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INFORMED SOURCE SEPARATION FOR TURBOFAN BROADBAND NOISE USING NON-NEGATIVE MATRIX FACTORIZATION

Sarah ROUAL[1,2*] **Claude SENSIAU**[1] **Gilles CHARDON**[2]

[1] Safran SA, Safran Tech, Pôle Energie et Propulsion, 78117, Châteaufort, France
[2] Université Paris-Saclay, CNRS, CentraleSupélec, Laboratoire des signaux et systèmes, 91190, Gif-sur-Yvette, France.

ABSTRACT

The rising concern in the aircraft industry regarding engine noise has led to the use of source separation techniques to target future noise reduction efforts. This paper investigates the use of Non-negative Matrix Factorization (NMF) as an automatic source separation method for engine noise, using an array of microphones. Turbofan broadband noise is a complex mixture of sounds generated by individual sources which have a specific spectrum and directivity. The objective of this study is to assess the separation performance of the method and the relevance of additional expert knowledge in the form of a regularization term. The method was applied to a set of simulated engine noises at a certification flight point, and the resulting separated sources were analyzed for their spectral and spatial characteristics. Results indicate that NMF can effectively separate the individual sources of engine noise, even when the sources have similar characteristics. In the case of low power sources, information is missing and regularization significantly improved the separation performance. NMF appears to be a promising method for source separation of turbofan broadband noise. Further validation should be obtained from a more complex corpus with better spectral resolution.

Keywords: *NMF, Source Separation, Turbomachinery*

**Corresponding author*: *sarah.roual@centralesupelec.fr.*

1. INTRODUCTION

The reduction of turbofan noise has become a priority for aircraft engine manufacturers due to the rise in air traffic and more stringent noise pollution regulations. While Computational Aeroacoustics algorithms (CAA) [1] are commonly used to estimate noise sources, they can be computationally expensive for small wavelengths. An alternative approach is to use microphone arrays placed around the turbofan for acoustic testing. Various source localization methods [2] are available, including deconvolution techniques, inverse methods such as SODIX [3]. These methods are based on fitting the data to acoustical sources, and to this end necessitates the propagation model between the sources and the microphones. Data-driven Blind Source Separation (BSS) techniques such as ICA (Independent Component Analysis) [4] and NMF (Nonnegative Matrix Factorization) [5] can perform source separation without knowledge of the propagation model. ICA is based on the assumption that the sources to be separated are independent. However, in double-flow turbofan, the upstream and downstream components of the fan are correlated sources that cannot be separated using ICA. NMF only assumes non-negative components and purely additive mixing [6], and has recently been applied in wind turbine noise estimation [7]. This article proposes applying NMF with regularization to separate four sources (inlet and exhaust fan components, core and jet) within the overall noise of a simulated turbofan, taking into account prior knowledge of the spectral shape of some components.

2. BACKGROUND

In a typical experiment, a constant-rate turbofan noise is measured with an array of N microphones spanning angles 20-160 degrees. Acoustical power is estimated across F frequency bands for each microphone, and the resulting powers were assembled to create the matrix $\mathbf{V} \in \mathbb{R}_+^{F\times N}$. The columns of $\mathbf{V}$, denoted as $\mathbf{v}_n$, represent the spectral mixture of the K sources present in the signal captured by the n-th microphone. These sources comprise the four sources in the turbofan, as well as a source for measurement noise.

Assuming that the spectrum of each source is the same for all microphones, then the matrix $\mathbf{V}$ can be factorized as

$$\mathbf{V} \approx \mathbf{WH} \tag{1}$$

where $\mathbf{H} \in \mathbb{R}_+^{K\times N}$ is the directivity matrix of each source, assumed to be frequency-independent. The columns of matrix $\mathbf{W} \in \mathbb{R}_+^{F\times K}$ hold spectra of each source, normalized with $\sum_{f=1}^{F} w_{fk} = 1$ to preserve amplitude in $\mathbf{H}$. It should be noted that $\mathbf{V}$ and $\mathbf{WH}$ are not necessarily equal, as the coefficients of $\mathbf{V}$ are estimated power spectral densities (PSDs), while coefficients of $\mathbf{WH}$ are theoretical PSDs, derived from the assumed spectral and directional characteristics of the sources.

2.1 Non-negative Matrix Factorization

Without further assumption, the factorization of Eq. (1) is impossible to recover. In NMF, the additional knowledge that the coefficients of $\mathbf{W}$ and $\mathbf{H}$ are nonnegative is exploited. The problem to be solved is to minimize the cost function:

$$\begin{aligned}(\hat{\mathbf{W}},\hat{\mathbf{H}}) &= \underset{\mathbf{W},\mathbf{H}}{\operatorname{argmin}}\ D(\mathbf{V}|\mathbf{WH}).\\ \text{subject to} &\quad \mathbf{W} \geq 0, \mathbf{H} \geq 0\end{aligned} \tag{2}$$

