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COMPARISON OF BOUNDARY ELEMENT BASED AND PLANE WAVE APPROXIMATION COMPUTATIONS OF TARGET ECHO STRENGTHS

L. Pacaut[1,2*] **J-F. Mercier**[2] **G. Serre**[1]
S. Chaillat[2]
[1] Naval Group, Ollioules, France
[2] POEMS, CNRS, Inria, ENSTA Paris, Institut Polytechnique de Paris, France

ABSTRACT

In naval defence applications, the knowledge of the Target echo strength (TES) of a submarine is of major interest, in order to optimize the scattered pressure that can be measured by an active sonar. In this contribution, we consider a rigid target and compute the TES using two methods: (i) the solution of the Helmholtz equation by reformulating it into a boundary integral equation with either a full space Green's function or a tailored Green's function, and (ii) the use of a plane wave approximation, well-suited for medium to high frequencies. In the first case, the use of a tailored Green's function adapted to the presence of a target reduces the cost of the numerical model. However, an integral equation still has to be solved. It is not the case with the plane wave approximation where the boundary pressure is not calculated but is considered proportional to the incoming wave. Numerical tests are performed to compare the efficiency and accuracy of each approach with respect to available numerical models developed on the submarine model "BeTSSi" – for Benchmark Target Strength Simulation –, under rigid hypothesis.

Keywords: *target echo strength, Green's functions, underwater noise, boundary element method, plane wave approximation*

****Corresponding author***: *louise.pacaut@ensta-paris.fr.*

1. INTRODUCTION

The two main ways to detect a submarine nowadays are (i) to measure the proper sound produced by the ship using a passive SONAR or (ii) to measure the scattered pressure resulting from its excitation by a plane wave, using an active SONAR. The quantity used to study the scattered pressure, in the latter case, is the Target Echo Strength (TES). In order to reduce the TES of a submarine, a precise computation of the scattered pressure with reasonable computation costs is of major interest. Various methods have already been proposed based on the Boundary Element Method (BEM) [1], or on Kirchhoff and other approximations [2]. Here, we focus on two methods: (i) the solution of Helmholtz equation using a boundary element method with tailored Green's function and (ii) the use of a plane wave approximation. Then, we validate each formulation over a sphere case. Finally, we apply these methods to the industrial Betssi submarine, under rigid hypothesis.

2. BOUNDARY INTEGRAL REPRESENTATION FORMULATIONS

Formulations using free field and tailored Green's functions. Considering an obstacle surrounded by a fluid domain Ω excited by a generic source S, the total pressure p is solution of the Helmholtz equation

$$(\Delta + k_0^2)p(\mathbf{x}) = -S(\mathbf{x}), \quad \mathbf{x} \in \Omega \tag{1}$$

with k_0 the wave-number and Δ the Laplace operator. The scattered pressure $p_s = p - p_i$ satisfies

$$(\Delta + k_0^2)p_s(\mathbf{x}) = 0, \quad \forall \mathbf{x} \in \Omega, \tag{2}$$

with p_i the incident pressure incoming towards the obstacle.
The solution of the Helmholtz equation (2) is obtained by introducing an arbitrary Green's function G and using Green's theorem.

$$\forall \mathbf{x} \notin \Gamma,$$
$$p_s(\mathbf{x}) = \int_\Gamma \left(p_s(\mathbf{Y}) \frac{\partial G}{\partial n_Y}(\mathbf{x},\mathbf{Y}) - G(\mathbf{x},\mathbf{Y}) \frac{\partial p_s}{\partial n_Y}(\mathbf{Y}) \right) \mathrm{d}S_\mathbf{Y}, \tag{3}$$

where $\mathbf{n}$ is the normal pointing outward the obstacle, of surface Γ.
An option for G is to consider a Green's function G_T tailored to the obstacle (satisfying the rigid boundary condition) [3], solution of

$$\left\{ \begin{array}{lcll} (\Delta_\mathbf{z} + k_0^2) G_T(\mathbf{x},\mathbf{z}) + \delta(\mathbf{x}-\mathbf{z}) & = & 0 & \forall \mathbf{z} \in \Omega, \\ \dfrac{\partial G_T}{\partial n_\mathbf{Z}}(\mathbf{x},\mathbf{Z}) & = & 0 & \forall \mathbf{Z} \in \Gamma, \end{array} \right. \tag{4}$$

for any source located at $\mathbf{x} \in \Omega$. The choice of this Green's function simplifies (3) such that, with the Neumann boundary condition $\dfrac{\partial p}{\partial n} = 0$ over Γ in the rigid case,

$$p_s(\mathbf{x}) = \int_\Gamma G_T(\mathbf{x},\mathbf{Y}) \frac{\partial p_i}{\partial n_Y}(\mathbf{Y}) \mathrm{d}S_\mathbf{Y}. \tag{5}$$

