation. Different interpolation techniques are established, such as pseudo-splines or an interpolation via decomposition into spherical harmonic functions (spherical harmonics, SH) [7]. The performance of the interpolation techniques depends on the sampling grid used, but also on the radiation pattern of the sources themselves. The interpolation is mostly based on mathematical ansatz functions. Typically, no physically motivated constraints are considered.

In this paper, we present a directivity pattern interpolation method that is physically informed and uses the Euler equations as constraints. In detail, an adjoint-based monopole synthesis is performed in the time domain to reproduce experimental microphone measurements in an optimal sense. High-resolution directional characteristics can be determined from the resulting numerical twin. A validation is carried out based on synthtic data.

2. FRAMEWORK

To represent the acoustic source, the governing Euler equations are first extended by a source term $s_\mathrm{p}(x_i,t)$ in the energy equation, which is formulated in terms of pressure.

$$\partial_t \begin{pmatrix} \varrho \\ \varrho u_j \\ \frac{p}{\gamma-1} \end{pmatrix} + \partial_{x_i} \begin{pmatrix} \varrho u_i \\ \varrho u_i u_j + p\delta_{ij} \\ \frac{u_i p \gamma}{\gamma-1} \end{pmatrix} - u_i \partial_{x_i} \begin{pmatrix} 0 \\ 0 \\ p \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ s_\mathrm{p} \end{pmatrix}$$

Therein, ϱ describes the density, u_i the velocity in x_i direction, δ the Kronecker delta, p the pressure and γ the

****Corresponding author***: mathias.lemke@tu-berlin.de.*

isentropic exponent. The source term $s_{\mathrm{p}}(x_i, t)$ can be interpreted as a distribution of monopole sources on the computational grid. Einstein's summation convention for $i, j = [1, 2, 3]$ applies.

The fitting of the high-dimensional source term is done using an adjoint-based approach [8, 9]. In detail, the objective function

$$J = \frac{1}{2} \iint (p - p^{\mathrm{exp}})^2 \, \sigma(x_i, t) \, \mathrm{d}x_i \, \mathrm{d}t$$

is minimized. Therein p^{exp} denotes the microphone measurements. The additional weight $\sigma(x_i, t)$ defines where and when the objective function is evaluated, i.e., the microphone positions and sampling rate.

The optimization is done iteratively, see Figure 1. First, the Euler equations are solved forward in time. Then, the adjoint equations are computed backward in time using the direct solution. Based on the adjoint solution, the gradient $\nabla_{s_{\mathrm{p}}} J$ is determined and used to update the source distribution s_{p}^n using a gradient approach

$$s(x_i, t)_{\mathrm{p}}^{n+1} = s(x_i, t)_{\mathrm{p}}^{n} + \alpha \nabla_{s_{\mathrm{p}}} J.$$

Therein, α denotes a suitable step size and n the iteration number. The gradient is calculated for the entire computational region and the entire simulation time.

More details on the adjoint Euler equations and their use for sound reinforcement optimization and source localization tasks can be found in [9] and [10].

3. VALIDATION

To validate the approach, a synthetic configuration is investigated. In detail, the directivity of a circular piston is to be reconstructed from synthetic discrete measurements. The setup is analogous to [11], in which non-discrete, volumetric objectives are investigated. The considered computational domain covers $2 \times 2 \times 2$ m^3. The source is centered in the domain. A finite difference approach is used to discretize the domain. Compact spatial derivatives of 6th order and an explicit time integration method of 4th order are used. The computational grid comprises $192 \times 192 \times 192$ equidistantly distributed points. The sampling rate is 48 kHz. A 6th order spatial filter is used to avoid numerical instabilities. All boundaries are modeled as non-reflective using a characteristic approach [12]. Details on adjoint initial and boundary conditions can be found in [8]. The concrete implementation is realized with an in-house solver.

First, a CDPS (complex directivity point source) [13] approach is used to calculate a reference solution for the configuration shown in Figure 2. The frequency of the circular piston is chosen to 2 kHz. The resulting directivity is visualized in Figure 3.

