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HIGHLY EFFICIENT COMPUTATION OF HRTFS BY KRYLOV SUBSPACE-BASED MODEL ORDER REDUCTION FOR VIRTUAL ACOUSTICAL RENDERING

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ABSTRACT

Head-related transfer functions (HRTFs) describe the individual spatial filtering of our ears, head and torso. The finite element method (FEM) can be used to compute HRTFs numerically. However, geometrical complexity, densely sampled frequencies, and fine spatial discretization, lead to considerable computational costs of these numerical models. In this contribution, we propose a highly efficient method to compute HRTFs numerically based on model order reduction (MOR) of corresponding large-scale FEM models. Krylov subspace-based MOR derives compact yet highly accurate surrogate models. While the compact model's generation is computationally demanding, its solution is significantly faster than that of the corresponding reference FEM model. A scanned head geometry is used to evaluate this method in a frequency range of $1\,\mathrm{Hz}$ to $10.000\,\mathrm{Hz}$ in 25 logarithmic steps. The 935.292-dimensional FEM model is reduced to a dimension of only 100 with a maximum magnitude error of $0.0257\,\mathrm{dB}$ for frequencies of $6813\,\mathrm{Hz}$ or below. The FEM model solves in $4\,\mathrm{h}$ while the compact model takes only $37\,\mathrm{ms}$, excluding the creation process of $1\,\mathrm{h}$.

Keywords: *head-related transfer function, finite element method, model order reduction, virtual reality*

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1. INTRODUCTION

Head-related transfer functions (HRTFs) model spatial reception of sound and are crucial for binaural auralization [1]. They are commonly determined by either measurements or computer simulations. Either way, they require densely-sampled directions and frequencies, resulting in large data sets. Therefore, these data sets are often recast into more efficient representations for later application by using for instance spherical harmonics [2] or singular value-based approaches [3, 4]. Obtaining data via measurements or numerical simulations is an elaborate process. The former requires measurements from numerous directions, the latter introduces high computational costs. These costs arise from the fine mesh required for sufficient accuracy, which leads to a high-dimensional system of equations. Moreover, this system must be solved for a large number of frequencies, multiplying the computational burden.

A prominent way to eliminate this bottleneck is the concept of model order reduction (MOR). Starting from a large-scale numerical model, this methodology constructs a highly accurate surrogate model of significantly smaller dimension [5]. As a result, all subsequent computations are accelerated by several orders of magnitude. The general idea is to identify patterns in the solution and to restrict the solution to these patterns. These patterns can be identified by e.g. modal analysis [6] or Krylov subspace-based methods [7, 8]. Furthermore, the numerical analysis might deploy the finite element method (FEM) [7] or the boundary element method (BEM) [9]. MOR for acoustic systems is well-established with applications ranging from car interiors [7] to ribcages and sharks [10]. However, little work has investigated the potential of MOR in the context of HRTF computations.

Therefore, this work proposes MOR as an efficient approach to numerically determine HRTFs. The procedure is demonstrated for a numerical case study of near-field HRTFs for a scanned head geometry [11]. This paper is structured as follows: Section 2 describes the case study and the workflow of its mathematical modeling, including initial partial differential equations (PDEs), its transformation to a system of ordinary differential equations (ODEs) by the FEM and finally, a reduced order model. Section 3 presents a corresponding near-field HRTF computed by the FEM and by its reduced version. The conclusion is provided in Section 4.

2. MATHEMATICAL MODELING

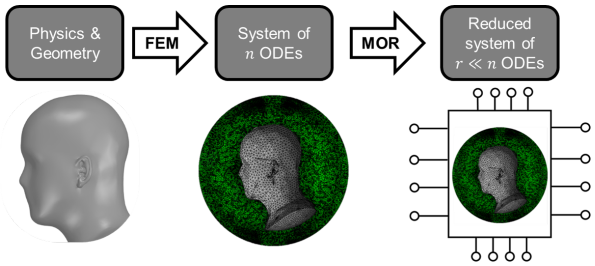

Figure 1. Workflow of mathematical modeling: The governing partial differential equation (Helmholtz equation) is spatially discretized by e.g. the FEM into a high-dimensional system of ODEs. In a subsequent step, MOR generates a highly accurate surrogate model of significantly smaller dimension.

Fig. 1 demonstrates the general workflow of mathematical modeling. The physicical setup is described by a linear Helmholtz equation [12] arising from the continuity relation and linearized Navier-Stokes equations:

$$\nabla^2 p + \frac{4}{3}\frac{\mu}{K}\nabla^2 \dot{p} - \frac{1}{c^2}\ddot{p} = -Q + \frac{4}{3}\frac{\mu}{\rho_0}\nabla^2 \dot{Q}, \qquad (1)$$

where p is the acoustic pressure, μ the dynamic viscosity, and K the fluid's bulk modulus. The sonic velocity is denoted by c, the density by ρ_0 and mass sources by Q. In general, this PDE cannot be solved analytically. Numerical methods such as the FEM or the BEM overcome this limitation by approximating the original PDE by a large-scale system of ODEs:

