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EIGENVALUE ASSIGNMENT OF A NONLINEAR ELECTROMAGNETIC SYSTEM BY THE RECEPTANCE METHOD: THEORY AND EXPERIMENT

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ABSTRACT

This paper presents experimental and numerical active vibration control of a nonlinear system for eigenvalue assignment and active damping, using receptance method. A nonlinear electromagnetic setup consist of two identical coils, magnets and a cantilever beam has been designed and built for this purpose. By varying electrical current and distance between the two coils, nonlinear cubic stiffness with various strengths obtained. Active damping control performed experimentally in order to validate the performance of control method on the designed nonlinear setup.

For eigenvalue assignment, first, receptance of the open loop system is experimentally measured by varying electrical currents and the coil distance to define the effect of parameter variations on the system's FRF. Next step involved developing an iterative Sherman-Morrison formula to obtain the feedback control gains for the eigenvalue assignment. This step was challenging due to the amplitude dependence and also instability and jump phenomena that exist in the nonlinear FRFs. Increasing excitation level causes strong nonlinear effects in closed loop receptance, as shown by simulation.

In the end, a stability analysis is conducted and discussed to prevent instability during the eigenvalue assignment process. Considered method effectively assigned poles and ensured the stability of the system under different conditions.

Keywords: *Experimental Pole Placement, Active Vibration Control, Nonlinear Electromagnetic System*

1. INTRODUCTION

This is not practical to assume that all systems can be represented by linear models in their mathematical description, as engineering systems often demonstrate nonlinear characteristics to some extent [1]. The behavior of nonlinear systems can be described by nonlinear differential equations where the principle of superposition is not applicable. Various forms of nonlinearities such as cubic stiffness, coulomb damping, piecewise stiffness, and friction-controlled backlash can be found in nonlinear systems [2-4]. Due to the dependence of nonlinear setups' behavior on the amplitude of the input signal, linear concepts such as linear control strategies, frequency response functions, and Nyquist stability analysis are not applicable to nonlinear structures [5], Thus, in order to define these systems, describing function method can be used. This method determines the ratio of amplitudes and phase angle between the fundamental harmonic components of the input and output sinusoid [6] and it is helpful in analyzing the stability of the nonlinear system [7].

Active control is a technique that involves applying external force to regulate the response of a structure, and is effective at suppressing vibrations, adapts to different conditions and has a wide range of applications. [9].The design of control systems is crucial for active control, which involves using sensor information to drive actuators. Inertial actuators can use velocity feedback, allowing the actuator to behave like an attached mass and reduce vibration [8], [10], and [11].

Active damping control is a technique that uses data on accelerations or forces to influence the dynamic response of mechanical structures. It is particularly effective for

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preventing resonance in systems with loads that change over time and have frequencies close to the natural frequencies of the structure. [11].

Relocating the eigenvalues of a nonlinear system is another challenging aim for vibration control because of the following reasons. Firstly, nonlinear systems often have an endless number of poles in response to harmonic input, also, controlling and stabilizing nonlinear systems is more complex due to bifurcation, chaos, limit-cycle oscillations, and jump effects. Finally, mathematical models of nonlinear structures are more complex, and in some cases, numerical methods are necessary for eigenvalue assignment.

Ram and Mottershead [12] developed the Sherman-Morrison receptance technique for linear systems, in which the system's eigenvalues assign based on measured receptances instead of modeling the mass, damping, and stiffness matrices. This approach offers advantages over traditional state-space methods. In this technique poles and zeros of the system can be determined without evaluating the system's dynamics characteristics, there is no need to estimate unmeasured states using an observer or to use model reduction. Also the receptance method is applicable to any input-output measured data [13].

In this paper the receptance method has been extended in order to control the poles of a cantilever beam subject to nonlinear electromagnetic field [14]. First an experimental setup has been designed and the mathematical model of the electromagnetic beam has been extracted, then hammer test has been performed and the impact of increasing the electric current and the distance between the coils and magnets on the nonlinearity of the system has been shown experimentally. In the further step, active damping theory and the receptance method and iterative solution has been investigated on the linear and nonlinear electromagnetic system in order to perform pole placement.