Subsequently, for the adjoint-based synthesis, the sound source is replaced by the source term $s_{\mathrm{p}}(x_i, t)$, see Figure 4. The volume in which s_{p} can be adapted is limited to a spherical radius of $r = 0.33$ m. This provides more than 130,000 degrees of freedom for optimization/synthesis.

The synthetic microphone data p^{exp} needed to fit the source term are taken from the previously determined reference solution. Different numbers of microphones are considered, from 1024 to 16, in bisecting steps. The arrangement of the synthetic microphones is chosen in such a way that all cover an equal area fraction of a spherical shell with a constant radius around the sources, see Figure 5.

The adjoint-based synthesis is performed for all numbers of microphones mentioned. The resulting numerical twin in each case, respecting the governing Euler equations, is used to determine the directivity of the reference source. To simplify the assessment, in each case the error to the reference is shown in a 2-dimensional plot that maps the error to the spherical shell. In addition, the mean error, defined as

$$\mathrm{error} = \frac{\sum_{1}^{R} \left| p'^2 - p'^2_{\mathrm{ref}} \right| \cdot w}{\sum_{1}^{R} p'^2_{\mathrm{ref}} \cdot w},$$

with $p' = p - p_{\mathrm{ambient}}$, w as weight for equal treatment of all microphones (spherical Voronoi function) and $\sum_1^R w = 1$, is calculated.

Figure 6 shows the results for 1024, 512, 256, and 128 microphones. For the first three syntheses, the error is below the perceptual threshold (JND) of 1 dB [14]. The fact that the results are partly improved when the number of microphones is reduced can be explained by the presence of local minima in the optimization process. In the case of 128 microphones used, the local and the mean error are significantly higher. This trend continues with a further reduction of microphones.

Looking at the resulting directional characteristics in detail, it becomes clear that the latter synthesis produces a solution that primarily considers the microphone positions, but not the space in between. This phenomenon can

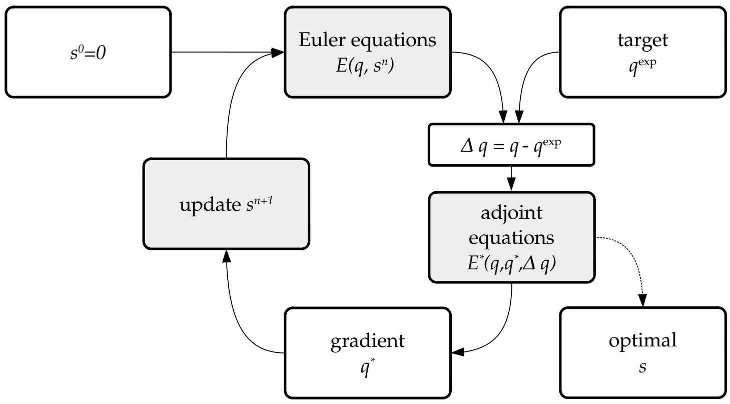

Figure 1. Iterative adjustment of the source term $s_{\mathrm{p}}(x_i, t)$.

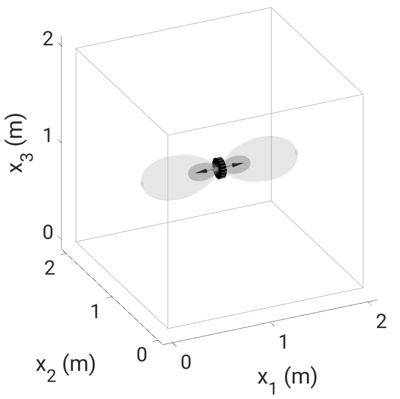

Figure 2. Synthetic setup to validate the adjoint-based approach. A circular piston at a frequency of 2 kHz is investigated.

be counteracted by reducing the available degrees of freedom, here by simply reducing the source volume.

