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TRANSMISSIBILITY AND INSULATION EVALUATION INSIDE A MULTI-DEGREES OF FREEDOM SYSTEM

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ABSTRACT

Nowadays electric cars are every day more and more present on the market and a crucial aspect from the customer's point of view is the acoustic comfort of the cabin. For this reason, it is of paramount importance to be able to predict how the excitation coming from the electric engine is transmitted through the various components of the vehicle. This research focuses on how Transfer Path Analysis through the Dynamic Substructuring technique applied to a car assembly can evaluate the frequency dependant insulation behavior of the Structure-borne noise between the source and the final receiver. The final goal is to understand the role in terms of dynamic stiffness of the various components in reducing the transmissibility at particular frequency ranges.

Keywords: *Transmissibility, Insulation, Dynamic Stiffness*

1. INTRODUCTION

This paper puts its focus on the evaluation of the insulation of a multi-DOFs system composed by masses m_i, stiffnesses k_i and dampings c_i. With this work we want to understand which structural parameters are responsible for characteristical form of the insulation curve over frequency and ones only playing a role in the local dynamics of the system and therefore not affecting the curve characteristics. In most of the industrial contexts we end up with very complex systems all connected to each other which are hard to understand because of their complexity. An example could be the one shown in Fig. 1 where we have the simplified representation of an electric drive-unit connected to a (rear) axle carrier connected to the main (trimmed) body of the car. One can measure or simulate the FRFs of the systems's substructures and then combine them using Dynamic Substructuring (see for instance [1–4]). The goal of those complex studies is to asses the final noise level coming from the contribution of each sub-component of the assembly. However it's rather difficult to establish how a certain element is contributing to the signal reduction through the system.

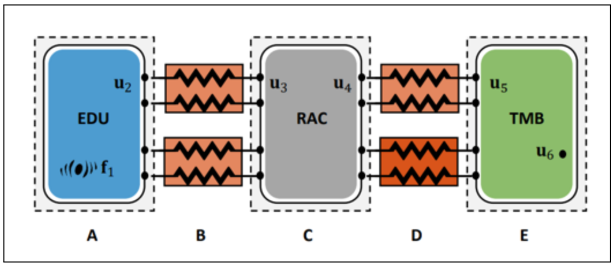

Figure 1: Simplified representation of a car assembly: engine, rear axle carrier, car body

The idea of this study is to represent each of those complex subsystems with very simple elements and start assessing general rules about a plausible behavior of the insulation between two points. Some properties and definitions about transmissibility can be found in [5–7]. With a very simplified example of a multi-DOFs system it is indeed easier to understand the system's behaviour and derive some rules of thumb about the most relevant parameters that would decrease or increase the transmissibility.

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2. GENERAL DEFINITIONS

For a single DOF system characterised by a mass m and connected to the ground with a stiffness k, a damping c and excited with a force f causing a displacement x we can define the following quantities:

- compliance as the ratio between displacement and force $H(j\omega) = X/F$
- mobility as the ratio between velocity and force $V(j\omega) = \dot{X}/F$
- accelerance as the ratio between acceleration and force $Y(j\omega) = \ddot{X}/F$
- dynamic stiffness as the inverse of the compliance and ratio between force and displacement $Z(j\omega) = F/X$

where j is the imaginary complex number and ω is the angular frequency.

Compared to the other quantities, like shown in Fig. 2, it is particularly convenient to plot the dynamic stiffness curves in a double-logarithmic graph. Here we can immediately highlight the system's stiffnesses as horizontal lines, the masses as oblique lines, which we refer as mass-lines, and the systems resonances and anti-resonances as vertical lines in correspondence of the crossing of the previous two. As a reference, the reader should keep in mind for later discussion that mass-lines grow with the power of 2 of the angular frequency ω.