2. MATHEMATICAL MODEL OF THE EXPERIMENTAL SETUP

The experimental setup designed for this study consists of a cantilever beam, with its free end connected to a pair of identical magnets. Additionally, a pair of identical coils is mounted next to the magnets to create a magnetic field and having access to nonlinear stiffness with creating cubic stiffness for the system. The advantage of the designed setup is that it allows for variation in the strength of the nonlinear stiffness by adjusting the distance between the two coils and the electrical current passing through them. In addition, the electromagnetic part of the setup can be moved along the length of the beam, allowing for the measurement of the hammer test on the beam at various points along its length. Fig. 1 shows the designed setup used in this study, which allows for the measurement of various parameters related to the behavior of the cantilever beam with nonlinear stiffness.

Figure 1. Nonlinear designed setup consists of cantilever beam, a pair of identical coils and magnets, a load speaker, and an accelerometer.

The pair of Mundorf copper coils used in this setup are of type L71-3,30, with a maximum current rating of 3 Amp based on their datasheet, and the disc magnets are neodymium (type F359, N42). In order to derive a mathematical model of the system, the equation of motion for a general linear system can be considered as Eqn. (1), in which the linear mass, damping, and stiffness terms are respectively described by m, c, and k_1, and an excitation force is assumed to be represented by $p(t)$. Since aim of this study is to control the only one mode of the beam, the model is considered to be single degree-of-freedom for simplicity. However, multi degree-of-freedom will be considered in future work. Eqn. (1) shows mathematical model of an electromagnetic cantilever beam.

$$m\ddot{y}(t) + c\dot{y}(t) + k_1 y(t) + \frac{dU(y)}{dy} = p(t) \qquad (1)$$

Term of $U(y)$ in Eqn. (1) shows the potential energy of the electromechanical system, and the part of $\frac{dU(y)}{dy}$ is a function of the electrical current (I_c), parameters of the coils and magnets as defined by Eqn. (2).

$$\frac{dU(y)}{dy} = H_{c1} I_c y + H_{c2} I_c y^3 \qquad (2)$$

It is possible to calculate H_{c1} and H_{c2}, as mentioned in Eqn. (2), using the Eqn. (3) as described in [14].

$$H_{c1} = \left(\frac{-2(\frac{3}{2}\mu\mu_0 r_c^2 N)}{(r_c^2 + h^2)^{\frac{5}{2}}} + \frac{10(\frac{3}{2}\mu\mu_0 r_c^2 N)h^2}{(r_c^2 + h^2)^{\frac{7}{2}}} \right) \qquad (3)$$

$$H_{c2} = \left(\frac{5(\frac{3}{2}\mu\mu_0 r_c^2 N)}{(r_c^2 + h^2)^{\frac{7}{2}}} - \frac{70(\frac{3}{2}\mu\mu_0 r_c^2 N)h^2}{(r_c^2 + h^2)^{\frac{9}{2}}} + \frac{105(\frac{3}{2}\mu\mu_0 r_c^2 N)h^4}{(r_c^2 + h^2)^{\frac{11}{2}}} \right)$$

The parameters in Eqn. (3) are introduced in the table 1. Eqn. (4) can be used to represent the linear and nonlinear equivalent stiffness of the system by utilizing the information provided in Eqn. (2) and (3).

$$k_{linear} = k_1 + H_{c1} I_c \qquad k_{Nonlinear} = H_{c2} I_c \tag{4}$$

Eqn. (5) can be used to express the describing function of the aforementioned system.

$$G_D(I_c, y) = H_{c2} I_c y^3 \tag{5}$$

By separating linear and nonlinear stiffness, the equation of motion in Eqn. (1) can be rewritten as Eqn. (6).

$$m\ddot{y}(t) + c\dot{y}(t) + k_{linear} y(t) + k_{Nonlinear} y^3 = p(t) \tag{6}$$

This equation represents an open-loop Duffing oscillator expression with cubic stiffness in the time domain. Tab. 1 displays the mechanical properties of the designed system and the numerical values used for the simulation study [14].

