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THE BICHROMATOR AS THE QUINTESSENCE OF A SAXOPHONE AND THE QUESTION OF HARMONICITY

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ABSTRACT

Reed wind instruments are divided into two families: those with a cylindrical bore (clarinet) and those with a conical bore (saxophone, oboe, bassoon). Apart from the timbre, the main features of the behavior of a clarinet can be obtained with a resonator modeled with a single mode (monochromator), while that of a saxophone can be obtained with a resonator modeled with two modes (bichromator), whose proper frequencies ratio is close to 2. This system has been largely studied by J. Gilbert et al. In particular, it has been demonstrated that when the inharmonicity is small, the oscillation on the fundamental frequency is obtained by an inverse bifurcation. Moreover, perfect harmonicity represents an optimum in terms of threshold pressure. Now a question remains: to what extent does the inharmonicity impact playability? This question is investigated in the present paper using both experiments and advanced numerical modeling.

Keywords: *Harmonicity - Saxophone - Bifurcation - Stability.*

1. INTRODUCTION

In the 1980s, Jean Kergomard, with Pierre Boulez and Ircam, worked on a quarter-tone wind instrument project. The principle was to branch on the instrument, near the mouthpiece, a small narrow tube whose effect was to shift the range by a quarter tone, thus allowing easy access to micro intervals without using complicated fingerings. This system worked rather well for the flute and the clarinet and had resulted in a patent. Unfortunately, the device did not experience the expected development. Nevertheless, Jean Kergomard had commissioned us (i.e. Joël Gilbert and Jean-Pierre Dalmont) to design a similar device for the saxophone and the bassoon. The problem turned out to be much more difficult than for cylindrical instruments because it quickly became clear that shifting the first impedance peak would not be enough because that would increase the inharmonicity to a point that makes the instrument unplayable. This produced, in certain situations, pseudo-periodic sounds combining the first two eigenfrequencies. This work allowed us to wonder about the functioning of reed instruments with conical bores. Examination of the impedance curves shows that conical instruments have resonance frequencies that are almost harmonic, following the series f_1, $2f_1$, $3f_1$ etc. with a first impedance peak of amplitude often lower than the second. On the other hand, the clarinet works on a series f_1, $3f_1$, etc. and the first peak has a larger amplitude than all the others. We have come to the conclusion that, just as many aspects of clarinet operation can be explained with a single-mode resonator model (the monochromator), many aspects of saxophone operation can be explained with a two-mode resonator model: the so called bichromator. Investigating the dynamics of this model ([1], then [2] and [3]) has made it possible to understand part of the specificities of the saxophone and more generally of reed instruments with conical bore.

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2. BICHROMATOR WITH PERFECTLY HARMONIC RESONANCES

Analysis of the impedance curves of reed instruments with conical resonators shows that the amplitude of the first impedance peak is higher than all the others for the notes at the top of the first register (beyond the fingering of A on a saxophone or an oboe) while it is lower than the second for the fingerings at the bottom of the first register. It was therefore relevant to focus on the importance of the relative amplitude of these two peaks on the oscillations.
In [1], appears a diagram of first importance (Fig. 1). This diagram is a plot of the following equation which gives the condition for a direct bifurcation [1]:

$$C < \frac{-2B^2}{3(Y_2 - Y_1)} \quad , \qquad (1)$$

where B and C are the second and third coefficient of the Taylor expansion of the non-linear characteristics $U(p)$ with U the volume velocity and p the pressure in the mouthpiece. $Y_1 = \frac{1}{Z_1}$ and $Y_2 = \frac{1}{Z_2}$ are the admittances of the two modes.

This shows that when $Z_1 >> Z_2$ and $C < 0$ (which is always the case for the saxophone), the Hopf bifurcation responsible for the emergence of oscillations is direct (top corner left on Fig. 1) as in the case of the clarinet. This means that when the pressure is gradually increased, the reed destabilizes for a given pressure called "direct pressure threshold" p_{th1} and begins to oscillate first with a low and then growing amplitude (see Fig. 2a). The diagram in Fig. 1 shows that when Z_1 is only slightly larger than Z_2, the bifurcation becomes inverse, i.e. when the pressure is gradually increased, the reed destabilizes at the "direct pressure threshold" p_{th1} but begins to oscillate with a large amplitude (blue arrow on Fig. 2b). It is then possible to reduce the pressure until the sound dies out at a pressure lower than the direct threshold pressure, known as the inverse pressure threshold or subcritical threshold p_{sc} (red arrow on Fig. 2b),
The analysis in [1] shows that if $Z_2 > Z_1$ the bifurcation is always direct because $C < 0$ (Eqn. (1)). This result is surprising to say the least: indeed, this situation corresponds to the notes of the bottom of the first register and there is no reason for the bifurcation to become direct in a sudden manner when $Z_2 = Z_1$. The question was investigated in [2], where it has been shown that the direct bifurcation actually corresponds to the branch of the inverted Helmholtz motion which is only observed at high level,