$$\Sigma = \begin{cases} \boldsymbol{M}\,\ddot{\boldsymbol{p}} + \boldsymbol{C}\,\dot{\boldsymbol{p}} + \boldsymbol{K}\,\boldsymbol{p} = \boldsymbol{B}\,\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{C}\,\boldsymbol{p} \end{cases}, \qquad (2)$$

where $\boldsymbol{p} \in \mathbb{R}^n$ is the state vector of nodal pressures, $\boldsymbol{M} \in \mathbb{R}^{n\times n}$ the mass matrix, $\boldsymbol{C} \in \mathbb{R}^{n\times n}$ the damping matrix and $\boldsymbol{K} \in \mathbb{R}^{n\times n}$ the stiffness matrix. $\boldsymbol{B} \in \mathbb{R}^{n\times p}$ contains fluidic loads scaled by the input $\boldsymbol{u} \in \mathbb{R}^p$. The output matrix $\boldsymbol{C} \in \mathbb{R}^{q\times n}$ assembles the user-defined outputs $\boldsymbol{y} \in \mathbb{R}^q$. Numerically computing HRTFs requires a harmonic analysis of this system, which reads

$$\left[-\omega^2\,\boldsymbol{M} + i\,\omega\,\boldsymbol{C} + \boldsymbol{K}\right]\hat{\boldsymbol{p}} = \boldsymbol{B}\,\hat{\boldsymbol{u}}, \qquad (3)$$

where $\hat{\boldsymbol{p}}$ and $\hat{\boldsymbol{u}}$ contain the corresponding complex amplitudes of nodal pressures and loads, respectively. This system of algebraic equations needs to be solved for each angular frequency ω of interest. Therefore, high-resolution HRTFs imply a significant computational effort.

A well-established methodology to drastically reduce this burden is MOR, which constructs highly efficient yet accurate surrogate models. The idea is to identify patterns in the solution space and to approximate the solution as some combination of these patterns. Mathematically, each pattern corresponds to a vector of nodal pressures. These patterns are collected into a matrix $\boldsymbol{V} \in \mathbb{R}^{n\times r}$ as its column vectors. Pressure distributions can now be approximated by combining these patterns according to:

$$\boldsymbol{p} = \boldsymbol{V}\,\boldsymbol{p}_r. \qquad (4)$$

Here, $\boldsymbol{V}$ contains the r identified patterns as its column vectors and $\boldsymbol{p}_r$ their corresponding weights. $\boldsymbol{V}$ is also referred to as the reduced basis and $\boldsymbol{p}_r$ as the reduced state vector. Introducing Eqn. (4) into the original system in Eqn. (2) and projecting it onto $\boldsymbol{V}$ results in a reduced system:

$$\Sigma = \begin{cases} \boldsymbol{M_r}\,\ddot{\boldsymbol{p}}_r + \boldsymbol{C_r}\,\dot{\boldsymbol{p}}_r + \boldsymbol{K_r}\,\boldsymbol{p}_r = \boldsymbol{B_r}\,\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{C_r}\,\boldsymbol{p}_r \end{cases}$$
$$\text{with: } \{\boldsymbol{M_r}/\boldsymbol{C_r}/\boldsymbol{K_r}\} = \boldsymbol{V}^T\,\{\boldsymbol{M}/\boldsymbol{C}/\boldsymbol{K}\}\,\boldsymbol{V} \qquad (5)$$
$$\boldsymbol{B_r} = \boldsymbol{V}^T\,\boldsymbol{B},\ \boldsymbol{C_r} = \boldsymbol{C}\,\boldsymbol{V}$$

All matrices and vectors except for input and output are reduced versions of their counterparts in Eqn. (2). A prominent methodology to compute the reduced basis are Krylov subspace-based methods, which are well-suited

