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ONE PEDAL NOTE CAN HIDE ANOTHER

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ABSTRACT

The so-called pedal note, frequently used in many orchestral scores for instance on trombone parts, is the first regime (lowest frequency) played by bass brass instruments. Because their frequency is far from any acoustic resonance frequency of the instrument (a few semitones away), pedal notes have been intriguing scientists for decades. Various hypotheses have been proposed throughout history to explain this characteristic. Almost a century ago, it was commented on by [1], and has been later explained as the consequence of nonlinear coupling of higher acoustic modes whose frequencies are close to harmonics of the playing frequency [2]. We show here that using advanced numerical methods for nonlinear dynamical systems allows us to finally demonstrate that standard models of sound production in brass instruments explain the emergence of pedal notes. This analysis also explains why pedal notes are played preferably at fortissimo level rather than pianissimo. Finally, we show how most recent studies have revealed, for certain brass instrument geometries like the tuba family, the existence of so-called ghost notes lying between the pedal note and the second register.

Keywords: *pedal note, ghost note, brass musical instrument, bifurcation diagram*

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1. INTRODUCTION

"Take a tenor trombone, substitute its usual mouthpiece with a tenor saxophone mouthpiece, the adaptation isn't perfect but it's good enough (...) And surprise, the lowest note obtained, in the first position, will be the D, i.e. the D0 pedal [1] below the usual B♭ pedal, so beware : one pedal can hide another."

The paragraph above is translated from the very first paper of Joël Gilbert in a journal for trombone players [3]. At that time, he has just completed his internship after his master degree in acoustics, and is not even a PhD student. This highlights that trying to shed light on the question of the pedal note produced by brass instruments was one of his scientific goals from the outset. A goal that will remain with him for the rest of his career. In this article, we outline the milestones in his work on this subject.

2. A MYSTERIOUS REGIME

The experiment described in the introduction above is mentioned as early as the beginning of the 20^{th} century by [1], who mentions "strange peculiarities in the low register" [2] for brass instruments. However Joël Gilbert proposed with Aumond a refined version of this experiment [4]: the input impedance of each instrument is measured, and the sounding frequencies are quantitatively compared to the resonance frequencies of the impedance peaks. Moreover, the experiment is reproduced numerically by carrying out time domain simulations of ele-

[1] The pedal note is the lowest note that can be played on a brass instrument.

[2] Original version in french: "les instruments à embouchure de cor présentent d'étranges particularités dans le grave".

mentary physical models of the sound production. This is done either when the trombone is played with a brass mouthpiece (vibrating lips) or with a clarinet mouthpiece (vibrating reed). Even if both experimental and numerical results are conclusive, there is still a long way to go before an explanation for the observed phenomena is proposed. However, the authors already conclude their paper by an intuition - decisive for the next 15 years - that a modal formulation of the models could be the key to a better understanding of the phenomena: "This method could be used to do a numerical morphing from a resonator having a set of harmonic resonances (saxhorn-like resonator) to a set of incomplete harmonic peaks (trombone-like resonator) by slightly moving down the first resonance frequency out of the harmonic series."

3. THE BEGINNING OF AN EXPLANATION

Significant progress was made when the possible solutions of such a modal model were analyzed with the tools of nonlinear dynamical systems. Indeed, a few years later, [5] demonstrated that in the case of the trombone, the emergence of a note well above the frequency of the first impedance peak (the pedal note), could be explained by a linear stability analysis (LSA) of the equilibrium solution [3] : "LSA clearly indicates that for low enough acoustic resonance frequencies in the bore input impedance, the frequency of the emerging oscillation is far beyond the resonance frequency of the instrument (...) This result from LSA is quite unexpected: the pedal note of the trombone seems to result from a coupling between the lips and the nearest acoustic mode below the playing frequency, just like for the other oscillation regimes". Thus, higher bore resonances are not necessary to explain the emergence of the pedal note, which is an unexpected result given the speculations made in previous studies [2]. However, this approach raises a new mystery when applied to instruments of the saxhorn family, since it suggests the production of a new regime called the "ghost note" (verified experimentally since then), but not the pedal note.

4. A DEEPER UNDERSTANDING OF PHENOMENA THANKS TO THE BIFURCATION ANALYSIS

The most recent answers to date concerning pedal notes and ghost notes regimes have been proposed in [6, 7].

[3] i.e. when the player blows into the instrument without any resulting self-sustained oscillation (no sound produced)

Bifurcation diagrams of a modal model of brass instrument including the action of the player are computed. To do so, periodic solutions are represented using a collocation approach, and solution branches are computed using a prediction-correction continuation approach [8].

It has been found that in the case of the trombone, the pedal note corresponds to a branch of periodic solutions emerging from the equilibrium branch through an inverse Hopf bifurcation (see figure 1). The solutions on the branch close to the equilibrium are unstable, and stable solutions exist only beyond a finite amplitude. This corresponds to the musician's feeling that the pedal note is played more preferably in the *forte* nuance than in the *piano* nuance. Moreover, the oscillation frequencies of the accessible stable regimes differ by less than a semitone. As such, it is considered that only one note is accessible to the musician.

The picture is more complex in the case of the saxhorn family (see figure 2 in the case of the B♭ contrabass tuba). Indeed, the branch of periodic solutions emerging from the equilibrium (grey line) is not the pedal note anymore. This is shown in the lower subplot, where the oscillation frequency is higher than the expected frequency of the note B♭0 (29.1 Hz in the tempered scale). This branch corresponds to the ghost note. The pedal note, however, corresponds to the stable part of the lime green line. It can be noted that this branch is not connected to the "principal branch" (namely the branch arising from the Hopf point H1 which marks the loss of stability of the equilibrium). This explains why it was not possible for previous authors using only linear stability analysis to predict a pedal note for this family of instruments, but only the ghost note (for instance [5]).