To compute G_T, we introduce the free field Green's function G_0 and we use the Green's theorem for the problem (4), such that G_T satisfies

$$G_T(\mathbf{x},\mathbf{z}) = G_0(\mathbf{x},\mathbf{z}) - \int_\Gamma \left(G_T(\mathbf{x},\mathbf{Y}) \frac{\partial G_0(\mathbf{z},\mathbf{Y})}{\partial n_\mathbf{Y}} \right) \mathrm{d}S_\mathbf{Y}. \tag{6}$$

Introducing the doubler layer potential which to any field ϕ associates the function

$$(\mathcal{D}\phi) : \mathbf{x} \in \Omega \mapsto \int_\Gamma \partial_{n_\mathbf{Y}} G_0(\mathbf{x},\mathbf{Y}) \phi(\mathbf{Y}) \mathrm{d}\mathbf{Y} \tag{7}$$

and its trace

$$\lim_{\mathbf{x} \to X^+} (\mathcal{D})(\mathbf{x}) = (\frac{1}{2} I + D)(\mathbf{X}), \tag{8}$$

the tailored Green's function is solution of the integral equation [3]

$$(\frac{1}{2} - D) G_T(\mathbf{x},\mathbf{Z}) = G_0(\mathbf{x},\mathbf{Z}), \quad \forall \mathbf{Z} \in \Gamma, \forall \mathbf{x} \in \Omega. \tag{9}$$

(9) shows that the computation of the tailored Green's function does not depend on the type of source p_i (incidence for instance), but only on the geometry of the obstacle. Once G_T computed, only the low-cost integral representation (5) has to be calculated when changing the source.

Plane Wave Approximation. To consider high frequencies, large meshes are needed. But they also lead to large computational costs for BEM solvers. Approximations, such as the Plane Wave Approximation (PWA) are therefore useful to evaluate sound levels in reasonable computational times at high frequencies. The PWA, described in [4], relies on the imposed relation

$$p_s(\mathbf{Y}) = \rho c v_n^s(\mathbf{Y}), \quad \forall \mathbf{Y} \in \Gamma, \tag{10}$$

where v_n is the fluid velocity normal to the boundary, ρ its density and c its celerity. In a rigid case, $v_n = 0 = v_n^s + v_n^i$ leads to $p_s = -\rho c v_n^i$.

Using Green's theorem for the Helmholtz equation satisfied by p_i, it can be shown that, for $\mathbf{x} \in \Omega$,

$$\int_\Gamma \left(p_i(\mathbf{Y}) \frac{\partial G_0(\mathbf{x},\mathbf{Y})}{\partial n} - G_0(\mathbf{x},\mathbf{Y}) \frac{\partial p_i(\mathbf{Y})}{\partial n} \right) \mathrm{d}S_\mathbf{Y} = 0. \tag{11}$$

Adding (11) to (3), and using the Neumann boundary condition, lead to

$$p_s(\mathbf{x}) = \int_\Gamma p(\mathbf{Y}) \frac{\partial G_0}{\partial n_Y}(\mathbf{x},\mathbf{Y}) \mathrm{d}S_\mathbf{Y}. \tag{12}$$

For a plane incident wave $p_i = p_0 e^{i(k e_{inc}.x - \omega t)}$ carried by the unit vector e_{inc}, we get $\rho c v_i = e_{inc} p_i$ and thus

$$p_s(\mathbf{x}) = \int_\Gamma p_i(\mathbf{Y})(1 - e_{inc}.n_Y) \frac{\partial G_0}{\partial n_Y}(\mathbf{x},\mathbf{Y}) \mathrm{d}S_\mathbf{Y}, \tag{13}$$

The advantage of (13) is that it does not require any system inversion. It can be computed directly without any inversion, contrary to formulation (5). But this is done at the price of some loss in the accuracy due to the PWA approximation (multiple reflections are note considered for instance). The goal of this contribution is to evaluate the accuracy of (13).

3. ACCURACY FOR THE CASE OF A RIGID SPHERE

We consider an obstacle that consists of a rigid unit sphere. The source is a plane wave. We focus on the relative error defined by

$$err = \frac{|p_{num} - p_{ana}|}{|p_{ana}|}, \tag{14}$$

with $p_{num} = p_i + p_s^{num}$, $p_{ana} = p_i + p_s^{ana}$, p_s^{num} and p_s^{num} being the numerical and analytical scattered pressures [5]. The considered fluid is water. The mesh for low to medium frequencies has 2 886 nodes and 5 768 elements. The mesh for high frequencies has 132 402 nodes and 264 800 elements. Both meshes are designed to have at least 10 points per wavelength at each frequency.

Validation in the far field domain of (5) at low and medium frequencies. Figure 1 shows the validation of the tailored Green's function and therefore of (5), thanks to a comparison with an analytical result obtained by decomposing on spherical Bessel functions [5]. We see a good agreement between the numerical and analytical results, since the relative error is smaller than 0.15%.

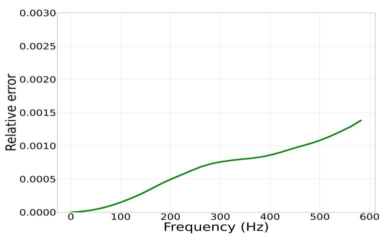

Figure 1. Relative error between formulation (5) with G_T and the analytical solution averaged on 5 random points in Ω.