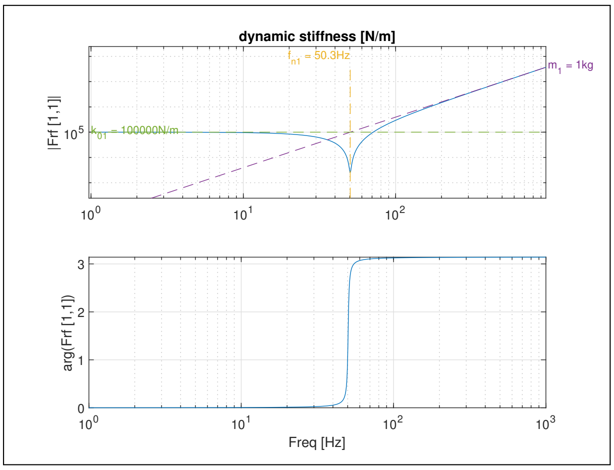

Figure 2: Dynamic stiffness of a 1-DOF system with phase

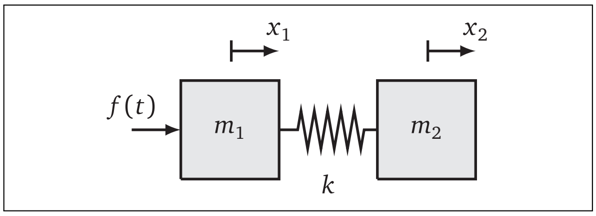

Figure 3: representation of a two mass system in free-free boundary conditions

2.1 Insulation definition

Let's now consider a 2-DOF system like in Fig. 3. Here the system's equation can be written as:

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (1)$$

Where X_i is the forced response of the i^{th} DOF, F_i is the excitation acting on the i^{th} DOF and H_{is} is the frequency response function between the s^{th} excited DOF and the i^{th} DOF where we measure the response.

Here, we can define the insulation between the DOF 1 and the DOF 2 as ratio of responses like:

$$G_{I,1\rightarrow 2}(j\omega) = \frac{X_1}{X_2} = \frac{H_{11}(j\omega)F_1 + H_{12}(j\omega)F_2}{H_{21}(j\omega)F_1 + H_{22}(j\omega)F_2} \quad (2)$$

In case of a single excitation ($F_2 = 0$) this expression reduces to the ratio of two FRFs and all the other terms simplify:

$$G_{I,1\rightarrow 2}(j\omega) = \frac{X_1}{X_2} = \frac{H_{11}(j\omega)}{H_{21}(j\omega)} \quad (3)$$

It is indeed possible to express the insulation between two poins as the ration of the forced responses or as the ratio of FRFs.

2.2 Forced response

To evaluate the forced response of a n-DOF system we need to solve:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}(t) \quad (4)$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are respectively the $n \times n$ mass, damping and stiffness matrices, while $\boldsymbol{u}$ and $\boldsymbol{f}(t)$ are the $n \times 1$ unknown displacement vector and the force excitation vector.

Following the state-space assumption we can write $\boldsymbol{y} = \begin{Bmatrix} \boldsymbol{u} \\ \dot{\boldsymbol{u}} \end{Bmatrix}$ and $\dot{\boldsymbol{y}} = \begin{Bmatrix} \dot{\boldsymbol{u}} \\ \ddot{\boldsymbol{u}} \end{Bmatrix}$ and obtain a $2n$ equation system in $2n$ unknowns which would look like

$$\begin{bmatrix} \boldsymbol{C} & \boldsymbol{M} \\ \boldsymbol{M} & \boldsymbol{0} \end{bmatrix} \dot{\boldsymbol{y}} + \begin{bmatrix} \boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M} \end{bmatrix} \boldsymbol{y} = \begin{Bmatrix} \boldsymbol{f}(t) \\ \boldsymbol{0} \end{Bmatrix} \tag{5}$$

here we can write Eqn. (5) in a more compact form and obtain Eqn. (6):

$$\boldsymbol{A}\dot{\boldsymbol{y}} + \boldsymbol{B}\boldsymbol{y} = \boldsymbol{f} \tag{6}$$

which we can numerically solve to obtain the unknown $2n \times 1$ vector of displacements and therefore $\boldsymbol{u}$.