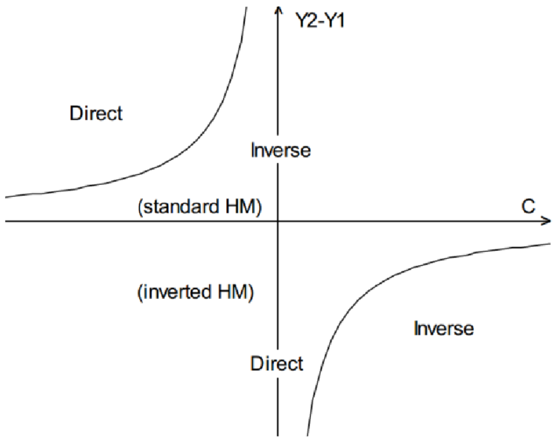

Figure 1. Diagram showing the regions where the bifurcation is direct as well as those regions where it is inverse (Eqn. (1)). Abscissa is for coefficient C, cubic term of the Taylor expansion of the non-linear characteristics $U(p)$. $Y_2 - Y_1$ is the admittance difference between the second and first modes (from [2]). HM stands for Helmholtz Motion.

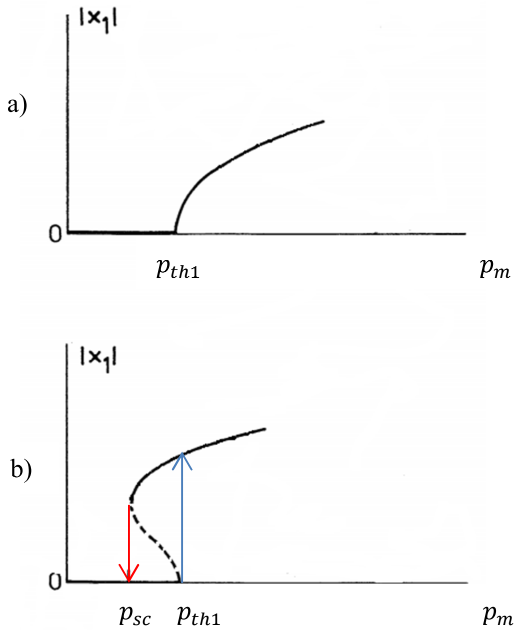

Figure 2. Diagram showing the amplitude $|x_1|$ of the first harmonic as a function of the mouth pressure for two situations: a) direct bifurcation b) inverse bifurcation (dotted line: unstable solution) (from [1]).

beyond the extinction threshold of the Helmholtz motion [4-6].

The branch of the Helmholtz motion is connected to the branch of the second regime (at the octave) whose direct threshold is p_{th2}. In this situation p_{th2} is lower than the direct threshold of the fundamental p_{th1} (Fig. 3): indeed the higher the impedance, the lower the threshold. The Helmholtz motion thus emerges from an inverse bifurcation with a brief passage over the octave. These results have an obvious practical consequence: it is difficult or even impossible to play pianissimo in the low register with a saxophone.

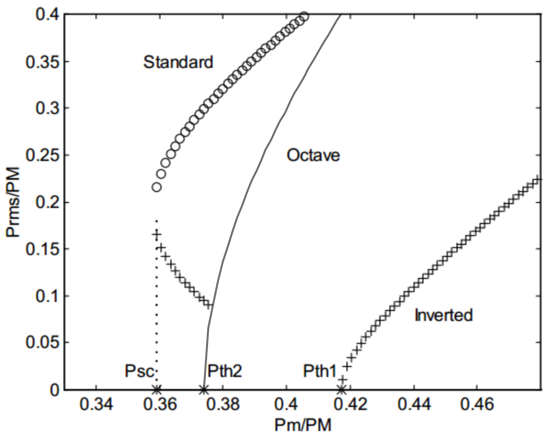

Figure 3. Bifurcation diagram for $Z_2 > Z_1$. RMS pressure with respect to the mouthpressure p_m. All quantities are normalized by the minimum closing pressure p_M. +++ unstable solutions (from [2]).