Equation (2) measures the fit between the measured field $\mathbf{V}$ and the model, defined as the Itakura-Saito (IS) divergence :

$$D(\mathbf{V}|\mathbf{WH}) = \sum_{f=1}^{F}\sum_{n=1}^{N} d_{\mathrm{IS}}(v_{fn}|[\mathbf{WH}]_{fn}). \tag{3}$$

In NMF, the IS-divergence is frequently utilized since it serves as a Maximum Likelihood Estimation criterion in the case of fitting theoretical PSDs to the estimated PSDs, assuming that the sources are Gaussian. This interpretation is demonstrated in [6]. The IS-divergence is defined between two strictly positive scalars:

$$d_{\mathrm{IS}}(x,y) = \frac{x}{y} - \log\frac{x}{y} - 1. \tag{4}$$

The Itakura-Saito divergence also possesses a scale-invariant property, which enables it to effectively handle acoustic signals with high dynamic range.

To solve the minimization problem of NMF as defined in (2), the matrices $\mathbf{W}$ and $\mathbf{H}$ are updated iteratively. Among various available algorithms, the Multiplicative Update [6] is selected because it guarantees convergence and inherently maintains non-negativity :

$$\begin{aligned}\mathbf{H} &\leftarrow \mathbf{H}\cdot\frac{\mathbf{W}^T[(\mathbf{WH})^{\cdot -2}\cdot\mathbf{V}]}{\mathbf{W}^T[(\mathbf{WH})^{\cdot -1}]}\\ \mathbf{W} &\leftarrow \mathbf{W}\cdot\frac{[(\mathbf{WH})^{\cdot -2}\cdot\mathbf{V}]\mathbf{H}^T}{[(\mathbf{WH})^{\cdot -1}]\mathbf{H}^T)},\end{aligned} \tag{5}$$

where $\cdot$ denotes the Hadamard product.

2.2 Regularization term

The Itakura-Saito divergence is effective in capturing significant variations between frequency or microphone channels, but may not be optimal for weaker sources. Regularization techniques, incorporating prior knowledge of a source's spectral shape, can enhance system performance. In this study, the well-defined and predictable jet source was used to impose a constraint on the cost function (6). An analytical model [8] was employed to generate a reference for the regularization term.

The regularized optimization problem is

$$\begin{aligned}(\hat{\mathbf{W}},\hat{\mathbf{H}}) &= \underset{\mathbf{W},\mathbf{H}}{\operatorname{argmin}}(D(\mathbf{V}|\mathbf{WH}) + R_{\boldsymbol{\lambda}}(\mathbf{W},\overline{\mathbf{W}})).\\ &\text{subject to } \mathbf{W}\geq 0, \mathbf{H}\geq 0.\end{aligned} \tag{6}$$

The regularization term

$$R_{\boldsymbol{\lambda}}(\mathbf{W},\overline{\mathbf{W}}) = \frac{1}{2}\sum_{k=1}^{K}\lambda_k\sum_{f=1}^{F}(w_{fk}-\overline{w}_{fk})^2 \tag{7}$$

quantifies the distance between the estimated $\mathbf{W}$ and the known prior template $\overline{\mathbf{W}}$, and is weighted by a vector $\boldsymbol{\lambda}$ that indicates the importance of the regularization term for each source.

The iterations of $\mathbf{H}$ remain unchanged. The same reasoning as in [6] is employed to derive the iterations associated with the cost function (6) :

$$\mathbf{W} \leftarrow \mathbf{W}\cdot\frac{[(\mathbf{WH})^{\cdot -2}\cdot\mathbf{V}]\mathbf{H}^T + \overline{\mathbf{W}}^T\mathrm{diag}(\boldsymbol{\lambda})}{[(\mathbf{WH})^{\cdot -1}]\mathbf{H}^T + \mathbf{W}^T\mathrm{diag}(\boldsymbol{\lambda})} \tag{8}$$

3. EXPERIMENT

3.1 Design of the experiment

In this study, NMF is applied to a simulated corpus of broadband noise component mixtures, under the assumption that predictable tonal components can be separated beforehand [9]. Spectral signatures and directivities of the different components are simulated using [8]. The aircraft is at Approach which corresponds to a noise certification point.

After obtaining the factorized matrices $\mathbf{W}$ and $\mathbf{H}$ using NMF, the field matrix $\hat{\mathbf{V}}^{(k)}$ for each source k can be computed by combining the k-th column of $\mathbf{W}$ with the k-th row of $\mathbf{H}$. The experimental protocol described Fig. 1 consists of estimating the field of a source k, denoted by $\hat{\mathbf{V}}^k$, and comparing it to the true field of the same source, denoted by $\mathbf{V}^k$, for five different engines.

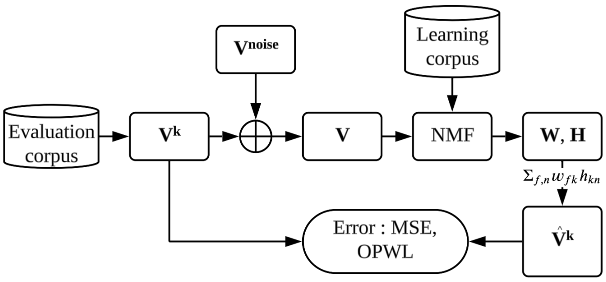

Figure 1: Block diagram summarizing the experiment.