Table. 1 Mechanical properties of designed setup [14]

Property	Value	Units
Young's Modulus of the Beam	70	GPa
Area moment of inertia I_m	66×10^{-12}	m^4
Mass of Beam m_b	0.034	Kg
Length, Width, Thickness of Beam l_b, b_b, t_b	0.54, 0.01, 0.	m
Mean Radius of Coils r_c	0.0155	m
Magnetic Dipole Moment μ	3.08	Am^2
Permeability of Free Space μ_0	$4\pi \times 10^{-7}$	NA^{-2}
Number of Turns in Coils N	485	
Total Mass m	0.104	Kg
Static Stiffness k_{linear}	32.84	Nm^{-1}

3. RECEPTANCE METHOD FOR ACTIVE DAMPING AND POLE PLACEMENT OF THE NONLINEAR SYSTEM

The feedback control force of $u(t)$ is a linear combination of the control gains multiplied by velocity and displacement, as written in Eqn. (7).

$$u(t) = -g_v \dot{y}(t) - g_d y(t) \tag{7}$$

If g_v and g_d are defined as velocity and displacement feedback gains respectively, the equation for the closed-loop system can be obtained by adding the feedback control force to the open-loop system as in Eqn. (8).

$$m\ddot{y}(t) + c\dot{y}(t) + k_{linear} y(t) + k_{Nonlinear} y^3 = p(t) + u(t) \tag{8}$$

Feedback control adds a control force to nonlinear systems for desired pole values and stable performance with reducing the vibration. This enables active damping and pole placement, expressed by combining Eqn. (7) and (8) as Eqn. (9) for precise control of system dynamics.

$$m\ddot{y}(t) + (c + g_v)\dot{y}(t) + (k_{linear} + g_d)y(t) + (k_{Nonlinear})y(t)^3 = p(t) \tag{9}$$

After taking the Fourier transform of Eqn. (9), in order to extract and plot displacement amplitude, harmonic balance method has been used with considering harmonic assumptions for input and output as $p = P\sin(\omega t - \varphi)$ and $y = Y\sin(\omega t)$ respectively.

$$m\omega^2 Y \sin(\omega t) + (c)Y\omega\cos(\omega t) + (k_{linear})Y\sin(\omega t) + k_{Nonlinear} Y^3 \sin(\omega t)^3 = P\sin(\omega t - \varphi) \tag{10}$$

Through the equating the coefficients of $\cos(\omega t)$and $\sin(\omega t)$ in Eqn. (10), two equations can be obtained, that addition of the squares of extracted equations can define as Eqn. (11).

$$\frac{9}{6} k_{Nonlinear}^2 Y^6 + \frac{3}{2} k_{Nonlinear}(k_{linear} - m\omega^2)Y^4 + \left[(k_{linear} - m\omega^2)^2 + \left(\omega(c)\right)^2\right] Y^2 - P^2 = 0 \tag{11}$$

Also in order to extract the closed-loop FRF, Eqn. (10) can be obtained after taking the Fourier transform of Eqn. (9), which is the first-order FRF, denoted as $\Lambda_1(j\omega, Y)$.

$$\Lambda_1(j\omega, Y) = \left(-\omega^2 m + j\omega c + k_{linear} + \frac{3}{4} k_{Nonlinear} Y^2\right)^{-1} \tag{12}$$

Eqn. (12), describes how a nonlinear system responds to monoharmonic excitation. It is similar to a linear FRF with the difference of amplitude dependency. Consider the equation of motion of linear system in frequency domain define as Eqn. (13).

$$H(s) = s^2 m + cs + k_{linear} \tag{13}$$

By adding control gains to nonlinear FRF, the first-order closed-loop receptance is obtained as Eqn. (14).

$$\overline{\Lambda_1}(s, Y) = \left(s^2 m + (c + g_v)s + (k_{linear} + g_d) + \frac{3}{4} k_{Nonlinear} Y^2\right)^{-1} \tag{14}$$

By utilizing the Sherman-Morrison formula, Eqn. (15) can be written to define the closed-loop receptance as a function of the open-loop receptance and the feedback gains [17] .

$$\overline{\Lambda_1}(s,Y) = \Lambda_1(s,Y) - \frac{\Lambda_1(s,Y)(g_d + s g_v)\Lambda_1(s,Y)}{1 + (g_d + s g_v)\Lambda_1(s,Y)} \quad (15)$$

The Sherman-Morrison formula can be used for extracting the closed-loop receptance through the use of the open-loop receptance and feedback gains. However, in order to perform active damping, the displacement gain are equal to zero.
Fig. 2 presents a closed-loop block diagram of the active damping method, in which, in the case of a linear system, the describing function of $N(s, Y)$ would be excluded.

