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"CELESTE" RANKS IN PIPE ORGANS AND ACCORDIONS: TONAL TIMBRE AND CONSONANCE OF DETUNED UNISON INTERVALS

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ABSTRACT*

Two simultaneously sounding tones differing in frequency by a few Hertz generate a waveform, whose amplitude modulation relates to the psychoacoustic quantity "fluctuation strength". Following the historic approaches of sensory consonance, any deviation from a pure interval yields dissonance. However, imperfect intonation is quite common in musical performance; some instruments are even slightly detuned by intention. Examples are various flat and sharp CELESTE ranks in the Pipe Organ or the VIOLIN double-reed and MUSETTE triple-reed stop of the Accordion.

Undulating sounds are pictured as pleasant, shimmering, or celestial. This work explains why mistuned dyads may still appear as consonant sounds. Moreover, it shows that the tonal timbre of harmonic complex tones can change noticeably and periodically with the beat cycle. The usual practice of CELESTE tuning Pipe Organs and Accordions has been analyzed to get an overview. Combining this information with data on the just noticeable frequency difference of our hearing allows deriving general tuning progression rules.

Keywords:
Beats, Consonance, Pipe organ, Accordion

1. INTRODUCTION

Some musical instruments generate intentional beats by producing two tones simultaneously, whose frequencies differ by a few Hertz. In the Pipe Organ, the term CELESTE rank refers to a set of pipes tuned sharp or flat with respect to true pitch. Several examples occur within the families of organ tone: a flat-tuned FLUTE (often named UNDA MARIS), the DIAPASON CELESTE (VOCE UMANA in the Italian organ tradition) and, most common, the STRING CELESTES (VOIX CELÉSTE, VIOLIN CELESTE). Pipe Organs do not make use of CELESTES in reed ranks; instead, these occur in Reed Organs and Accordions. The VIOLIN stop of the Accordion combines two reed ranks (true, sharp) and the MUSETTE stop consists of three ranks (tuned flat, true and sharp).

The beat frequency (BF) is usually set between 0.5 Hz and 15 Hz; a gradual rise upon pitch is considered desirable.

Although the beating tone arises due to a detuned unison interval, it sounds consonant and additionally a chorus effect emerges (as known from strings playing unison).

The following sections address four questions:

- What is the BF, or, what <u>are</u> the BF of a complex tone?
- Why do mistuned intervals sound consonant?
- Does a CELESTE rank modify the tonal timbre?
- What are the tuning progression rules for CELESTE ranks?

2. THE BEAT FREQUENCY OF COMPLEX TONES

Two pure tones with similar frequencies $f-\Delta f/2$ and $f+\Delta f/2$ generate the BF $\Delta f \ll f$. In case of two harmonic complex tones, frequencies of the form $n\cdot(f-\Delta f/2)$ and $m\cdot(f+\Delta f/2)$ occur yielding multiple BF, such as $n\Delta f$ for $n=m$ and integers n and m. The lowest BF, Δf, corresponds to that of the primary beats, most clearly perceived [1] in holistic listening. Focusing on any lower partial (spectral listening) allows to recognize the corresponding secondary BF $n\Delta f$ of this harmonic (i.e. the n-th partial). This is, of course, easier for those partials with the highest sound pressure levels (SPL). Fig. 1 depicts the harmonic partials of the sound spectra of two organ pipes (both pitch C_5), one belonging to the rank VIOL DI GAMBA 8´ and the other from the VIOL CELESTE 8´. Both ranks are part of the "St. Anne´s" digital pipe organ sample set, from which their long-term average sound spectra were extracted (see [2] for details). This pipe organ is tuned to $f(A_4) = 436.5\,\text{Hz}$ and

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thus $f(C_5) \approx 519\,\text{Hz}$. However, to exemplify the beat effects over time with "smooth" numbers, Fig. 1 plots both waveforms as if the frequencies of the two pipes were 499.0 Hz and 501.0 Hz yielding to exactly 2.0 Hz beat frequency.