2.3 FRF definition

The frequency response function (FRF) is defined as the ratio between response and excitation as function of excitation frequency. For a n-DOF system, we can define the FRF between the s^{th} excited DOF and the i^{th} DOF where the response is measured, as a superposition of eigenmodes like in Eqn. (7)

$$H_{is}(\omega) = \sum_{r=1}^{n} \left(\frac{\phi_{ir}\phi_{sr}}{j\omega - \lambda_r} + \frac{\phi_{ir}^* \phi_{sr}^*}{j\omega - \lambda_r^*} \right) \tag{7}$$

Where ϕ_{ir} is the i^{th} component of the r^{th} eigenvector, λ_r is the r^{th} eigenvalue and the symbol $*$ represents the complex conjugate number of a certain value.

3. CURVE TREND

In this chapter, after defining the insulation, we want to have a closer look at the trend of the terms inside Eqn. (2) and at their frequency dependency.

3.1 Insulation evaluation from forced responses ratio

The first way to determine the insulation from one DOF to the other is by ratio of forced responses. This operation is particularly useful and straightforward:

- during measurements, where we simply divide the signal acquired by two (or more) accelerometers positioned on the DOFs we want to measure;
- when we have more than one force excitation that would lead to an expression similar to Eqn. (2) but perhaps even more complicated.

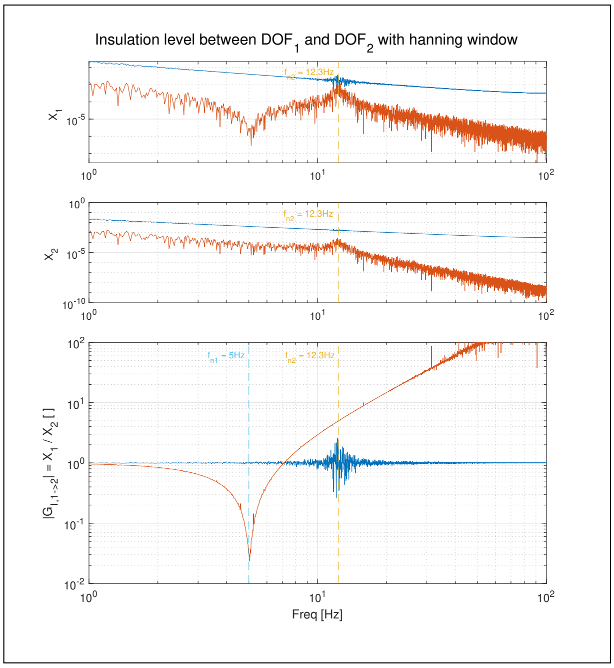

Figure 4: Insulation between DOF 1 and 2

After evaluating the time-domain forced responses of a system (like in Eqn. (6)), we can perform the Fourier Transform to obtain the forced responses in frequency domain. Then, according with Eqn. (3), we get to the insulation between two DOFs from the ratio of two forced responses in frequency domain. Fig. 4 shows an example of such operation applied to the 2-DOF system of Fig. 3.

Here one can spot immediately that, without an appropriate window on the time-domain signals of the response, the results are completely unreadable (see the blue curves inside Fig. 4). While, after windowing the response signals, we manage to perform a better FFT having made our signal periodic, and the resulting insulation (orange curve) is now clear (see also a comparison with the later discussed Fig. 7).

3.2 Insulation evaluation from FRF ratio

Another way to determine the insulation, in case the system's forced excitation is unique, is from the ratio of FRFs.

3.2.1 Driving dynamic stiffness behavior

Let's now have a closer look at the numerator of Eqn. (3) which is the driving point receptance. Like explained before, we can invert this quantity and represent it in double logarithmic graph to highlight the relations with the mass and stiffness terms. Fig. 5 shows the trend of such dynamic stiffness for a system like Fig. 3. In this example we chose $m_1 = 2kg$, $m_2 = 10kg$, $k_{12} = 10N/mm$.