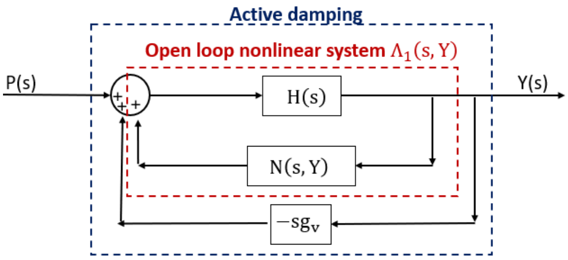

Figure 2. Closed loop block diagram of the active damping.

Active damping counteracts the effects of inherent damping in any system. In contrary, for pole placement, both the velocity and displacement are required, which can be computed based on the characteristic equation of the system as Eqn. (16).

$$\begin{pmatrix} g_v \\ g_d \end{pmatrix} = \begin{bmatrix} \mu_1 \Lambda_1(\mu_1, Y) & \Lambda_1(\mu_1, Y) \\ \mu_2 \Lambda_1(\mu_2, Y) & \Lambda_1(\mu_2, Y) \end{bmatrix}^{-1} \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad (16)$$

In Eqn. (16), μ_1 and μ_2 are the desired poles (eigen values) that we wish to assign. Fig. 3 is an illustration of a closed-loop feedback controller in order to perform pole placement or eigenvalue assignment of the nonlinear system.

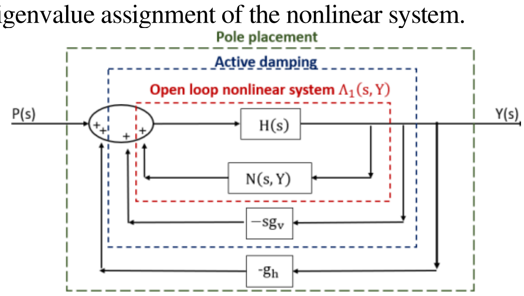

Figure 3. Closed loop block diagram of the pole placement.

To assign the desired poles of linear and nonlinear systems, the required control gains are obtained using a three-step iterative approach applied to the Sherman-Morrison receptance [11]. These three steps have been explained below.

- Finding first- order open- loop FRF $\Lambda_1(\mu_1, Y)$, in this step both theory or experimental measurements are acceptable.
- Estimating initial values of g_v and g_d by applying extracted $\Lambda_1(\mu_1, Y)$ from first step to Eqn. (16).
- Adjusting closed-loop displacement amplitude using Eqn. (15).

With having final values of control gains and adding these values to Eqn. (12), frequency response function of closed-loop system can be derive based on Eqn. (17).

$$\frac{9}{6}k_{Nonlinear}^2 Z^3 + \frac{3}{2}k_{Nonlinear}(k_{linear} + g_d - m\omega^2)Z^2 + \left[(k_{linear} + g_d - m\omega^2)^2 + \left(\omega(c + g_v)\right)^2\right]Z - P^2 = 0 \quad (17)$$

In which $Z = Y^2$. In order to ensure the stability of explained method, local stability analysis has been checked in the following section.

4. STABILITY ANALYSIS

There are some methods include the nonlinear Nyquist criterion, phase plane trajectory approach, circle and Popov criteria, and Lyapunov exponents for stability analysis in nonlinear systems [16], that the "Nonlinear Nyquist Criterion" method is used to analyze a nonlinear system in this paper. To determine the location of closed-loop poles, it is necessary to define the transfer function of the closed-loop system in terms of the open-loop system and controller. Combination of the open-loop plant matrix $H(s)$, the transfer function between the actuator input $u(t)$ and the accelerometer outputs $y(t)$, and the controller matrix $G(s)$ in a system with single degree of freedom, create the control loop. The closed-loop transfer function of $\Lambda_1(s, Y)$ in the system defined as Eqn. (18):

$$\Lambda_1(s,Y) = \left(1 + G(s)\Lambda_1(s,Y)\right)^{-1}\Lambda_1(s,Y) \quad (18)$$

Based on Eqn. (18) the closed-loop poles on any system can define by solving characteristic equation of the system as Eqn. (19):

$$det\left(1 + G(s)\Lambda_1(s,Y)\right) = 0 \quad (19)$$

The Nyquist contour of $G(s)\Lambda_1(s, Y)$ can be used to measure the closed-loop system's relative stability [15].
To check the existence of limit cycle, by combining describing function and Nyquist theory, Eqn. (20) can be used to determine the requirement for limit-cycle oscillations in the nonlinear system.

$$1 + N(j\omega.Y)H(j\omega) = 0 \; or \; H(j\omega) = \frac{-1}{N(j\omega.Y)} \quad (20)$$

The intersection of the Nyquist plot and the negative inverse of the describing function determines if there's a limit-cycle oscillation. In this case, limit-cycle is stable if it goes in a straight line from infinity to the origin and unstable if it goes from the origin to infinity.