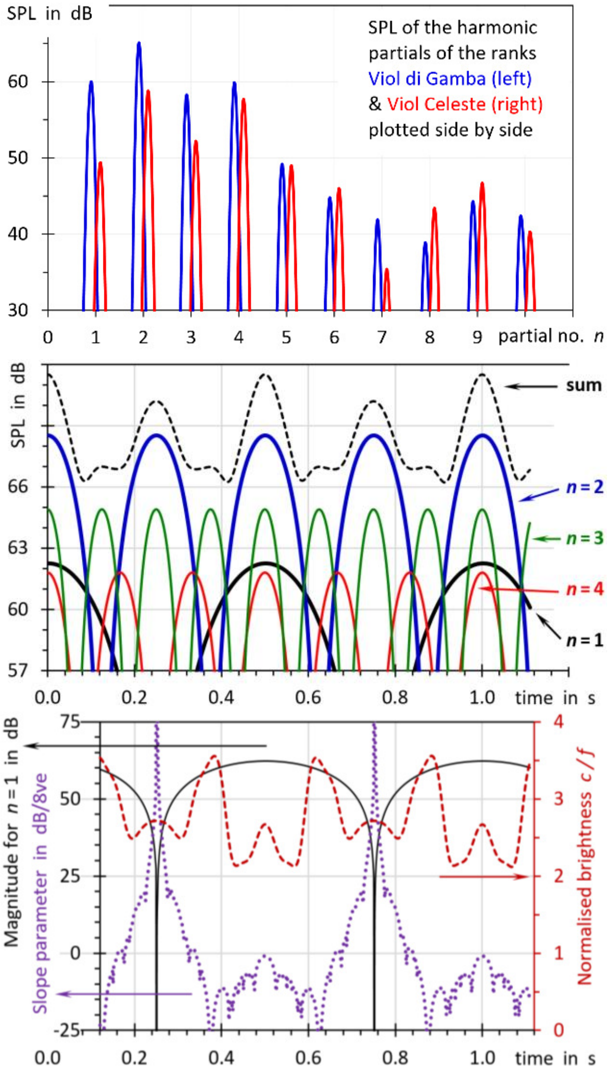

Figure 1. The sound spectra of the VIOL DI GAMBA 8´ and the VIOL CELESTE 8´ (top) for $\Delta f = 2.0\,\text{Hz}$. The SPL varies periodically over time (middle). The two selected timbre parameters in the bottom chart and their variation in time are discussed in Ch. 4.

To tackle the initial question what the BF of a complex is, the middle chart of Fig. 1 provides two answers: The total SPL (dashed curve) varies with $\Delta f = 2.0\,\text{Hz}$, which is the BF in the condition of holistic listening. However, in spectral listening the BF refers to the frequency of the varying SPL of that partial on which the listener focuses his attention. Note that the BF of the fundamental ($n = 1$) equals the BF of the complex tone (black curves).

3. THE REASON FOR CONSONANCE

Various models of tonal consonance for dyads have focused on the frequency ratios of their fundamentals [3, 4]. According to these compactness approaches, intervals with frequency ratios of small natural numbers (like 1:1, 2:1, 3:2 etc.) are regarded consonant [3], while a slight frequency deviation would suffice for dissonance. Other theories use auditory roughness to explain consonance, because a maximum of dissonance occurs when the frequency difference of the two tones corresponds to ≈27 % of the critical bandwidth (Fig. 10 in [3]). A recent study has proven that the model for sensory consonance significantly improves when combining the compactness and the roughness approach rather than just adhering to one of them [4].
Moreover, taking account for the just noticeable frequency difference creates a transition region, a few Hertz wide, in which the consonance of pure intervals gradually changes into dissonance [4]. Including the difference limen is the key to understand why musical intervals, which are rarely tuned perfectly in practice (e.g. in musical performance), can still sound consonant. For example, a stretched octave consisting of two pure tones with frequencies of 499 Hz and 1002 Hz still appears consonant. The beats ($\Delta f = 4\,\text{Hz}$) are too slow to cause auditory roughness [5], but they identify the octave interval as mistuned, while the undulating sound is perceived with a certain fluctuation strength.