3. SLIGHTLY INHARMONIC BICHROMATOR

In practice, the harmonicity of the resonance frequencies is never perfectly satisfied, especially at the top of the first register, because the length of the truncation is no longer small compared to the wavelength. The truncated cone with equivalent volume is then no longer a good approximation of the stepped cone. The problem of the slightly inharmonic bichromator has been treated in [3] where it is also observed that the lowest oscillation thresholds are obtained for perfectly harmonic resonances thus confirming the prescription of Bouasse-Benade [7-8]. Moreover, it appears that the almost non-existent octave stability range in the harmonic case takes on a significant proportion. It is observed that there is a pressure range in which the two regimes are stable (Fig. 4). In practice, this means a higher risk of inadvertent emission of the octave. On the other hand, this phenomenon can be advantageously used to make it possible to play the note and its octave with the same fingering as is the case for the baroque oboe for example. Indeed, on this instrument, there is no register key and the octave is obtained by a subtle modification of the embouchure.

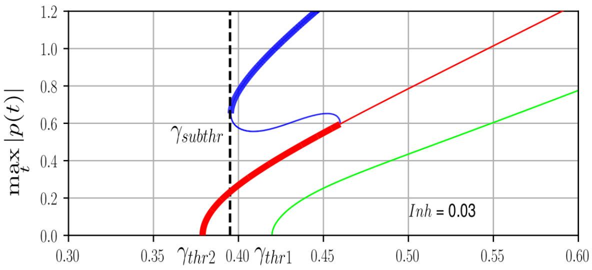

Figure 4. Bifurcation diagram for $\frac{Z_2}{Z_1} = \frac{3}{2}$ and $Inh = \frac{f_2}{2f_1} - 1 = 0.03$. Thick/thin lines: stable/unstable oscillations. Blue: fundamental Helmholtz motion. Red: octave. Green: inverted fundamental Helmholtz motion (from [3]).

4. QUANTITATIVE ANALYSIS OF THE INFLUENCE OF THE INHARMONICITY AND THE AMPLITUDE RATIO OF THE TWO MODES

Beyond the qualitative observations in [3], we would like to be able to easily deduce the possible playing frequency from the impedance curve. In the present work, with the methods used in [3], we aim at evaluating the playing frequency as a function of both the inharmonicity and the amplitude ratio of the two modes. The results will be compared with those obtained with the “sum function” of Wogram and Worman [9-10]. We can also compare with Grothe's proposal who weights the reduced frequency f_n/n of each mode by their amplitudes. We also investigate to what extent studying the dependence of stability ranges of the sound regimes on the harmonicity and relative amplitude of modes can help predict playing difficulties.

5. EXPERIMENTAL VALIDATION

In [12], a first experimental validation was attempted with an artificial mouth and a saxophone body extended with cylindrical tubes of different lengths. It turns out that the hypothesis of a linear resonator is not tenable with a saxophone because the non-linear losses at the side holes [13-14] are in practice never negligible. Indeed the comparison between theory and experiment on Fig. 5 emphasize the fact that the dynamics is more reduced in practice than in the theory. This work will be resumed with resonators made up of two cylinders, the last cylinder being extended by a network of large lateral holes. Whatever the results of these experiments, the existence of non-linear losses in the instruments with lateral holes must lead us to be cautious about the results of the numerical calculations and in particular for instruments with small lateral holes.

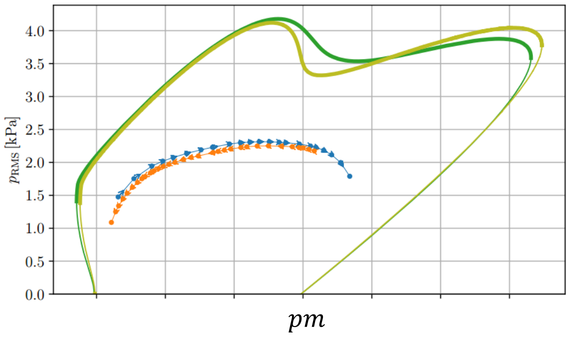

Figure 5. Experimental and theoretical bifurcation diagram for fingering *C* of an alto saxophone. Light/dark green: experiment with/without reed dynamics. Blue/orange marks: experiments with increasing/decreasing mouthpressure (from [12]).

6. CONCLUSION

The approach proposed by Joël Gilbert, consisting in approaching the saxophone by a two-mode model, has proven to be extremely fruitful for the understanding of its operation. The model has not been fully exploited yet, and with current digital tools can be used to unveil behavior laws to extrapolate playing frequencies and anticipate some emission difficulties.

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