5. RESULTS

As mentioned before in Section 2, the designed setup has the ability to adjust the strength of nonlinearity by varying the electrical current of coils and also the distance between two coils and magnets. To investigate the effect of the strength of magnetic field on the system's nonlinearity, the frequency response function (FRF) of the nonlinear system was plotted under different conditions.

Open-loop characterization:
The hammer test was conducted to experimentally measure the input force and output acceleration of the system. Data accuracy was ensured by repeating each test 20 times and calculating the final FRF using the averaging method to reduce experimental noise. Fig. 4 shows the frequency response function of the nonlinear system under various conditions of displacement between the system's coils and magnets and their electrical current.

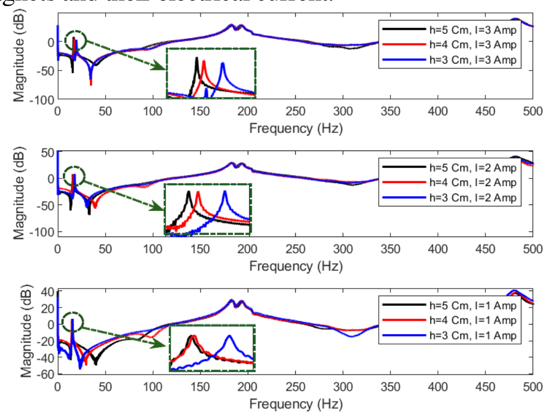

Figure 4. Experimental FRF of nonlinear system in various displacements and constant current.

As it is obvious in Fig. 4 the FRF is similar to FRF of linear systems does not show any hardening or softening effect. The reason is that with performing hammer test, the effect of hardening would be canceled because of the averaging from final Furrier transform results. In order to observe hardening effect and jumping phenomenon, shaker tests will be perfumed in future. However in this study, in order to show the influence of variation of displacement and electrical current of coils, the experimental result has been compared with numerical simulation as shown in Fig. 5.

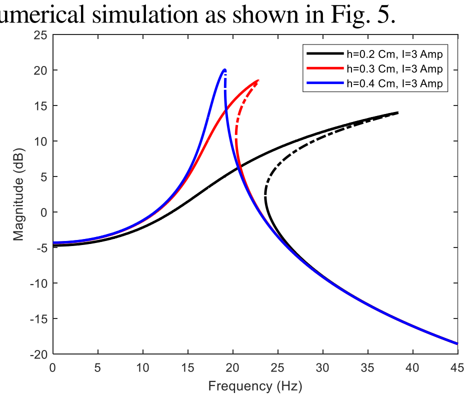

Figure 5. Illustration of numerical FRF of the open loop system for different values of displacement, in a constant current of $I_c = 3$ Amp

Based on Figs. 4 and 5, for fixed value of electrical current, with increasing the displacement between coils, system acts more similar to the linear system because of the small nonlinear stiffness.
Also the extracted open loop FRFs for different values of electrical current with constant displacement have been plotted in Fig. 6.

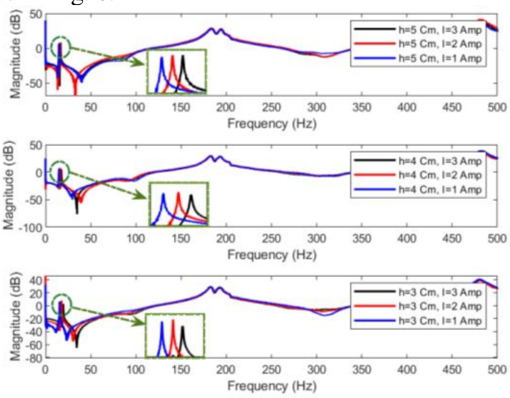

Figure 6. Experimental FRF of nonlinear system in various currents and constant displacement.

Also, in order to validate the experimental tests, the results have been compared with the numerical ones in Fig. 7. Both Figs. 6 and 7 illustrate that for a fixed displacement, by increasing the electrical current between coils, system shows intensive nonlinear behavior because of the large nonlinear stiffness.