4. HOW BEATS MODIFY TONAL TIMBRE

4.1 Fluctuation strength

The basic model of fluctuation strength F correlates the frequency Δf with the SPL difference ΔL caused by the undulation [5]. Note that fluctuation strength and magnitude ΔL are proportional to each-other ($F \sim \Delta L$) and that a BF of $\Delta f = 4\,\text{Hz}$ generates maximum fluctuation strength [5].
With complex harmonic tones instead of pure tones, the fluctuation strength can decrease due to the contribution of even partials ($n = 2, 4, 6, \ldots$). The reason is that the nodes of the latter do not coincide with those of the fundamental, thus reducing the magnitude ΔL. This is visible by comparing the black curves of the SPL in the top chart of Fig. 1. Keeping in mind that $F \sim \Delta L$, the fluctuation strength of undulating complex tones without odd partials (as with STOPPED FLUTE ranks) or of tones with comparably weak

harmonics (as in an OPEN FLUTE of wide pipe-scale) is larger than for DIAPASON ranks, STRING ranks, or Accordion reeds, all of them rich in harmonic content. Since STRING ranks are preferred over FLUTE ranks to obtain the CELESTE effect in Pipe Organs, it seems that organ builders are generally not aiming for maximum fluctuation strength, rather than for the periodic alteration of tonal timbre.

4.2 Tonal timbre

A harmonic complex tone contains the frequencies $f_n = nf$ of several partials. In a dyad of two complex tones with similar frequencies (as in Fig. 1) each harmonic n undulates with its individual BF $n\Delta f$. The SPL of all odd harmonics reach a common minimum twice per cycle; at these times, only the even partials contribute to the spectrum. This yields a periodic change of the spectral centroid c and to a variation of the slope parameter s, defined by Eqn (1):

$$s = \sum_n \frac{1}{n^q} \cdot (L_{n-1} - L_n) \Big/ \sum_n \frac{1}{n^q} \quad \text{with} \quad q = 1.729 \qquad (1)$$

Since the slope parameter is calculated from the magnitudes of the sound spectrum at the harmonic partials L_n, its value is sensitive to extinguished partials (where $L_n \to -\infty$), resulting in ripples visible in the bottom chart of Fig. 1.
The timbre parameter c is noticeable as brightness and the slope parameter s refers to the string quality of the sound [2]. As these parameters change periodically with time, the related sound qualities vary correspondingly. For the selected combination of the VIOL DI GAMBA 8´ and the VIOL CELESTE 8´ (Fig. 1) the normalised spectral centroid calculates to $c/f = 2.67$ without beats and varies within the range $2.12 \ldots 3.55$ with beats. In this example, one would notice a loudness fluctuation in addition to the variation of tonal timbre. A closer investigation of the bottom chart in Fig. 1 reveals that two main maxima occur in the course of c/f within one beat cycle, while the main maximum of the slope parameter s appears only once per period.
It seems that organ builders aim for this kind of timbre fluctuation ("shimmering"), when including a STRING CELESTE. In contrast, a FLUTE CELESTE can only introduce the effect of loudness fluctuation, but it cannot noticeably vary the timbre, as it lacks the harmonic content to do so.
Playing a single tone on a FLUTE rank (for which $c/f \approx 1.0$ above C_2 [6], Fig. 5) with the tremulant (a device to periodically vary the wind pressure of the pipe organ) can sound similar to a FLUTE CELESTE. In both cases the loudness varies periodically, whereas the changes of the timbre parameters $c/f(t)$ and $s(t)$ are too small to be noticeable by ear. In contrast to the vibrato of a tremulant the CELESTE ranks facilitates adjusting the BF across the tonal compass.

5. PITCH-DEPENDENT TUNING OF CELESTES

The dependency between frequency and pitch is formulated using the pitch category p counting the semitone steps (cf. Tab. 1–4 in the Appendix). For a musical scale in 12-tone equal temperament this results in Eqn. (2).

$$f(p) = (440\ \text{Hz}) \cdot 2^{(p-57)/12} \qquad (2)$$

$$\Delta f(f, \Delta p) = f \cdot (2^{\Delta p/(1200\ \text{ct})} - 1) \qquad (3)$$

The pre-factor in Eqn. (2) is the frequency of A_4 with the pitch category $p(A_4) = 57$. Small pitch differences Δp are usually given in cent values. Thus, when working with electronic tuners it might be helpful to convert the BF $\Delta f(p)$ into cent values (unit: ct) and vice-versa according to Eqn. (3).