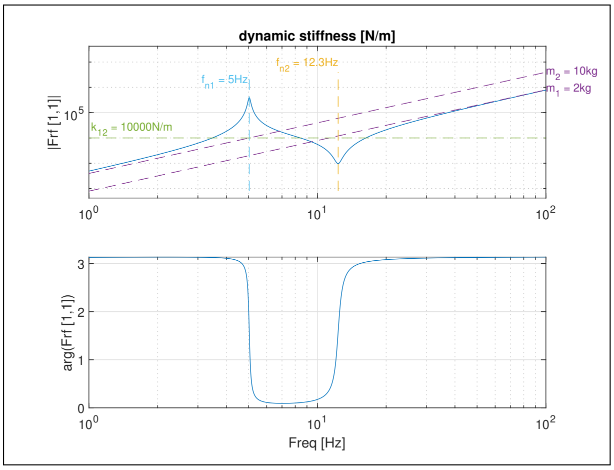

Figure 5: Driving point dynamic stiffness of a 2-DOF system with phase

What we can observe, is that the driving dynamic stiffness of the body 1 is at lower frequencies following the mass line of the heavier body m_2 until a certain anti-resonance. Starting from that frequency, following the stiffness level of k_{12}, the decoupling begins, which means the two bodies are moving independently one with respect to the the other. Here, the dynamic stiffness decreases and finds its minimum in correspondence to the resonance peak and then follows its own mass-line m_1. The driving dynamic stiffness of body 2 would just follow for the entire frequency range the mass-line m_2 with the only disturbance of a resonance and anti-resonance peak. It is interesting to note that the decoupling begins with the anti-resonance and it completes with the resonance, after that frequency the two bodies can be considered dynamically decoupled.

3.2.2 Transfer dynamic stiffness behavior

For the transfer dynamic stiffness term the situation is a little bit different (see Fig. 6): here the dynamic stiffness Z_{21} follows the mass line m_2 and only detaches from it in correspondence of the natural frequency. After this local minimum, being an FRF a superposition of eigenmodes, it starts increasing with the power of 4 of the angular frequency ω and therefore the curve trend is steeper than one of the mass-line m_2

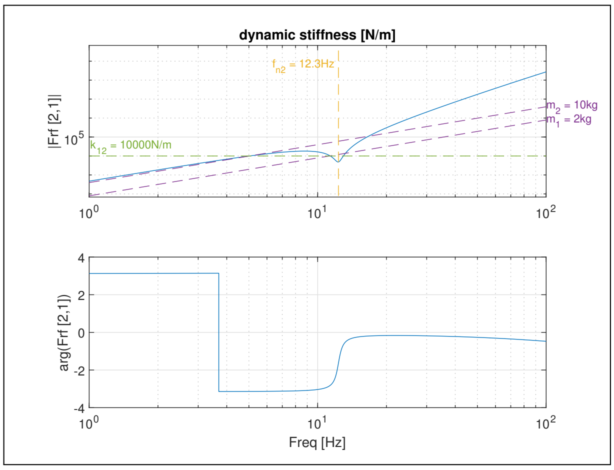

Figure 6: Transfer dynamic stiffness of a 2-DOF system with phase

3.2.3 Insulation curve

Following the definition in Eqn. (3) it is possible to obtain the insulation between two DOF as a ratio of FRFs, more in particular as a ratio of dynamic stiffnesses. Such result is proposed in Fig. 7, here it is interesting to note that

- in the low frequency range there's no insulation since the two above mentioned curves are both following the mass line of m_2;
- the insulation decreases in correspondence of the first anti-resonance and after that frequency starts increasing, which means that in the high frequency range the body 2 is decoupled from the body 1 and its displacement is lower than the displacement of the body 1;
- the resonance doesn't play any role in the determination of the insulation curve.

3.3 Application: insulation of a 3-DOF system

Here an application representing the system in Fig. 1 using 3 point masses, 2 springs and 2 dampers is briefly shown. Fig. 8 is showing the insulation curve between the engine and the rear axle carrier of this 3-mass system.