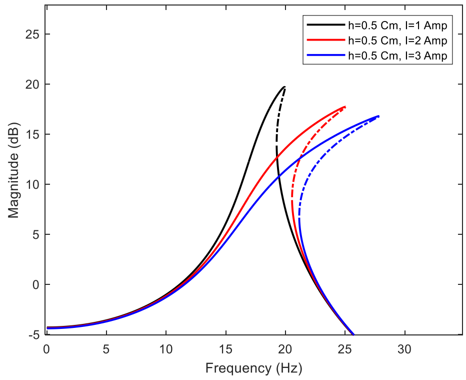

Figure 7. Numerical FRF of the nonlinear system for different values of electrical current, and constant displacement of $\mathrm{h} = 0.5$ Cm.

Fig. 8 shows a very good comparison on the location of the poles of the nonlinear designed system in two conditions of the weakest and strongest nonlinear stiffness.

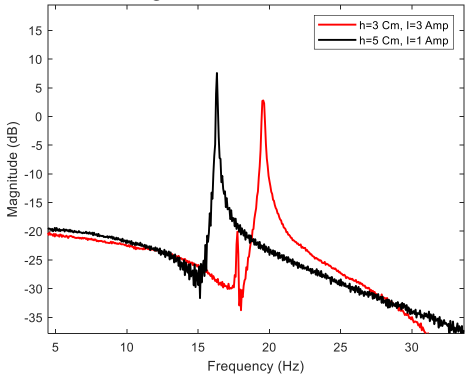

Figure 8. comparison the FRF of the nonlinear system in two condition of weakest and strongest nonlinear stiffness.

Active damping experiment:

To experimentally perform active damping strategy in a linear system, a Dayton load speaker has been connected to the beam. This allows for the implementation of a feedback control method. Fig. 9, describes the experimental result of performing active damping method on the linear setup with positive and negative gains.

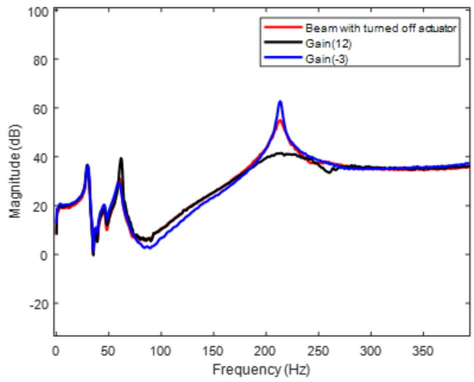

Figure 9. FRF of experimentally performing active damping method on the linear setup with positive and negative control gains.

The study aimed to reduce the amplitude of the system's second mode using a positive gain of 12, as shown in Fig. 9. The Sherman-Morrison receptance method was iteratively applied to perform pole placement on the system. Nonlinear system pole placement was demonstrated using Simulink software with a sine wave input. A new set of desired closed-loop complex-conjugate poles with values of $\mu_{1,2} = -0.85 \pm 35i$ are to be assigned. The required feedback gains are calculated using Eqn. (16) to be $g_d = 31.92$ and $g_v = 0.0728$. Fig. 10. Shows both control gains based on the number of iteration.

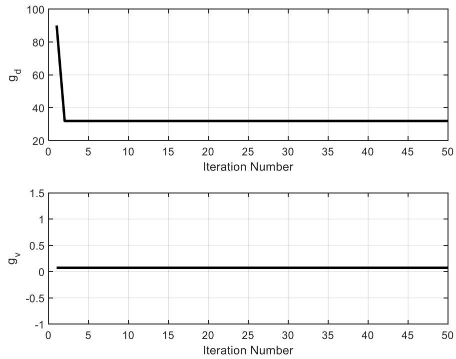

Figure 10. Displacement and velocity control gains based on the number of iteration.

The numerical result of the pole placement for this nonlinear system with considering sine wave as an input force has been presented in Fig. 11. This figure displays both the open-loop and closed-loop receptances which plotted using the iterative Sherman-Morrison receptance method.

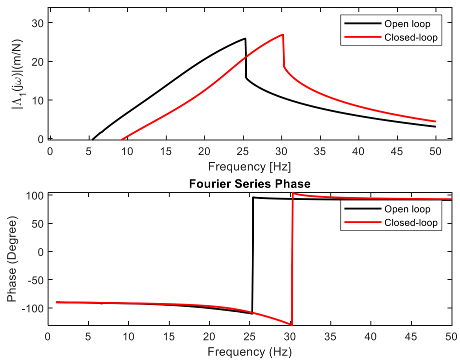

Figure 11. pole placement of the open-loop and closed-loop nonlinear system.