5.1 CELESTES in the Pipe Organ

There are four basic ideas to tune a CELESTE rank (Fig. 2):
a) Aim for a constant BF (e.g. $\Delta f = 2$ Hz) of the CELESTE with the unison rank across the tonal compass. As a result, the octaves in the CELESTE rank are slightly compressed (i.e. flat), but still tolerable though. Since the BF is constant upon pitch, the effect is similar to that of a vibrato with the frequency Δf – especially for FLUTE ranks as their higher partials are too weak [2] to generate a timbre fluctuation.
b) Adjust the BF at a reference tone, say A_4, to the desired value. Then keep the CELESTE rank "dry", i.e. in tune with itself so that its octaves remain pure intervals. Organ builder E.M. Skinner proposed this method for CELESTE ranks, tuned sharp by 1.7 Hz at A_4 (i.e. 1.0 Hz at C_4 [7]).
Apart from these recipes, other variants are in use, such as:
c) Take the pitch of the pure major third from the frequently available TIERCE 1³⁄₅´ rank for tuning (e.g. use the F_2 pipe of the TIERCE to tune A_4 of the CELESTE rank). The result is a flat CELESTE ($\Delta f = -3.5$ Hz at A_4) with pure octaves, but its beats are often considered too fast in the treble.
d) A reasonable compromise between methods a) and b) is to double the BF every 1.5 octaves (and $\Delta f \approx 1.7$ Hz at A_4).
In general, the BF of CELESTE ranks $\Delta f(p)$ is either constant (case a), or it increases exponentially upon pitch p.

5.2 CELESTES of the Accordion

Accordions commonly provide beats as characteristic feature and several traditions for CELESTES styles exist on a scale from "dry" to "wet" (referring to $\Delta f = 0.5 \ldots 7$ Hz at A_4). Note that knowing the BF at A_4 alone is insufficient to describe a tuning curve $\Delta f(p)$, for which the BF increases upon pitch. If the beats accelerate with a constant rate (i.e. $\Delta f(p) \sim p$) this results in straight lines (Fig. 3). The BF can also rise exponentially upon pitch, either across the whole compass or in a part of it. Moreover, there are other tuning curves in use. Note that comparing Δf values (given in Hz) to cent values (as used in electronic tuners) one needs to convert these quantities first according to Eqn. (2) and (3).

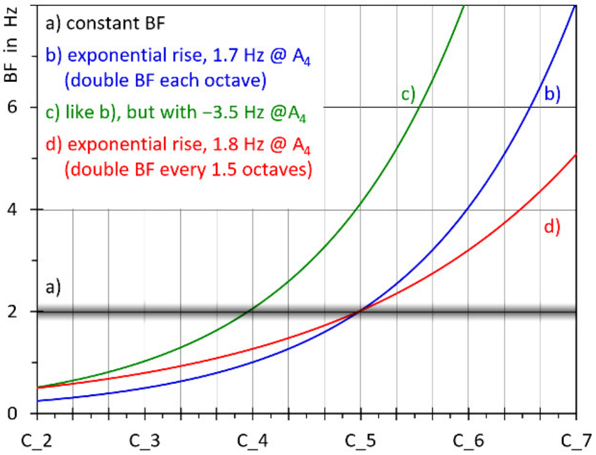

Figure 2. Four approaches to tune CELESTE ranks in the Pipe Organ as described under a) – d) in the text

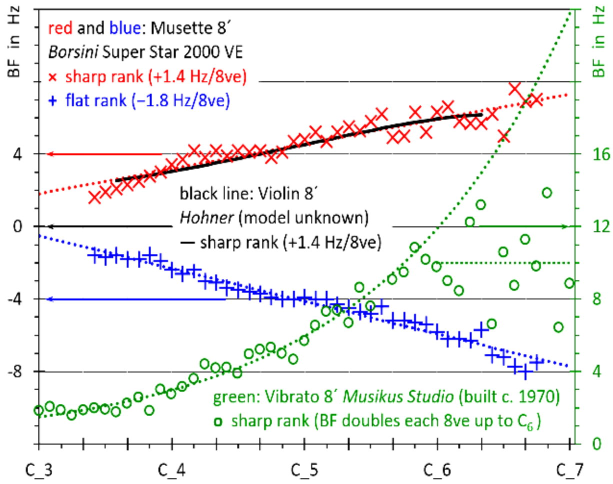

Figure 3. Tuning curves in two different Accordions and a Harmona (a reed organ with Accordion reeds)

In summary, two important cases occur:

e) The BF $\Delta f(p)$ increases linearly upon pitch, or,

f) the BF grows exponentially (as in d), i.e. $\Delta f(p)$ doubles its value after the same pitch distance, e.g. each octave.