Figure 7 shows the results for 64, 32 and 16 discrete microphones. Again, the error increases with reduced number of microphones, but still shows practically usable results especially for the case with 64 microphones. The local errors are lower near the microphone positions.

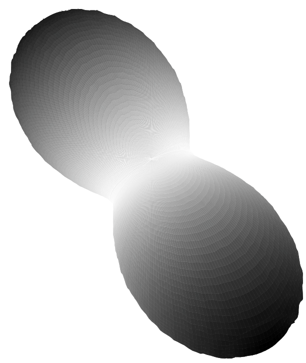

Figure 3. Reference directional characteristic of the investigated circular piston.

3.1 Comparison with spline-interpolation

The parameter r has a clear influence on the quality of the interpolation. Figure 8 shows the progress of the mean error of the physics-informed method in comparison to the thin plate pseudo-spline interpolation [7]. In the case of an adjusted radius r (number of available monopoles for the synthesis) a comparable quality is found. For the cases with 64 and 32 microphones r is chosen to 165 mm and for the case with 16 microphones to 100 mm. Without this adaption, the mean error is worse. However, for larger numbers of microphones the effect becomes negligible as

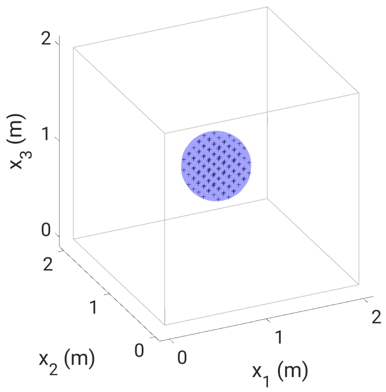

Figure 4. Replacement of the reference sound source by a multitude of grid-based monopoles modeled by the source term $s_{\mathrm{p}}(x_i, t)$.

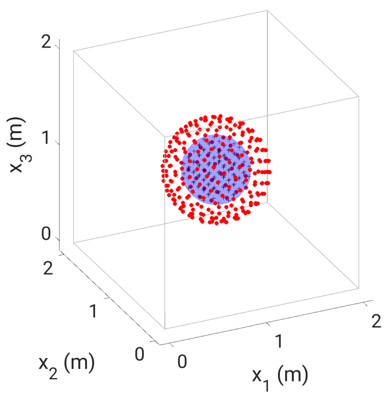

Figure 5. Uniform positioning of synthetic microphones around the replacement source(s) on a sphere with constant radius.