The first two graphs show respectively the dynamic stiffnesses Z_{21} and Z_{11}, here like before it is interesting to

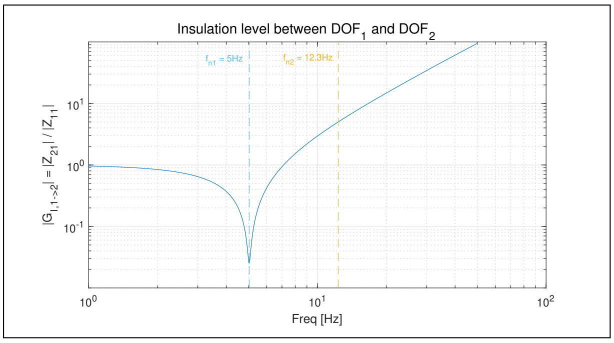

Figure 7: Insulation between DOF 1 and 2 of a 2-DOF system

note how the crossing between certain static stiffness and mass lines determines the position of certain resonances or anti-resonances. Therefore, one can immediately understand which system's parameter should be changed when wanting to shift the anti-resonances and obtain a particular level of insulation in the frequency range of interest.

4. CONCLUSIONS AND OUTLOOKS

This paper presented a quick overview over the concept of transmissibility and insulation. Firstly the concept of insulation and transmissibility was introduced, then two different methods for analysing such concept were presented, either from a ratio of forced responses or from a ratio of FRFs. The first approach emulates an experiment: starting from the dynamic properties of a system, we introduced a force excitation and then we numerically solved the system to obtain the forced responses in time domain of all the DOFs. Being such procedure more straightforward and easier to implement inside an experimental context (we only need to do the ratio between 2 acquired signals) there are few drawbacks like the need of a proper windowing on the time-domain signals.

Being the insulation a property of the system and not of the excitation, the second approach obtains the same results from a ratio of FRFs. Knowing the mass, stiffness and damping properties of a system allows us to solve the homogeneous equation of Eqn. (4) and write the systems's FRFs as a superposition of eigenmodes. The main advantage is that plotting such FRFs as dynamic stiffnesses in a double logarithmic scale gives an immediate feeling of which parameters inside our multi-DOF system play a relevant role on the determination of an insulation curve.

It is in fact the trend of the dynamic stiffness curves

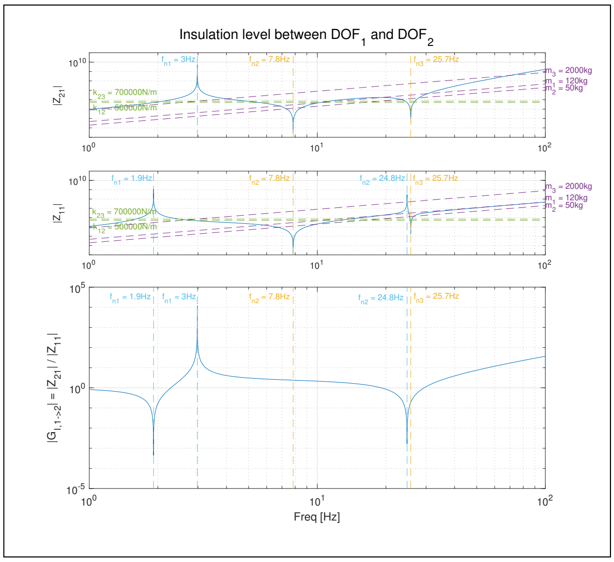

Figure 8: Insulation between DOF 1 and 2 of a 3-DOF system representing a car assembly

that really determines the level and the peaks of an insulation curve. Here the reader can then derive himself some general rules of thumb that would help him to better understand dynamic stiffness curves of complex systems. Also one can get an immediate feeling of which paramenters or components are worth our engineering effort when we want to reduce the transmissibility in a certain frequency range.

5. ACKNOWLEDGMENTS

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