As a summary, the bifurcation diagrams are in accordance on many points with the musicians' experience:

- For instruments with a predominantly cylindrical bore profile like the trombone, the pedal note is the only playable note around the expected frequency. [6]
- The ghost note can be played exclusively on bass brass instruments with a predominantly-expanding bore profile such as tubas, euphoniums or saxhorns. In that case, it stands between the pedal note – the lowest natural note playable, or first regime – and the instrument's second regime. More precisely, for the different instruments of the saxhorn family, the ratio between the frequency of the recorded pedal note and ghost note has been found in accor-

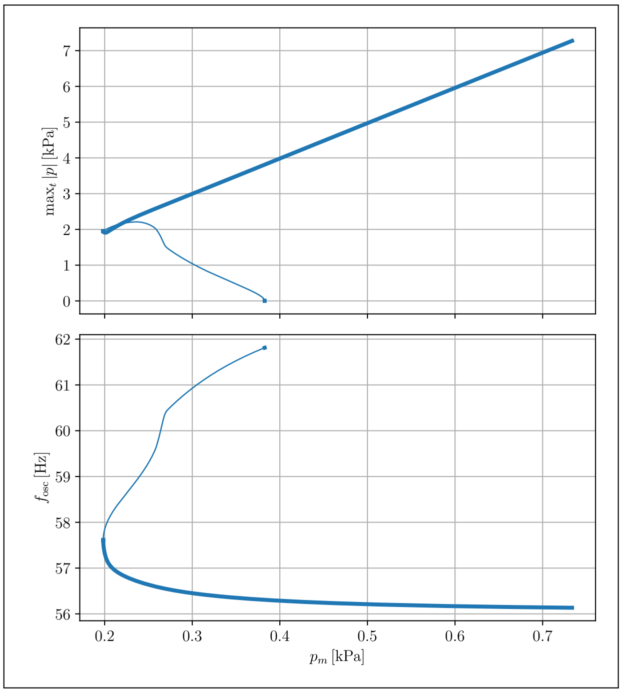

Figure 1. Typical partial bifurcation diagram for the physical model of a bass trombone around the first Hopf bifurcation. Top and bottom plots represent respectively the maximum amplitude of the periodic solutions branches and their oscillation frequency vs. the blowing pressure in the case of a trombone. The line thickness indicates whether the branch portion is stable (thick line) or unstable (thin line). The expected note is B♭1 whose frequency is 58.27 Hz in the tempered scale. Figure from [6].

dance with the value predicted by the bifurcation diagram [7].

This agreement between the picture displayed by bifurcation diagrams and the experiments is all the more interesting as the physical model used is very simple, with a single degree-of-freedom model for lips.

5. CONCLUSION

Bifurcation analysis provides a global view of the dynamics of a musical instrument. It appears to be a suitable tool for understanding the production of the pedal note by brass instruments.

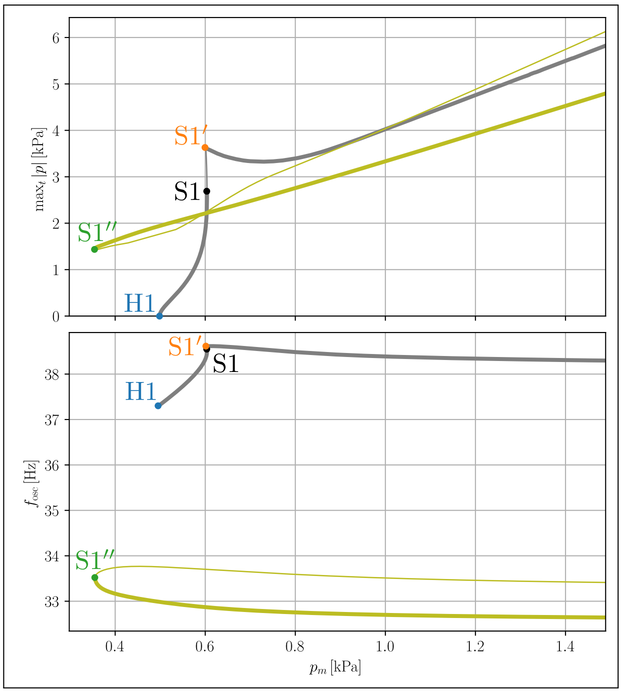

Figure 2. Typical partial bifurcation diagram of a B♭ contrabass tuba around the first Hopf bifurcation. Top and bottom plots represent respectively the maximum amplitude of the periodic oscillation and their oscillation frequency with respect to the blowing pressure. The line thickness indicates whether the branch portion is stable (thick line) or unstable (thin line). The point H1 corresponds to the Hopf bifurcation point at which a stable oscillation regime arises from the equilibrium. This corresponds to the ghost note. S1 corresponds to the saddle-node bifurcation point at which the stable regime arising from the Hopf point destabilises. S1′ corresponds to the saddle-node bifurcation point beyond which periodic solutions become stable again. S1″ corresponds to the saddle-node bifurcation point at which the stable regime of the pedal note arises from the isolated branch (lime green). Figure from [7].

This tool also makes it possible to revisit earlier work and understand why approaches based on time integration or linear stability analysis, although they provided some interesting insights, could only lift one corner of the veil on these ”strange peculiarities” mentionned a century ago by Bouasse [1].

6. IN MEMORIAM

In memory of Joël Gilbert (1963-2022) and his career-long efforts to advance this research topic.

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