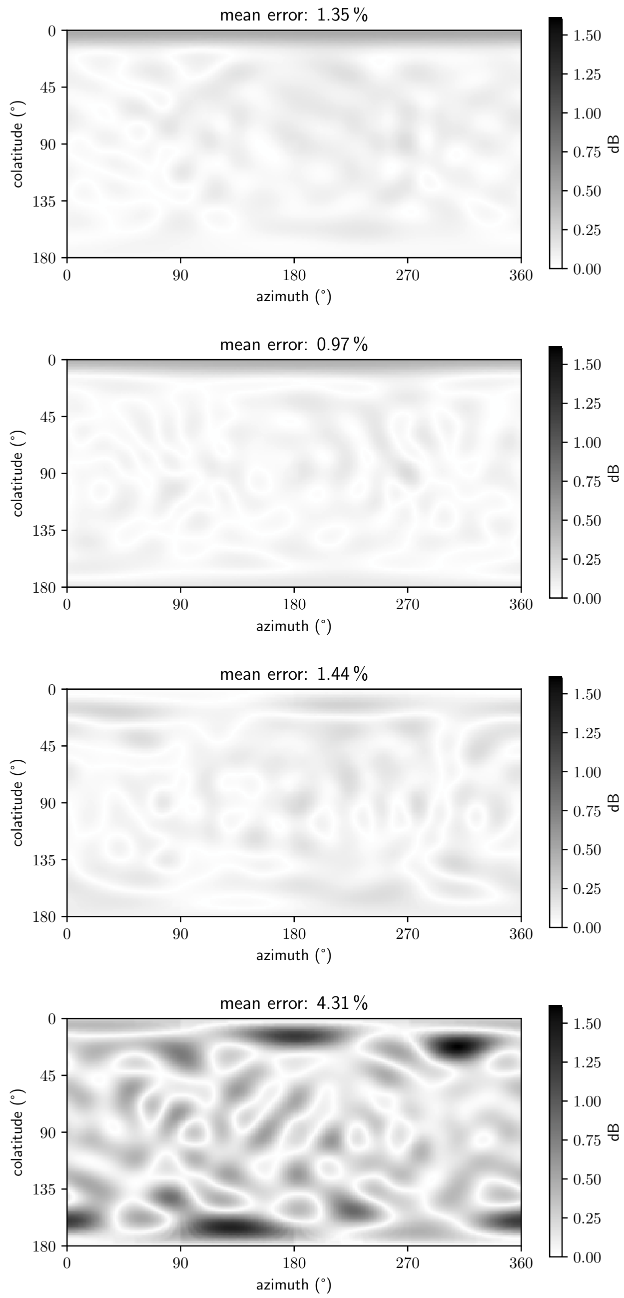

Figure 6. Interpolation error compared to reference solution for 1024, 512, 256 and 128 microphones (top to bottom).

the local errors are below the perceptual threshold.

3.2 Effect of measurement errors

By using the Euler equations as constraints for the optimization, the method is relatively insensitive to measurement inaccuracies. To analyze the influence of measurement noise, the case with 256 microphones is reanalyzed. In detail, the synthetic measurement data used

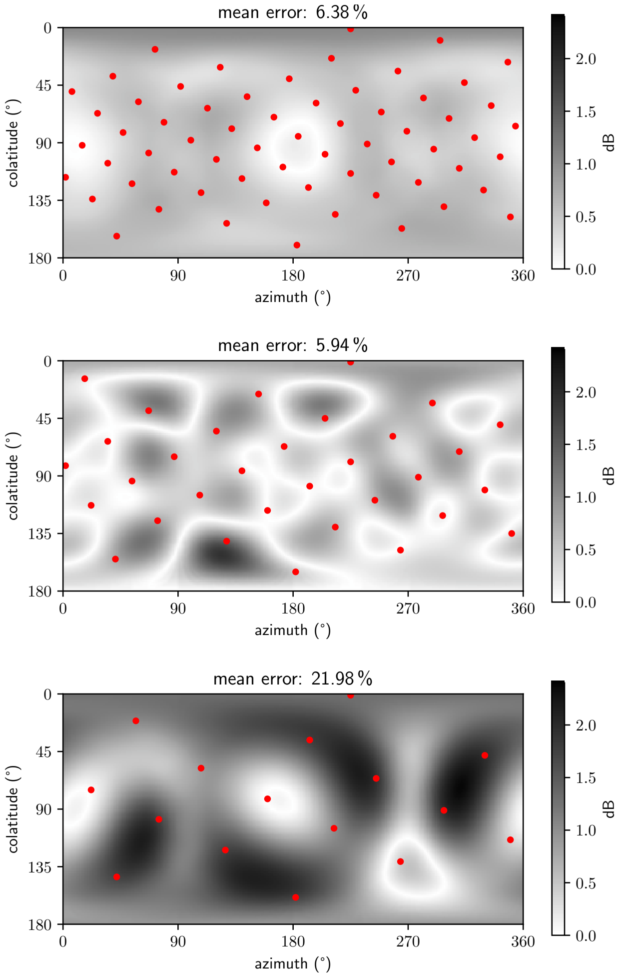

Figure 7. Interpolation error compared to reference solution for 64, 32 and 16 microphones (from top to bottom). The red markers denote the synthetic microphone positions on the sphere.

are overlayed by white noise with a signal-to-noise ratio (amplitude-based) of 4:1, with respect to the maximum signal of all microphones (a) and the maximum signal of each microphone independently (b). Figure 9 shows the interpolation errors. In the main radiation direction, both cases show good results with errors below the perceptual threshold. However, larger deviations in the radiation pattern become visible at small polar angles. For case (a), where the signal-to-noise ratio in this radiation direction is about 1:1 the error is above the perceptual threshold.