Limiting the BF in the treble, e.g. at ≈ 10 Hz (Fig. 3) or 15 Hz [8] might be desirable to avoid auditory roughness caused by secondary beats. Studies on different Accordions including listening tests indicated that $\Delta f(p)$ should increase linearly in the bass and exponentially in the treble ("mixed type" in Fig. 4), or follow an S-shaped curve [8].

5.3 Proposals for CELESTE tuning

The choice of suitable BF is a matter of the reverb time of the room, of personal taste and it depends on the style of music. Moreover, single notes seem to tolerate more detuning than chords, and CELESTE ranks with high harmonic content are usually tuned "wetter" than FLUTE ranks.

One can derive an upper bound for $\Delta f(p)$ using the just noticeable variation of frequency (JNVF, [5]) of our hearing as orientation. To obtain a frequency-dependent expression for the JNVF an empirical function was fitted to depicted data [5] resulting in Eqn. (4), valid for $50\,\text{Hz} < f < 20\,\text{kHz}$:

$$\lg\left(\frac{JNVF(f)}{\text{Hz}}\right) = \sum_{i=-1}^{4} a_i \cdot \left(\lg\left(\frac{f}{\text{Hz}}\right)\right)^i \tag{4}$$

where f is the frequency and a_i are coefficients given by $a_{-1} = 7.76$, $a_0 = -19.6$, $a_1 = 20.2$, $a_2 = -9.57$, $a_3 = 2.12$, and $a_4 = -0.17$ yielding $R^2 = 0.99999$.

This empirical function is shown in Fig. 4 as dotted curves.

A second requirement is that the BF of the CELESTE shall steadily increase upon pitch while neither exceeding the JNVF nor a maximum of 15 Hz (or even less) [8].

These two criteria allow commenting on the methods a) - e):

a) It is possible to keep the BF $\Delta f(p)$ at a constant value, if $\Delta f(p) \leq 3.5$ Hz, but this misses the chance that each tone (of a chord) beats with its own, individual frequency.

b) For a five octave key compass $\Delta f(p)$ increases too much. Moreover, octave dyads unnecessarily share common BF.

c) With $\Delta f(A_4) = 3.5$ Hz the BF exceeds the JNVF above C_5 already. This might be still tolerable for Accordions, but considered unpleasant for tuning a Pipe Organ CELESTE.

d) Doubling every 1.5 octaves is a good choice insofar as complex tones played in octaves will not share the same BF and the latter are distinct in frequently used chords.

e) A linear course for $\Delta f(p)$ will work reasonably below C_5; however, due to the relation between pitch and frequency, exponential laws appear to be the "natural" choice for the BF and is recommended, at least for larger pitch ranges.

Fig. 4 depicts different possibilities for tuning curves. The top red curve is a proposal for Accordions based on listening tests ([9], valid for $E_2 \ldots A\#_7$) approximated by Eqn. (5).

$$\frac{BF_{[9]}(f)}{\text{Hz}} = \sum_{i=0}^{4} b_i \cdot \left(\frac{f}{\text{Hz}}\right)^i \tag{5}$$

with $b_0 = 1.15$, $b_1 = 4.42 \cdot 10^{-3}$, $b_2 = -3.34 \cdot 10^{-6}$, $b_3 = 1.63 \cdot 10^{-9}$, and $b_4 = -2.21 \cdot 10^{-13}$, $R^2 = 0.99981$.