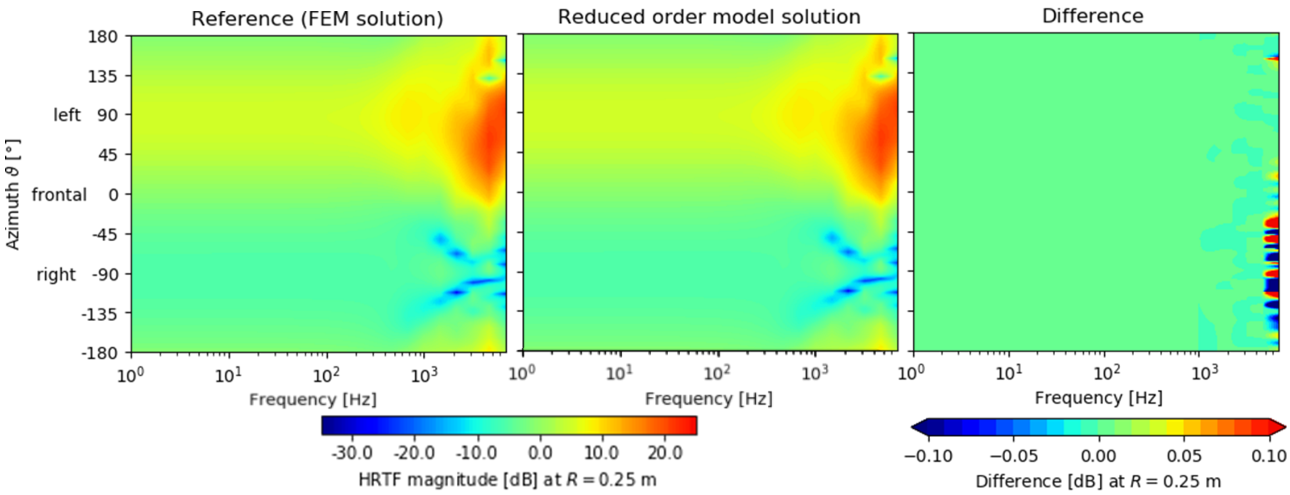

Figure 2. Right ear's near-field HRTF (radius $0.25\,\mathrm{m}$) computed by FEM-analysis (left) and its reduced order model (middle) with a maximum magnitude error of $0.0257\,\mathrm{dB}$ for frequencies between $1\,\mathrm{Hz}$ and $6813\,\mathrm{Hz}$. This error reaches $9.35\,\mathrm{dB}$ for the last sample at $10\,\mathrm{kHz}$ due to the expansion point of $s_0 = 0\,\mathrm{Hz}$. In addition, the difference in sound pressure level is displayed (right). Note the modified scale from $-0.1\,\mathrm{dB}$ to $0.1\,\mathrm{dB}$.

for large-scale models and have excellent approximation quality in the frequency domain [5]. They ensure that the Taylor-expanded transfer function of the original model and its reduced counterpart match in the first coefficients. Please note that the Taylor expansion is a local approximation and done around a chosen expansion point s_0 in the frequency domain. However, a reduced basis may contain column vectors from several frequencies to ensure reliable results in the frequency range of interest. A variant particularly for systems as in Eqn. (2) is the second-order Arnoldi method (SOAR) [13], which is also used here.

3. NUMERICAL CASE STUDY

A scanned head geometry [11] as indicated in Fig. 1 serves as a numerical case study. The head is enclosed by a sphere of radius $0.25\,\mathrm{m}$ with absorbing boundary conditions, which restricts the derived HRTFs to the near field. The pinnae are meshed with an elementsize of $1\,\mathrm{mm}$, while the global elementsize of $5\,\mathrm{mm}$ in combination with quadratic elements ensures reliable results for frequencies up to $11.44\,\mathrm{kHz}$. According to the reciprocity principle, a wall velocity of $1\,\frac{\mathrm{mm}}{\mathrm{s}}$ is applied to the blocked ear. The FEM model translates into a second-order system as in Eqn. (2) of dimension $n = 935.292$. This system is subsequently reduced by SOAR to a dimension of $r = 100$ around an expansion point $s_0 = 0\,\mathrm{Hz}$. Both the original FEM model and its reduced counterpart are evaluated for 25 logarithmically-spaced frequencies from $1\,\mathrm{Hz}$ to $10.000\,\mathrm{Hz}$. Fig. 2 presents the right ear's HRTF computed by the original FEM model and its reduced version as well as the latter's deviation. Due to the Taylor expansion at $s_0 = 0\,\mathrm{Hz}$, the approximation error is negligible for low frequencies, but rises towards $10\,\mathrm{kHz}$. This error can be tuned by choosing a different expansion point or also multiple expansion points. The reduced model's dimension impacts the error as well. Tab. 1 provides the computational times of both approaches. Please note that the efficiency gained by MOR increases with the number of evaluated frequencies.

Table 1. Comparison of the computational times for analyzing comprising 25 frequencies (Intel® Xeon® CPU E5-2687W v4 3.0 GHz and 64 GB RAM).

Model	Dimension	CPU time
Original	$935,292$	4:11:21 h
Reduced	100	1:10:21 h (Write, MOR) $37\,\mathrm{ms}$ (Simulation)

4. CONCLUSIONS AND OUTLOOK

This work proposes Krylov subspace-based MOR to efficiently compute HRTFs from scanned human geometries. This concept is successfully demonstrated for a FEM model of a scanned head, although BEM models are equally feasible. The computational time is reduced by 72 % while hardly compromising on accuracy. This speed up increases drastically with the number of frequencies to be evaluated. In addition, accuracy can be adjusted by increasing the reduced model's dimension and by optimizing the choice of expansion point(s).

To achieve a reasonable model dimension, the case study is limited in terms of maximum frequency and enclosure dimension. Furthermore, projection-based MOR relies on system matrices, which might be difficult to obtain in case of commercial software. MOR by Krylov subspaces lacks an a priori error bound. Therefore, accuracy is computed by a comparison to the original model's solution; however, evaluating few frequencies suffices.

Future work will investigate the potential of parametric MOR, which is yet to be unleashed. This methodology preserves parameters in symbolic form with the reduced order model, enabling efficient parametric studies. This translates into highly efficient HRTF-customization to different individuals, without the need for adjusting the original model and reducing it again.

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