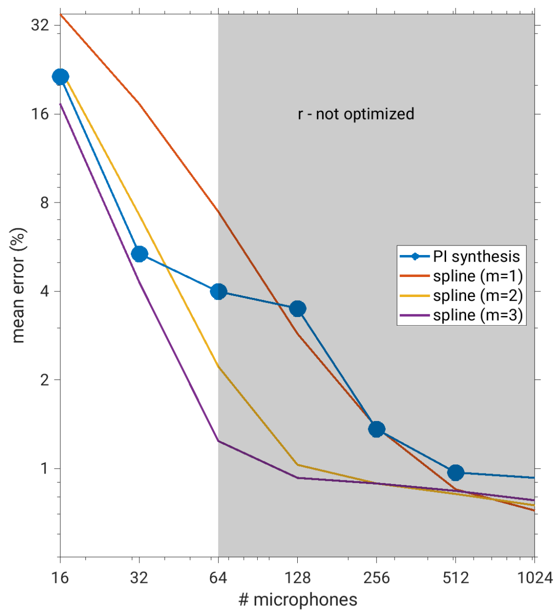

Figure 8. Progress of the mean error with respect the number of microphones used for the physics-informed analysis (blue) compared to thin plate pseudo-spline analyses with spline orders m = [1-3].

For case (b), the error is well below 1 dB and thus only slightly worse than the original case without noise.

4. SUMMARY

A physically informed method for interpolating directional characteristics of complex sound sources based on discrete microphone measurements was presented. In detail, an adjoint-based approach is used to generate a numerical twin of the sound source using grid-based monopoles.

The method respects the Euler equations and allows the free arrangement of the microphones used. Moreover, precise positioning (centering) of the source with respect to the microphones is not necessary. The method is relatively insensitive to noise because of the use of the constraint that acts as a physics-informed filter. The interpolation quality is comparable to established procedures.

In particular, the method is appropriate if physical radiation characteristics are required, for example, to avoid

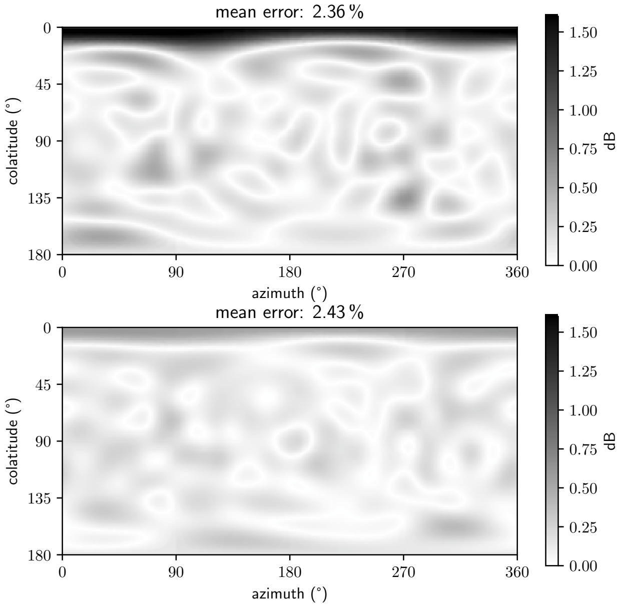

Figure 9. Interpolation error compared to the reference solution for 256 microphones considering artificial measurement noise with a signal-to-noise ratio of 4:1 concerning the maximum amplitude of all microphones (top, a) and each microphone (bottom, b).

nonphysical artifacts. Furthermore, it is suitable if a centering of the source needed for spherical harmonics interpolation is hard to realize or if the used microphone positions hinder a spline-based interpolation.

In upcoming work, the approach will be further validated. In addition, complex boundary and environmental conditions will be taken into account during synthesis and the position of the microphones will be optimized for the approach.

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