Accuracy of the PWA in the far field at high frequencies. Figure 2 shows the relative error between the Plane Wave Approximation (13) and the analytical results. The PWA gives results with a mean error of around 5% on sound pressure levels. This is a satisfactory approximation of high frequency problems.

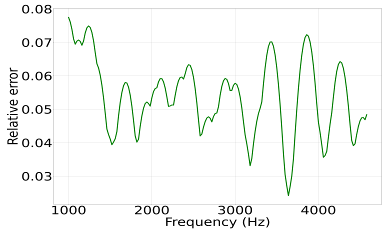

Figure 2. Relative error between the PWA formulation (13) and the analytical solution averaged on 5 random points in Ω.

4. EFFICIENCY FOR THE BETSSI RIGID CASE

Betssi, for Benchmark Target Strength Simulation, is a submarine developed by FWG in Germany for which various Target Echo Strength (TES) computations were performed (see, for instance, [2]).

The TES determines the reflectivity of an obstacle for an incoming plane wave, and is used here to test formulations (5) and (13). It is defined by:

$$TES = 20\log_{10}(|\frac{p_s}{p_i}|) + 20\log_{10}(|R_{obs} - r_0|), \tag{15}$$

where p_s is the scattered pressure taken at an observer point, p_i the incoming plane wave, R_{obs} the distance of the observer point from the obstacle and r_0 the center of the obstacle. Here, $R_{obs} = 800$ m is considered to be "as infinity". Since the formulation (5) with G_T is well suited for varying source and fixed receiver, a multi-static TES is computed to check formulation (5). In this case, the observer is placed at $\varphi_{obs} = \pi/2$ and the incidence angle of the plane wave varies (Figure 3). To validate the PWA, we focus on the mono-static case ($\varphi_{obs} = \varphi_{PW}$) which is more commonly used.

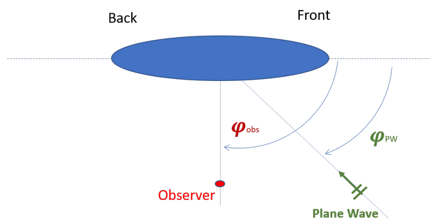

Figure 3. Angles associated to the observer and the incident plane wave around BETSSI (schematic).

Results at 200 Hz - Formulation with G_T. Figure 4 shows the comparison between formulation (5) and a

BEM formulation taken as reference, at 200 Hz. This formulation is (3) using the free field Green's function G_0. For this frequency, the mesh has 18 865 nodes and 34 346 elements leading to 10 points per wavelength. The results are satisfying: the reference formulation with G_0 and formulation (5) with G_T are close to each other.

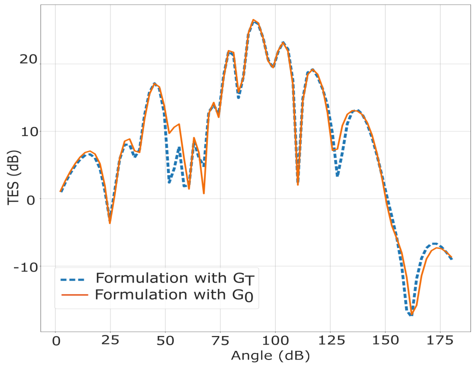

Figure 4. Comparison between the multi-static TES computed using (5) with G_T and BEM formualtion with G_0 (taken as reference) at 200 Hz.

Results at 3 kHz - Plane Wave Approximation. PWA is interesting at high frequency when numerical costs are high. To test the validity of the PWA formulation (13), we focus on 3 kHz frequency, for which we have a reference solution in the literature. The mesh has 1 954 516 nodes and 3 909 028 elements, leading to 17 points per wavelength. Figure 5 compares the PWA formulation (13) to a reference [1] and it shows that the approximation gives satisfactory results at high frequency with low computation costs.

5. CONCLUSION

For low to medium frequencies, we have tested that the formulation (5)based on boundary integral equations with G_T gives accurate results. This method does not rely on the source type and only depends on the geometry of the obstacle and the fluid characteristics. At high frequencies, the plane wave approximation can be useful to determine a first order approximation with limited computation costs.

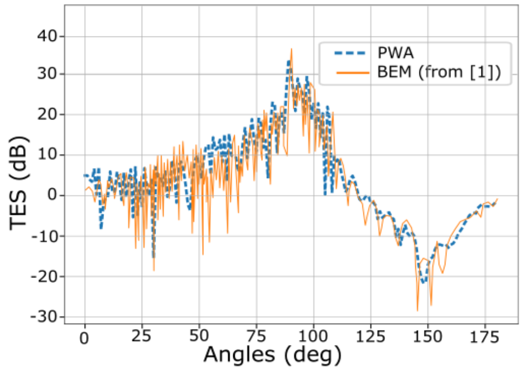

Figure 5. Comparison between the mono-static TES computed with the PWA formulation (13) and a BEM computation from [1] at 3 kHz.

6. ACKNOWLEDGMENTS

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