Since it is not possible to plot the hardening part of nonlinear systems in Simulink study, thus, in order to validate the simulation results, the equation extracted from the harmonic balance method (Eqn. (17)) was solved numerically, and the corresponding frequency response function was plotted in Fig. 12.

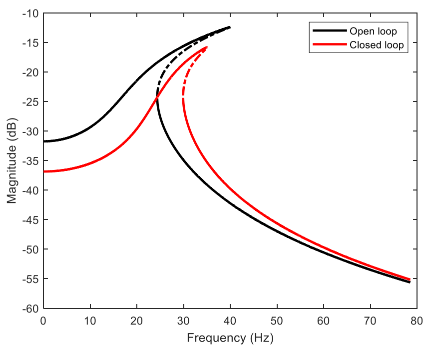

Figure 12. Numerical result of open loop FRF and closed loop pole placement of nonlinear system.

As is obvious from both Fig. 11. and 12, the poles of the system have been moved to the desired position correctly. Additionally, Fig. 11 shows the jumping phenomenon and hardening effect in the considered nonlinear system.

In order to check the existence of limit cycle and its stability, as mentioned in previous section, Fig. 13 shows Nyquist block diagram of the open loop system and negative inverse of describing function.

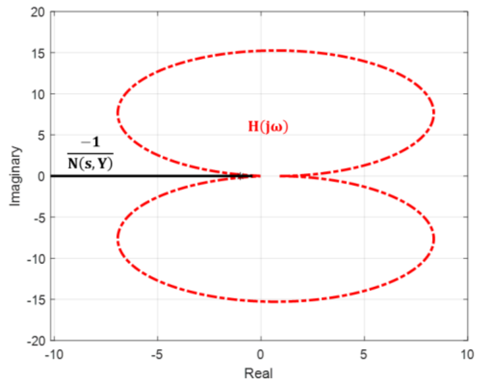

Figure 13. Indirect Nyquist block diagram of the nonlinear system in order to check the limit cycles.

Intersection between two plots shows the limit cycle exist in this nonlinear system. Since the limit cycle with variation of amplitude moves from infinity to the origin, it remains stable. Also Fig. 14. shows the real part of the eigenvalues of the closed-loop nonlinear system with variation of the amplitude of the system.

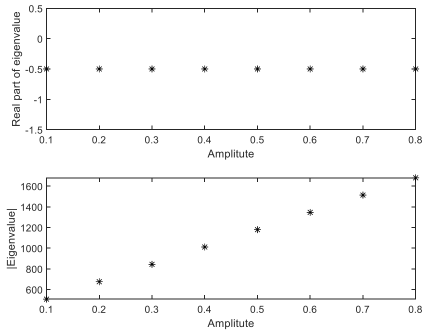

Figure 14. Illustration of eigenvalues of the closed loop nonlinear system based on amplitude

As it is obvious in Fig.14, the real part of the eigenvalues of this system in the variation of chosen amplitude are always negative which means that the closed-loop system is locally stable.

6. CONCOLUSION

This paper presents the application of the receptance method to control the poles of a nonlinear system experimentally using a nonlinear electromagnetic cantilever beam that designed and built for this purpose. The nonlinear stiffness in the electromagnetic system is performed using a pair of identical magnets and coils. The modeled nonlinear stiffness can vary by adjusting the input electrical current and the

distance between the two coils. The experimental and analytical results demonstrate that by increasing the distance and decreasing the electric current between the two coils, the system behaves similarly to a linear system because the nonlinear magnetic field weakens.
Active damping was performed experimentally to evaluate the performance of the designed control method and the used inertial actuator on the nonlinear setup. The poles of the nonlinear electromagnetic beam were analytically assigned for the experimental setup. To achieve this, experiments were conducted to analyze the system's behavior.
The nonlinear system's poles were assigned using the linear feedback control method and the iterative Sherman-Morrison formula at various levels of excitation. The jump phenomenon of the nonlinear FRF was shown, and pole placement was performed for only one mode of a single degree of freedom nonlinear system in this study. The stability investigation of the first five modes of a multi-variable system will be studied in future works.

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