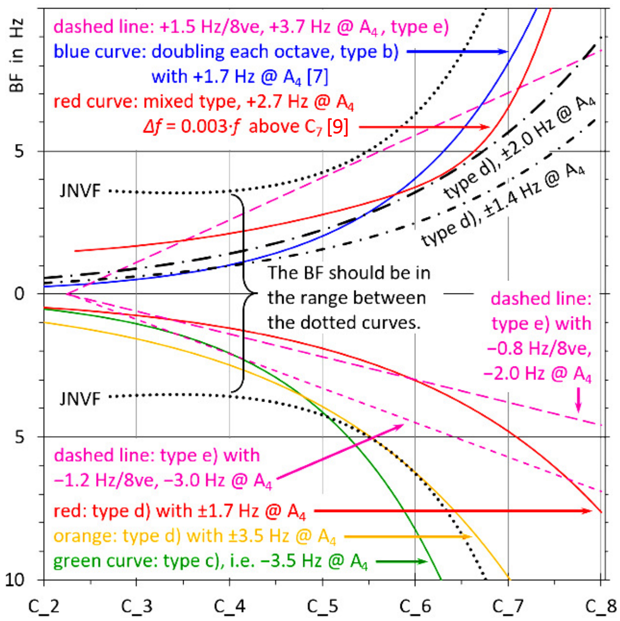

Figure 4. Proposed CELESTE tunings in the range $C_2 \ldots C_8$. Tuning can be flat or sharp (except for c). For tabulated values of "type d)" see Appendix.

5.4 Flat and sharp tuning of Pipe Organ CELESTES

In Pipe Organs, most CELESTE ranks are tuned sharp, flat tuning (as in the UNDA MARIS) is less common. Let´s assume, the audible frequency of a rank sounding together with its CELESTE equals $f + \Delta f/2$. Then, all intervals in a sharp CELESTE rank (i.e. $\Delta f > 0$) doubling their BF slower than once per 1.5 octaves (as in "type d"), are slightly compressed. However, as this deviation amounts to only 0.07 ct per semitone (calculated at A_4), it is not obvious, why organ builders frequently prefer sharp over flat tuning.
One possible answer is that the pipe-scale for the CELESTE pipes is often smaller, compared to the unison rank, which increases their spectral centroid c and thus their brightness.
Listening tests of harmonic complex tones with the same fundamental frequency f revealed that tones with a higher spectral centroid evoke a higher pitch height [10]. By tuning the CELESTE sharp, the brightness rises even a bit further. Note that this only holds true, if the CELESTE has a smaller pipe-scale than the unison rank <u>and</u> if the value $c/f > 1$ (as for STRING and DIAPASON ranks [2]). Since FLUTE ranks possess almost minimum brightness ($c/f \approx 1.0$, Fig. 5, [6]), FLUTE CELESTES can either be tuned flat or sharp.

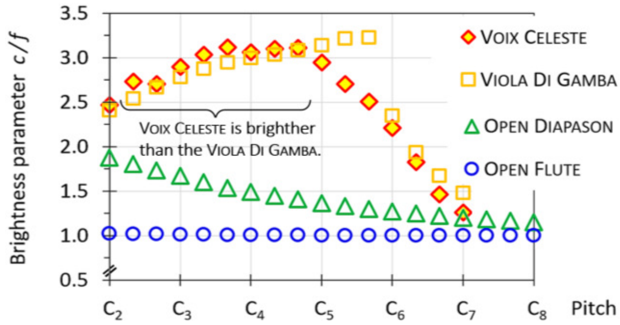

Figure 5. Pitch dependency of c/f for both STRING ranks in this work, compared with a standard OPEN FLUTE [6] and a standard OPEN DIAPASON rank [6].

5.5 Beat frequencies of more than one CELESTE

Some Pipe Organs contain more than one CELESTE rank; in the Accordion we find the 3-rank MUSETTE. If the CELESTE ranks shall match together, the ratios of their BF should be small integers. To illustrate what is possible with a flat, a unison and a sharp rank consider the tuning: $-\Delta f$, 0, $+\Delta f$. Then the BF for any two combined ranks are Δf or $2\,\Delta f$, for all three ranks they are Δf <u>and</u> $2\,\Delta f$. For all other BF ratios the two concurrent modulations of all three ranks together result in irregular sounding beats. Another viable option is to apply an "asymmetrical" detuning like 0, $+\Delta f$, $+2\,\Delta f$.