3.2 Modalities

The study utilizes data from five dual-flow turbofan engines, each measured using $N = 15$ microphones and $F = 24$ frequency bins. Prior to analysis, the data is preprocessed with omnidirectional white noise at a signal-to-noise ratio of 13 dB. In this study, the regularization is applied on the jet source only, with a regularization coefficient λ_{jet}, varied systematically from 0 to 30 with a step of 1. The other coefficients of $\boldsymbol{\lambda}$ are set at 0.

3.3 Metrics

To evaluate the separation performance, two complementary metrics are implemented :

The Mean Squared Error (MSE) quantifies the accuracy of the reconstructed fields in terms of their spectral and directivity shapes by measuring the discrepancy between the real power v^k_{fn} and the estimated power $w_{fk}h_{kn}$ of source k at frequency f and point n :

$$\text{MSE}(\mathbf{V}^k, \hat{\mathbf{V}}^k) = \frac{1}{NF} \sum_{n=1}^{N} \sum_{f=1}^{F} (v^k_{fn} - w_{fk}h_{kn})^2 \quad (9)$$

However, this penalizes errors in the lower power regions of the spectrum, which contribute less to the actual sound level of the source.

The Overall Power Level (OPWL) of each source indicates the total emitted power and ensures the accuracy of the represented sound level. In acoustics, the primary goal is to successfully estimate the true level of each source within a measured field.

4. RESULTS

The following section presents the outcomes of applying our method to the data of 5 turbofan engines at approach rate. In this phase, distinguishing the jet noise from the core noise is challenging due to their comparable spatial and spectral characteristics, with the core noise being slightly louder. The spectral and directional characteristics of each source are displayed in Fig. 2.

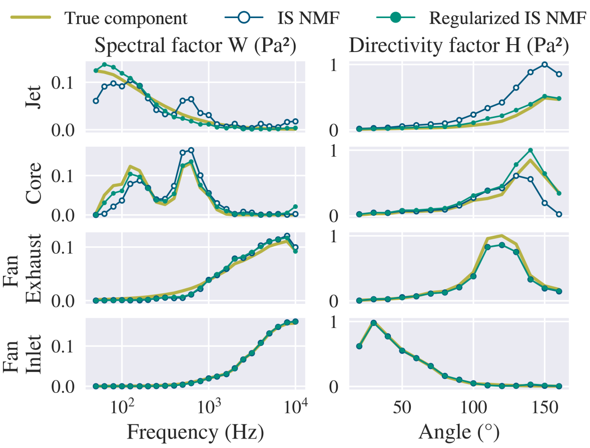

Figure 2: Comparative results of IS-NMF method with (coef. 10) and without regularization on a representative case

In the results, the jet source exhibited core noise characteristics in its spectral profile, including two local maxima on the W component and an unusually high level at 150°. Applying regularization effectively separated the mixed jet and core components, showcasing its corrective capability, while incorporating weighted information enhanced adaptability and improved performance with partial source knowledge.

The sensitivity analysis in Figure 3 shows that the MSE error decreases for regularization coefficients above 5, with 10 being a suitable choice. No instability was observed within the range of 5 to 30, but poor templates can lead to increased error at higher values.

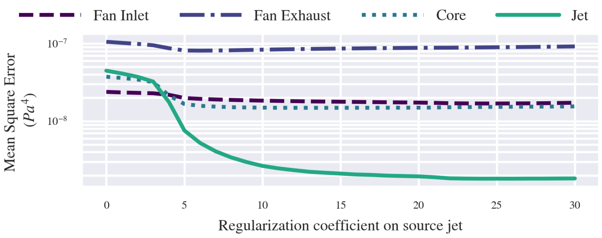

Figure 3: Study of sensitivity on the regularization coefficient.

The evaluation of OPWL errors for each source, as shown in Table [tab:OPWL], demonstrates the impact of regulrization.

Table 1: Source OPWL error (dB)

Source	Ref.	IS-MNF	Reg. IS-NMF
Jet	77	3.46	0.88
Core	80	1.1	0.7
Fan Inlet	91	0.04	0.02
Fan Exhaust	91	0.46	0.40

While all sources exhibited errors below 1 dB, the jet component notably experienced a significant 2.5 dB reduction in error, showcasing the effectiveness of the constraint in improving source estimation.

5. CONCLUSION

In conclusion, this study assessed NMF's performance in separating broadband noise sources in aircraft turbofan engines. The proposed method showed satisfactory results by leveraging prior knowledge of spectral shape. However, accurately predicting the spectrum of masked sources with inherent variability, especially when the core is masked, remains challenging. Further research is needed to address this issue, specifically by investigating and challenging the assumption of constant directivity across frequencies.

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