6. SUMMARY AND CONCLUSIONS

The undulating sound of musical instruments with CELESTE ranks sounds consonant, if $\Delta f <$ JNVF, in accordance with a recently revised theory on sensory consonance. This also explains why of a group of string instruments or a singing choir does not evoke a dissonant impression despite numerous small frequency deviations resulting in beats.
A CELESTE introduces a loudness fluctuation to the sound, however, there is another effect involved, too. As the SPL of the partials within complex tones become time-dependent, the tonal timbre varies in terms of brightness and string quality. The timbre variation is only important for sounds with high harmonic content such as STRING ranks or Accordion reeds. This might explain the sought-after effect provided by a STRING CELESTE rank in the Pipe Organ.
Comparing common tuning practice of CELESTE ranks in Pipe Organs and Accordions as well as taking the JNVF values of the auditory system into account allowed formulating tuning guidelines to obtain pleasant CELESTE ranks. CELESTES are preferably tuned sharp, if the corresponding unison rank owns less brightness. To combine CELESTES, their BF should equal simple ratios like 1:1 or 1:2.

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APPENDIX: TABLES FOR TUNING

The proposed tuning curves depicted in Fig. 4 might be easier to use with some tabulated numbers. Thus, the following tables provide the values for the curves of "type d", in which the BF doubles every 1.5 octaves. The BF and the pitch differences Δp were calculated according to Eqn. (2) and (3). Tab. 1, Tab 2, Tab. 3 and Tab. 4 refer to different quantities of Δf at A_4, namely $\Delta f(A_4) = 1.4$ Hz, 1.7 Hz, 2.0 Hz, and 3.5 Hz, respectively. In these tables, the pitch distances equal 0.75 octaves and thus the Δf values double every second row. Contrary to "type b)" and "type c)", the pitch difference Δp does not remain constant upon pitch.

Table 1. Tuning "type d)" for $\Delta f(A_4) = 1.4$ Hz

Pitch	f in Hz	Δf in Hz	p	Δp in ct
$F\#_2$	92.5	0.49	30	9.3
$D\#_3$	155.6	0.70	39	7.8
C_4	261.6	0.99	48	6.5
$\mathbf{A_4}$	440.0	**1.40**	57	5.5
$F\#_5$	740.0	1.98	66	4.6
$D\#_6$	1244.5	2.80	75	3.9
C_7	2093.0	3.96	84	3.3
A_7	3520.0	5.60	93	2.8

Table 2. Tuning "type d)" for $\Delta f(A_4) = 1.7$ Hz

Pitch	f in Hz	Δf in Hz	p	Δp in ct
$F\#_2$	92.5	0.60	30	11.5
$D\#_3$	155.6	0.85	39	9.5
C_4	261.6	1.20	48	8.0
$\mathbf{A_4}$	440.0	**1.70**	57	6.7
$F\#_5$	740.0	2.40	66	5.6
$D\#_6$	1244.5	3.40	75	4.7
C_7	2093.0	4.81	84	4.0
A_7	3520.0	6.80	93	3.3

Table 3. Tuning "type d)" for $\Delta f(A_4) = 2.0$ Hz

Pitch	f in Hz	Δf in Hz	p	Δp in ct
$F\#_2$	92.5	0.71	30	13.2
$D\#_3$	155.6	1.00	39	11.1
C_4	261.6	1.41	48	9.4
$\mathbf{A_4}$	440.0	**2.00**	57	7.9
$F\#_5$	740.0	2.83	66	6.6
$D\#_6$	1244.5	4.00	75	5.6
C_7	2093.0	5.66	84	4.7
A_7	3520.0	8.00	93	3.9

Table 4. Tuning "type d)" for $\Delta f(A_4) = 3.5$ Hz

Pitch	f in Hz	Δf in Hz	p	Δp in ct
$F\#_2$	92.5	1.24	30	23.1
$D\#_3$	155.6	1.75	39	19.5
C_4	261.6	2.47	48	16.4
$\mathbf{A_4}$	440.0	**3.50**	57	13.8
$F\#_5$	740.0	4.95	66	11.6
$D\#_6$	1244.5	7.00	75	9.7
C_7	2093.0	9.90	84	8.2
A_7	3520.0	14.00	93	6.9